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Principal n-Ideals which Form Generalized Stone Nearlattices

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Abstract

In this paper, we give several characterizations of those $P_n(S)$ which are generalized Stone nearlattices in terms of n-ideals. We show that when n is a central element of a nearlattice S and $P_n(S)$ is a sectionally pseudocomplemented distributive nearlattice, then $P_n(S)$ is generalized Stone if and only if for any $x \in S$, $<x>_n^+ \vee <x>_n^{++} = S$. Moreover, when $P_n(S)$ is sectionally pseudocomplemented distributive nearlattice, then we prove that $P_n(S)$ is generalized Stone if and only if each prime n-ideal contains a unique minimal prime n-ideal.

Keywords: Principal *n*-ideal; Minimal prime *n*-ideal; Central element; Generalized Stone nearlattice.

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1. Introduction

Generalized Stone lattices have been studied by many authors including [1], [2], [3], [4] and [5]. On the other hand, minimal prime ideals and generalized Stone nearlattices have been studied by [6]. In this paper, we generalize several important results on generalized Stone nearlattices in terms of *n*-ideals.

A nearlattice S is a meet semilattice with the property that any two elements possessing a common upper bound, have a supremum. Nearlattice S is distributive if for all $x, y, z \in S$, $x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$ provided $y \vee z$ exists. An element n of a nearlattice S is called *medial* if $m(x, n, y) = (x \wedge y) \vee (x \wedge n) \vee (y \wedge n)$ exists in S for all $x, y \in S$. A nearlattice S is called a *medial nearlattice* if $m(x, y, z)$ exists for all $x, y, z \in S$. An element s of a nearlattice S is called *standard* if for all $t, x, y \in S$,

$$t \wedge [(x \wedge y) \vee (x \wedge s)] = (t \wedge x \wedge y) \vee (t \wedge x \wedge s).$$

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The element s is called neutral if

(i) s is standard and

(ii) for all $x, y, z \in S$, $s \wedge [(x \wedge y) \vee (x \wedge z)] = (s \wedge x \wedge y) \vee (s \wedge x \wedge z)$.

In a distributive nearlattice every element is neutral and hence standard. An element n in a nearlattice S is called *sesquimedial* if for all $x, y, z \in S$, $\{[(x \wedge n) \vee (y \wedge n)] \wedge [(y \wedge n) \vee (z \wedge n)]\} \vee (x \wedge y) \vee (y \wedge z)$ exists in S. An element n of a nearlattice S is called an *upper element* if $x \vee n$ exists for all $x \in S$. Every upper element is of course a sesquimedial element. An element n is called a *central element* of S if it is neutral, upper and complemented in each interval containing it.

For a fixed element n of a nearlattice S, a convex subnearlattice of S containing n is called an n-ideal of S. For a medial element n of a nearlattice S, an n-ideal P of S is called *prime* if $P \neq S$ and $m(x,n,y) \in P (x, y \in S)$ implies either $x \in P$ or $y \in P$.

A prime n-ideal P is said to be a *minimal prime n-ideal* belonging to n-ideal I if (i) $I \subseteq P$ and (ii) There exists no prime n- ideal Q such that $Q \neq P$ and $I \subseteq Q \subseteq P$. A prime n-ideal P of a nearlattice S is called a *minimal prime n-ideal* if there exists no prime n-ideal Q such that $Q \neq P$ and $Q \subseteq P$.

Let L be a lattice with 0 and $a \in L$. Then a^* of L is called a pseudocomplement of a if $a \wedge a^* = 0$ and if $a \wedge x = 0$ for any $x \in L$ then $x \leq a^*$. A lattice L is called pseudocomplemented if every element of L has a pseudocomplement.

A nearlattice S with *0* is called *sectionally pseudocomplemented* if the interval $[0, x]$ for each $x \in S$, is pseudocomplemented. Of course, every finite distributive nearlattice is sectionally pseudocomplemented. A nearlattice S is called *relatively pseudocomplemented* if the interval $[a, b]$ for each $a, b \in S$, $a < b$ is pseudocomplemented.

A distributive nearlattice S with *0* is called a *generalized Stone nearlattice* if $(x]^* \vee (x]^{**} = S$ for each $x \in S$. A distributive nearlattice S with *0* is a generalized Stone nearlattice if and only if each interval $[0, x]$, $0 < x \in S$ is a Stone lattice.

For any n-ideal J of a nearlattice S,

$$J^+ = \{x \in S : m(x, n, j) = n \quad \textit{for all} \quad j \in J\}.$$

An n-ideal generated by a single element a is called *principal n-ideal*, denoted by $<a>_n$. The set of principal n-ideal is denoted by $P_n(S)$. When S is a distributive nearlattice then for any $a \in S$ we define

$$\begin{aligned} <a>_n &= \{y \in S : a \wedge n \leq y = (y \wedge a) \vee (y \wedge n)\} \\ &= \{y \in S : y = (y \wedge a) \vee (y \wedge n) \vee (a \wedge n)\} \end{aligned}$$

When n is an upper element, then $<a>_n$ is the closed interval $[a \wedge n, a \vee n]$.

We know that for a distributive nearlattice S with an upper element n, $P_n(S)$ is a distributive nearlattice with the smallest element $\{n\}$. Let $<a>_n \in P_n(S)$. By the interval $[\{n\}, <a>_n]$ in $P_n(S)$, we mean the set of all principal n-ideals contained in $<a>_n$. $P_n(S)$ is called sectionally pseudocomplemented if for each $<a>_n \in P_n(S)$, the interval $[\{n\}, <a>_n]$ in $P_n(S)$ is pseudocomplemented . That is , each principal n-ideal contained in $<a>_n$ has a relative pseudocomplement in $[\{n\},<a>_n]$ which is also a member of $P_n(S)$. We shall denote the relative pseudocomplement of $<b>_n$ in any interval by $<b>_n^0$, while $<b>_n^+$ denotes the pseudocomplement of $<b>_n$ in $I_n(S)$.

If $P_n(S)$ is a distributive sectionally pseudocomplemented nearlattice, then $P_n(S)$ is a generalized Stone nearlattice if for each $<a>_n \in P_n(S)$, the interval $[\{n\},<a>_n]$ in $P_n(S)$ is a Stone lattice.

For $b \le a \le n$, if $[b,n]$ is dual pseudocomplemented then a^{0d} denotes the relative dual pseudocomplement of *a* in $[b,n]$. If $[n,d]$ is pseudocomplemented then for $c \in [n,d]$, c^0 denotes the relative pseudocomplement of *c* in $[n,d]$. Two prime n-ideals *P* and *Q* of a nearlattice *S* are called comaximal if $P \vee Q = S$.

In this paper, we have given several characterizations of those $P_n(S)$ which are generalized Stone nearlattices in terms of n-ideals. we have also discussed on *O(P)* and *n(P)* and given some properties of *n(P)*. Moreover, when $P_n(S)$ is sectionally pseudocomplemented distributive nearlattice, then we have proved that $P_n(S)$ is generalized Stone if and only if each prime n-ideal contains a unique minimal prime n-ideal.

Following result is due to [7] which will be needed for the development of this paper.

Theorem 1.1. *For an element n of a nearlattice S, the following conditions are equivalent :*

(i) *n is central in S*

(ii) n *is upper and the map* $\Phi : P_n(S) \rightarrow (n]^d \times [n)$ *defined by* $\Phi(<a>_n) = (a \wedge n, a \vee n)$ *is an isomorphism, where* $(n]^d$ *represents the dual of the lattice* $(n]$. □

When n is a central element of S (then of course, n is upper, and so sesquimedial), then by Theorem 1.1, $P_n(S) \cong (n]^d \times [n)$. Thus we have the following result.

Theorem 1.2. *Let S be a nearlattice and $n \in S$ be a central element. Then $P_n(S)$ is sectionally pseudocomplemented if and only if $(n]$ is sectionally dual pseudocomplemented and $[n)$ is sectionally pseudocomplemented.* □

Corollary 1.3. *Let n be a central element and $P_n(S)$ be a* sectionally pseudocomplemented distributive nearlattice. Then for $\{n\} \subseteq <a>_n \subseteq <b>_n$, $<a>_n^0 = [a \wedge n, a \vee n]^0 = [(a \wedge n)^{0d}, (a \vee n)^0]$.

Proof. Since $P_n(S)$ is sectionally pseudocomplemented, so by Theorem 1.2, $(n]$ is sectionally dual pseudocomplemented and $[n)$ is sectionally pseudocomplemented. Here $b \wedge n \le a \wedge n \le n \le a \vee n \le b \vee n$.

Since $(a \wedge n)^{0d}$ is the relative dual pseudocomplement of $a \wedge n$ in $[b \wedge n, n]$ and $(a \vee n)^0$ is the relative pseudocomplement of $a \vee n$ in $[n, b \vee n]$, so,

$$[a \wedge n, a \vee n] \cap [(a \wedge n)^{0d}, (a \vee n)^0] = [(a \wedge n) \vee (a \wedge n)^{0d}, (a \vee n) \wedge (a \vee n)^0]$$
$$= [n, n] = \{n\}.$$

Now let $t \in < a >_n^0$. Then $[t \wedge n, t \vee n] \subseteq < a >_n^0$.

Thus, $\{n\} = [t \wedge n, t \vee n] \cap [a \wedge n, a \vee n]$
$= [(t \wedge n) \vee (a \wedge n), (t \vee n) \wedge (a \vee n)]$

and so $(t \wedge n) \vee (a \wedge n) = n = (t \vee n) \wedge (a \vee n)$.

This implies $(t \wedge n) \geq (a \wedge n)^{0d}$ and $(t \vee n) \leq (a \vee n)^0$.

Hence, $[t \wedge n, t \vee n] \subseteq [(a \wedge n)^{0d}, (a \vee n)^0]$ and so $< a >_n^0 \subseteq [(a \wedge n)^{0d}, (a \vee n)^0]$.

Therefore $< a >_n^0 = [(a \wedge n)^{0d}, (a \vee n)^0]$. □

If S is a distributive lattice with 0 and 1, then for a central element $n \in S$, $P_n(S) = F_n(S)$. Then $P_n(S)$ is pseudocomplemented if and only if $(n]$ is dual pseudocomplemented and $[n)$ is pseudocomplemented, as $F_n(S) \cong (n]^d \times [n)$. For any $n \leq b \leq 1$, b^+ denotes the pseudocomplement of b in $[n, 1]$, while for $0 \leq a \leq n$, a^{+d} denotes the dual pseudocomplement of a in $[0, n]$.

Corollary 1.4. *Let n be a central element of a lattice S with* 0, 1 *and $P_n(S)$ is a pseudocomplemented distributive lattice. Then for any $a \in S$,*

$$< a >_n^+ = [(a \wedge n)^{+d}, (a \vee n)^+]. \quad □$$

A distributive nearlattice S with 0 is *generalized Stone nearlattice* if for each $x \in S$, $(x]^* \vee (x]^{**} = S$. By [6], a distributive nearlattice S with 0 is a generalized Stone nearlattice if and only if each interval $[0, x]$, $0 < x \in S$ is a Stone lattice.

To prove Theorem 1.7 we need the following lemmas. Lemma 1.5 is trivial by Theorem 1.2

Lemma 1.5. *Suppose n is a central element of a distributive nearlattice S, and $P_n(S)$ is sectionally pseudocomplemented. Then $P_n(S)$ is generalized Stone if and only if $(n]$ is dual generalized Stone and $[n)$ is generalized Stone.* □

Lemma 1.6. *Suppose $P_n(S)$ is a sectionally pseudocomplemented distributive nearlattice. Let $x, y \in S$ with $< x >_n \cap < y >_n = \{n\}$. Then the following conditions are equivalent :*

(i) $< x >_n^+ \vee < y >_n^+ = S$;

(ii) *For any $t \in S$,* $< m(x,n,t) >_n^0 \vee < m(y,n,t) >_n^0 = < t >_n$

where $< m(x,n,t) >_n^0$ denotes the relative pseudocomplement of $< m(x,n,t) >_n$ in $[\{n\}, < t >_n]$.

Proof. (i)$\Rightarrow$(ii). Suppose (i) holds. Then for any $t \in S$,

$$< m(x,n,t) >_n^0 \vee < m(y,n,t) >_n^0$$
$$= (< x >_n \cap < t >_n)^0 \vee (< y >_n \cap < t >_n)^0$$
$$= ((< x >_n \cap < t >_n)^+ \cap < t >_n) \vee ((< y >_n \cap < t >_n)^+ \cap < t >_n) \text{ by [8, Lemma 1.4]}$$
$$= (< x >_n^+ \cap < t >_n) \vee (< y >_n^+ \cap < t >_n) \quad \text{by [8, Lemma 1.3]}$$
$$= (< x >_n^+ \vee < y >_n^+) \cap < t >_n$$
$$= S \cap < t >_n$$
$$= < t >_n .$$

Hence (ii) holds.

(ii)$\Rightarrow$(i). Suppose (ii) holds and $t \in S$.

By (ii), $< m(x,n,t) >_n^0 \vee < m(y,n,t) >_n^0 = < t >_n$.

Then using [8, Lemmas 1.3 and 1.4] and the calculation of (i)$\Rightarrow$(ii) above we get $(< x >_n^+ \vee < y >_n^+) \cap < t >_n = < t >_n$.

This implies $< t >_n \subseteq < x >_n^+ \vee < y >_n^+$ and so $t \in < x >_n^+ \vee < y >_n^+$.

Therefore, $< x >_n^+ \vee < y >_n^+ = S$. □

Theorem 1.7. *Let n be a central element of S, and $P_n(S)$ be a sectionally pseudocomplemented distributive nearlattice. Then the following conditions are equivalent :*

(i) $P_n(S)$ *is generalized Stone* ;

(ii) *For any* $x \in S$, $< x >_n^+ \vee < x >_n^{++} = S$;

(iii) *For all* $x, y \in S$, $(< x >_n \cap < y >_n)^+ = < x >_n^+ \vee < y >_n^+$;

(iv) *For all* $x, y \in S$, $< x >_n \cap < y >_n = \{n\}$ *implies that* $< x >_n^+ \vee < y >_n^+ = S$.

Proof. (i) $\Rightarrow$ (ii). Suppose (i) holds and, the $t \in S$ n for any $x \in S$, $m(x,n,t) \in < t >_n$ and so $< m(t,n,x) >_n \in [\{n\}, < t >_n]$.

Since $P_n(S)$ is generalized Stone , so $< m(t,n,x) >_n^0 \vee < m(t,n,x) >_n^{00} = < t >_n$.

Then by [8, Lemma 1.4],

$$< t >_n = (< m(t,n,x) >_n^+ \cap < t >_n) \vee (< m(t,n,x) >_n^{++} \cap < t >_n)$$
$$= ((< x >_n \cap < t >_n)^+ \cap < t >_n) \vee ((< x >_n \cap < t >_n)^{++} \cap < t >_n)$$

Thus by [8, Lemma 1.3],

$$< t >_n = (< x >_n^+ \cap < t >_n) \vee (< x >_n^{++} \cap < t >_n) .$$

Thus $< t >_n = (< x >_n^+ \vee < x >_n^{++}) \cap < t >_n$.

This implies $< t >_n \subseteq < x >_n^+ \vee < x >_n^{++}$ and so $t \in < x >_n^+ \vee < x >_n^{++}$.

Therefore $< x >_n^+ \vee < x >_n^{++} = S$.

(ii)$\Rightarrow$(iii). Suppose (ii) holds.

For any $x, y \in S$

$$(<x>_n \cap <y>_n) \cap (<x>_n^+ \vee <y>_n^+)$$
$$= (<x>_n \cap <y>_n \cap <x>_n^+) \vee (<x>_n \cap <y>_n \cap <y>_n^+)$$
$$= \{n\} \vee \{n\} = \{n\}$$

Now let $<x>_n \cap <y>_n \cap I = \{n\}$ for some n-ideal I.

Then $<y>_n \cap I \subseteq <x>_n^+$. Meeting $<x>_n^{++}$ with both sides,

we have $<y>_n \cap I \cap <x>_n^{++} = \{n\}$.

This implies $I \cap <x>_n^{++} \subseteq <y>_n^+$.

Hence $I = I \cap S$

$$= I \cap (<x>_n^+ \vee <x>_n^{++})$$
$$= (I \cap <x>_n^+) \vee (I \cap <x>_n^{++})$$
$$\subseteq <x>_n^+ \vee <y>_n^+$$

Therefore, $<x>_n^+ \vee <y>_n^+ = (<x>_n \cap <y>_n)^+$.

(iii) $\Rightarrow$ (iv). Let $<x>_n \cap <y>_n = \{n\}$ for some $x, y \in S$.

Then by (iii),

$$S = \{n\}^+ = (<x>_n \cap <y>_n)^+$$
$$= <x>_n^+ \vee <y>_n^+.$$

Thus (iv) holds.

To complete the proof we shall show that (iv) $\Rightarrow$ (i).

Suppose (iv) holds. Since $P_n(S)$ is sectionally pseudocomplemented, so by Theorem 1.2, $(n]$ is sectionally dual pseudocomplemented and $[n)$ is sectionally pseudocomplemented. Suppose $n \le b \le d$. Let b^0 be the relative pseudocomplement of b in $[n, d]$.

Now $b^0 \wedge b^{00} = n$.

Thus $<b^0>_n \cap <b^{00}>_n = [n, b^0 \wedge b^{00}] = [n, n] = \{n\}$.

Also $<b^0>_n, <b^{00}>_n \subseteq <d>_n$. Then by equivalent conditions of (iv) given in Lemma 1.6, we have $<m(b^0, n, d)>_n^0 \vee <m(b^{00}, n, d)>_n^0 = <d>_n$.

But $m(b^0, n, d) = b^0$ and $m(b^{00}, n, d) = b^{00}$ as $n \le b^0$, $b^{00} \le d$.

Since by Corollary 1.4, $<b^0>_n^0 = <b^{00}>_n$ and $<b^{00}>_n^0 = <b^{000}>_n = <b^0>_n$.

Therefore, $<d>_n = <b^{00}>_n \vee <b^0>_n$

$$= <b^0 \vee b^{00}>_n$$

which gives $b^0 \vee b^{00} = d$. This implies $[n, d]$ is a Stone lattice.

That is, [n) is generalized Stone.

A dual proof of above shows that (iv) also implies that $(n]$ is a dual generalized Stone lattice. Therefore, by Lemma 1.5, $P_n(S)$ is generalized Stone. □

Following corollary is an immediate consequence of above result.

Corollary 1.8. *Let* n *be a central element of a distributive lattice* L *with* 0 *and* 1 *and* *let $P_n(L)$ be a pseudocomplemented distributive lattice. Then the following conditions are equivalent :*

(i) $P_n(L)$ *is Stone* ;

(ii) *For all* $x \in L$, $<x>_n^+ \vee <x>_n^{++} = L$;

(iii) *For all* $x, y \in L$, $(<x>_n \cap <y>_n)^+ = <x>_n^+ \vee <y>_n^+$;

(iv) *For all* $x, y \in L$, $<x>_n \cap <y>_n = \{n\}$ *implies that* $<x>_n^+ \vee <y>_n^+ = L$. □

For a prime ideal P of a distributive nearlattice S with 0, we define

$$0(P) = \{x \in S : x \wedge y = 0 \quad \textit{for some} \quad y \in S - P\}$$

Clearly $0(P)$ is an ideal and $0(P) \subseteq P$. Note that $0(P)$ is the intersection of all the minimal prime ideals of S which are contained in P.
For a prime n-ideal P of a distributive nearlattice S, we write

$$n(P) = \{y \in S : m(y,n,x) = n \quad \textit{for some} \quad x \in S - P\}.$$

Clearly, $n(P)$ is an n-ideal and $n(P) \subseteq P$.

Lemma 1.9. *Let S be a distributive nearlattice with a medial element n and P be a prime* n- *ideal in S. Then each minimal prime* n- *ideal belonging to n(P) is contained in P.*

Proof. Let Q be a minimal prime n-ideal belonging to $n(P)$. If $Q \not\subseteq P$, then choose $y \in Q - P$. Since Q is a prime n-ideal, so by [9,Theorem 1.5], we know that Q is either an ideal or a filter. Without loss of generality suppose Q is an ideal. Now let

$$T = \{t \in S : m(y,n,t) \in n(P)\}.$$

We shall show that $T \not\subseteq Q$. If not, let $D = (S - Q) \vee [y)$.

Then $n(P) \cap D = \Phi$.

For otherwise, $y \wedge r \in n(P)$ for some $r \in S - Q$. Then by convexity, $y \wedge r \leq m(y,n,r) \leq (y \wedge r) \vee n$ implies $m(y,n,r) \in n(P)$. Hence $r \in T \subseteq Q$, which is a contradiction.

Thus by [9,Theorem 1.9], there exists a prime n-ideal R containing $n(P)$ disjoint to D. Then $R \subseteq Q$.

Moreover, $R \neq Q$ as $y \notin R$, this shows that Q is not a minimal prime n-ideal belonging to $n(P)$, which is a contradiction.

Therefore $T \not\subseteq Q$. Hence there exists $z \notin Q$ such that $m(y,n,z) \in n(P)$. Thus $m(m(y,n,z),n,x) = n$ for some $x \in S - P$. It is easy to see that $m(m(y,n,z),n,x) = m(m(y,n,x),n,z)$.

Hence $m(m(y,n,x),n,z) = n$. Since P is prime and $y, x \notin P$ so $m(y,n,x) \notin P$.

Therefore, $z \in n(P) \subseteq Q$, which is a contradiction.

Hence $Q \subseteq P$. □

Proposition 1.10. *For a medial element* n *if P is a prime* n- *ideal in a distributive nearlattice S, then n(P) is the intersection of all minimal prime* n- *ideals contained in P*.

Proof. Clearly *n(P)* is contained in any prime n-ideal which is contained in *P*. Hence *n(P)* is contained in the intersection of all minimal prime n-ideals contained in *P*.

Since S is distributive, so by [7, Corollary 2.1.10], *n(P)* is the intersection of all minimal prime n-ideals belonging to it.

By [8, Lemma 1.2], as each prime n-ideal contains a minimal prime n-ideal, above remarks and Lemma 1.9 establish the proposition. □

Following result has been proved by [5] for lattices. We generalize that result for nearlattices with the help of [10,Theorem 1.7].

Theorem 1.11. *Let* $P_n(S)$ *be a sectionally pseudocomplemented distributive nearlattice and n be central element in* S. *Then the following conditions are equivalent :*

(i) *For any* $x \in S$, $<x>_n^+ \vee <x>_n^{++} = S$,
equivalently, $P_n(S)$ *is generalized Stone* ;

(ii) *For any two minimal prime* n- *ideals P and Q,* $P \vee Q = S$;

(iii) *Every prime* n- *ideal contains a unique minimal prime* n- *ideal* ;

(iv) *For each prime* n- *ideal P, n(P) is a prime* n- *ideal.*

Proof. (i)$\Rightarrow$(ii). Suppose (i) holds.

Let $x \in P - Q$. Then $<x>_n \subseteq P - Q$. Now, $<x>_n \cap <x>_n^+ = \{n\} \subseteq Q$.

So $<x>_n^+ \subseteq Q$ as *Q* is prime.

Again, $x \in P$ implies $<x>_n^{++} \subseteq P$ by [8, Theorem 1.6].

Hence by (i), $S = <x>_n^+ \vee <x>_n^{++} \subseteq Q \vee P$. Therefore, $P \vee Q = S$.

(ii)$\Leftrightarrow$(iii) is trivial.

(iii)$\Rightarrow$(iv) is direct consequence of Proposition 1.10.

(iv)$\Rightarrow$(i). Suppose (iv) holds.

First we shall show that for all $x, y \in S$ with $<x>_n \cap <y>_n = \{n\}$ implies $<x>_n^+ \vee <y>_n^+ = S$. If it does not hold, then there exist $x, y \in S$ with $<x>_n \cap <y>_n = \{n\}$ such that $<x>_n^+ \vee <y>_n^+ \neq S$.

As *S* is distributive, so by [9, Theorem 1.9], there is a prime n-ideal *P* such that $<x>_n^+ \vee <y>_n^+ \subseteq P$. Then $<x>_n^+ \subseteq P$ and $<y>_n^+ \subseteq P$ imply $x \notin n(P)$ and $y \notin n(P)$.

By (iv), *n(P)* is prime n-ideal and so $m(x, n, y) = n \in n(P)$ is contradictory.

Thus for all $x, y \in S$ with $<x>_n \cap <y>_n = \{n\}$

implies that $<x>_n^+ \vee <y>_n^+ = S$.

Hence by equivalent conditions of Theorem 1.7, (i) holds. □

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