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Commutativity in Prime Γ–Near-Rings with Permuting Tri-derivations

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Abstract

The object of this paper is to introduce a permuting tri-derivation in a Γ-near-ring. We obtain the conditions for a prime Γ-near-ring to be a commutative Γ-ring.

Keywords: Γ-near-ring; Prime Γ-near-ring; Commutative Γ-ring; Permuting tri-derivation.

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1. Introduction

The derivations in near-rings have been introduced by Bell and Mason [1]. They investigated some basic properties of derivations in near-rings. Then Asci [2] obtained some commutativity conditions for a Γ-near-ring with derivations. Some characterizations of Γ-near-rings and some regularity conditions were obtained by Cho [3]. Kazaz and Alkan [4] introduced the notion of two-sided Γ-α-derivation of a Γ-near-ring and investigated the commutativity of prime and semiprime Γ-near-rings. Uckun *et al.* [5] worked on prime Γ-near-rings with derivations and they investigated the conditions for a Γ-near-ring to be commutative.

In this paper, the notion of a permuting tri-derivation in a Γ-near-ring is introduced. We investigate the conditions for a prime Γ-near-ring to be a commutative Γ-ring.

2. Preliminaries

A Γ-near-ring is a triple $(R, +, \Gamma)$ where

(i) $(R, +)$ is a group (not necessarily abelian),

(ii) Γ is a non-empty set of binary operations on R such that for each $\alpha\in\Gamma$, $(R, +, \alpha)$ is a left near-ring.

(iii) $x\alpha(y\beta z) = (x\alpha y)\beta z$, for all $x, y, z\in R$ *and* $\alpha, \beta\in\Gamma$.

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Exactly speaking, it is a *left* Γ*-near-ring* because it satisfies the left distributive law. We will use the word Γ*-near-ring* to mean *left* Γ*-near-ring*. For a Γ-near-ring R, the set $R_0 = \{x\in R : 0\alpha x = 0, \alpha\in\Gamma\}$ is called the *zero-symmetric part* of R. A Γ-near-ring R is said to be *zero-symmetric* if $R = R_0$. Throughout this note, R will be a zero-symmetric Γ-near-ring and R is called *prime* if $x\Gamma R\Gamma y = \{0\}$ implies $x = 0$ or $y = 0$. Recall that R is called *n-torsion-free*, where n is a positive integer, if $nx = 0$ implies $x = 0$ for all $x\in R$. The symbol $C(R)$ will represent the multiplicative center of R, that is, $C(x) = \{x\in R : x\alpha y = y\alpha x$ for all $y\in R, \alpha\in\Gamma\}$. For $x\in R$, the symbol $C(x)$ will denote the centralizer of x in R. As usual, for $x, y\in R$, $\alpha\in\Gamma$, $[x, y]_\alpha$ will denote the commutator $x\alpha y - y\alpha x$, while (x, y) will indicate the additive-group commutator $x + y - x - y$. An additive map $d : R \rightarrow R$ is called a *derivation* if the Leibniz rule $d(x\alpha y) = d(x)\alpha y + x\alpha d(y)$ holds for all $x, y\in R$, $\alpha\in\Gamma$. By a *bi-derivation* we mean a bi-additive map $D : R\times R \rightarrow R$ (i.e., D is additive in both arguments) which satisfies the relations $D(x\alpha y, z) = D(x, z)\alpha y + x\alpha D(y, z)$ and $D(x, y\alpha z) = D(x, y)\alpha z + y\alpha D(x, z)$ for all $x, y, z\in R$, $\alpha\in\Gamma$. Let D be symmetric, that is, $D(x, y) = D(y, x)$ for all $x, y\in R$. The map $d : R \rightarrow R$ defined by $d(x) = D(x, x)$ for all $x\in R$ is called the *trace* of D. A map $F : R \times R \times R \rightarrow R$ is said to be *permuting* if the equation $F(x_1, x_2, x_3) = F(x_{\pi(1)}, x_{\pi(2)}, x_{\pi(3)})$ holds for all $x_1, x_2, x_3\in R$ and for every permutation $\{\pi(1), \pi(2), \pi(3)\}$.

3. Permuting Tri-derivations and Commutativity

A map $f : R \rightarrow R$ defined by $f(x) = F(x, x, x)$ for all $x\in R$, where $F : R \times R \times R \rightarrow R$ is a permuting map, is called the *trace* of F. It is obvious that, in the case $F : R \times R \times R \rightarrow R$ is a permuting map which is also tri-additive (i.e., additive in each argument), the trace f of F satisfies the relation $f(x + y) = f(x) + 2F(x, x, y) + F(x, y, y) + F(x, x, y) + 2F(x, y, y) + f(y)$ for all $x, y\in R$. Since we have $F(0, y, z) = F(0 + 0, y, z) = F(0, y, z) + F(0, y, z)$ for all $y, z\in R$, we obtain $F(0, y, z) = 0$ for all $y, z\in R$. Hence we get $0 = F(0, y, z) = F(x-x, y, z) = F(x, y, z) + F(-x, y, z)$ and so we see that $F(-x, y, z) = -F(x, y, z)$ for all $x, y, z\in R$. This tells us that f is an odd function.

A tri-additive map $D : R \times R \times R \rightarrow R$ will be called a *tri-derivation* if the relations $D(x_1\alpha x_2, y, z) = D(x_1, y, z)\alpha x_2 + x_1\alpha D(x_2, y, z)$, $D(x, y_1\alpha y_2, z) = D(x, y_1, z)\alpha y_2 + y_1\alpha D(x, y_2, z)$ and $D(x, y, z_1\alpha z_2) = D(x, y, z_1)\alpha z_2 + z_1\alpha D(x, y, z_2)$ are fulfilled for all $x, y, z, x_i, y_i, z_i\in R$, $i = 1, 2$, $\alpha\in\Gamma$.

We need the following lemmas to obtain our main results.

Lemma 3.1 [6, Lemma 2.3] *Let R be a prime* Γ*- near-ring. If* $C(R) - \{0\}$ *contains an element z for which $z + z\in C(R)$, then* $(R, +)$ *is abelian.*

Lemma 3.2 [7, Lemma 2.2] *Let R be a* 3!*-torsion free* Γ*-near-ring. Suppose that there exists a permuting tri-additive map $F : R \times R \times R \rightarrow R$ such that* $f(x) = 0$ *for all $x\in R$, where f is the trace of F. Then we have $F = 0$.*

Lemma 3.3. *Let R be a* 3!*-torsion free prime* Γ*-near-ring and let $x\in R$. Suppose that there exists a nonzero permuting tri-derivation $D : R \times R \times R \rightarrow R$ such that* $x\alpha d(y) = 0$ *for all $y\in R$, $\alpha\in\Gamma$, where d is the trace of D. Then we have $x = 0$.*

Proof. Since we have $d(y + z) = d(y) + 2D(y, y, z) + D(y, z, z) + D(y, y, z) + 2D(y, z, z) + d(z)$ for all $y, z\in R$, $\alpha\in\Gamma$, the hypothesis gives

$$2x\alpha D(y, y, z) + x\alpha D(y, z, z) + x\alpha D(y, y, z) + 2x\alpha D(y, z, z) = 0 \text{ for all } y, z\in R, \alpha\in\Gamma. \quad (1)$$

Setting $y = -y$ in (1), it follows that

$$2x\alpha D(y, y, z) - x\alpha D(y, z, z) + x\alpha D(y, y, z) - 2x\alpha D(y, z, z) = 0 \text{ for all } y, z\in R, \alpha\in\Gamma. \quad (2)$$

On the other hand, for any $y, z\in R$, $d(z + y) = d(z) + 2D(z, z, y) + D(z, y, y) + D(z, z, y) + 2D(z, y, y) + d(y)$ and so, by the hypothesis, we have

$$2x\alpha D(y, z, z) + x\alpha D(y, y, z) + x\alpha D(y, z, z) + 2x\alpha D(y, y, z) = 0 \text{ for all } x, y, z\in R, \alpha\in\Gamma, \quad (3)$$

Since D is permuting. Comparing (1) with (2), we get $2x\alpha D(y, z, z) + x\alpha D(y, y, z) + x\alpha D(y, z, z) = x\alpha D(y, y, z) - 3x\alpha D(y, z, z)$ which means that $2x\alpha D(y, z, z) + x\alpha D(y, y, z) + x\alpha D(y, z, z) + 2x\alpha D(y, y, z) = x\alpha D(y, y, z) - 3x\alpha D(y, z, z) + 2x\alpha D(y, y, z)$ for all $x, y, z\in R$, $\alpha\in\Gamma$.

Now, from (3), we obtain

$$x\alpha D(y, y, z) - 3x\alpha D(y, z, z) + 2x\alpha D(y, y, z) = 0 \text{ for all } x, y, z\in R, \alpha\in\Gamma. \quad (4)$$

Taking $y = -y$ in (4) leads to

$$x\alpha D(y, y, z) + 3x\alpha D(y, z, z) + 2x\alpha D(y, y, z) = 0 \text{ for all } x, y, z\in R, \alpha\in\Gamma. \quad (5)$$

Combining (4) and (5), we obtain

$$x\alpha D(y, z, z) = 0 \text{ for all } x, y\in R, \alpha\in\Gamma, \quad (6)$$

since R is 6-torsion free.

Replacing $z = z + w$ to linearize (6) and using the conditions show that

$$x\alpha D(w, y, z) = 0 \text{ for all } w, x, y, z\in R, \alpha\in\Gamma. \quad (7)$$

Substituting $w\beta v$ for w in (7), we get $x\alpha w\beta D(v, y, z) = 0$ for all $v, w, x, y, z\in R$, $\alpha, \beta\in\Gamma$. Since R is prime and $D \neq 0$, we arrive at $x = 0$. This completes the proof of the theorem.

Lemma 3.4. *Let R be a Γ-near-ring and let $D : R \times R \times R\to R$ be a permuting tri-derivation. Then we have $[D(x, z, w)\alpha y + x\alpha D(y, z, w)]\beta v = D(x, z, w)\alpha y\beta v + x\alpha D(y, z, w)\beta v$ for all $v, w, x, y, z\in R$, $\alpha, \beta\in\Gamma$.*

Proof. Since we have $D(x\alpha y, z, w) = D(x, z, w)\alpha y + x\alpha D(y, z, w)$ for all $w, x, y, z\in R$, $\alpha\in\Gamma$, the associative law gives

$$D((x\alpha y)\beta v, z, w) = D(x\alpha y, z, w)\beta v + x\alpha y\beta D(v, z, w)$$
$$= [D(x, z, w)\alpha y + x\alpha D(y, z, w)]\beta v + x\alpha y\beta D(v, z, w) \text{ for all } v, w, x, y, z\in R, \alpha, \beta\in\Gamma \quad (8)$$

and

$$D(x\alpha(y\beta v), z, w) = D(x, z, w)\alpha y\beta v + x\alpha D(y\beta v, z, w)$$
$$= D(x, z, w)\alpha y\beta v + x\alpha[D(y, z, w)\beta v + y\beta D(v, z, w)]$$
$$= D(x, z, w)\alpha y\beta v + x\alpha D(y, z,w)\beta v + x\alpha y\beta D(v, z, w) \text{ for all } v, w, x, y, z\in R, \alpha, \beta\in\Gamma \quad (9)$$

Comparing (8) and (9), we see that $[D(x, z, w)\alpha y + x\alpha D(y, z, w)]\beta v = D(x, z, w)\alpha y\beta v + x\alpha D(y, z, w)\beta v$ for all $v, w, x, y, z \in R$, $\alpha, \beta \in \Gamma$.

The proof of the lemma is complete.

Now we are ready to prove our main results in this section.

Theorem 3.5. *Let R be a* 3!-*torsion free prime* Γ- *near-ring. Suppose that there exists a nonzero permuting tri-derivation* $D : R \times R \times R \to R$ *such that* $D(x, y, z) \in C(R)$ *for all* $x, y, z \in R$. *Then R is a commutative* Γ-*ring.*

Proof. Assume that $D(x, y, z) \in C(R)$ for all $x, y, z \in R$. Since D is nonzero, there exist $x_0, y_0, z_0 \in R$ such that $D(x_0, y_0, z_0) \in C(R) - \{0\}$ and $D(x_0, y_0, z_0) + D(x_0, y_0, z_0) = D(x_0, y_0, z_0 + z_0) \in C(R)$.

So $(R, +)$ is abelian by Lemma 3.1.

Since the hypothesis implies that

$$w\beta D(x, y, z) = D(x, y, z)\beta w \text{ for all } w, x, y, z \in R,\ \beta \in \Gamma, \tag{10}$$

we replace x by $x\alpha v$ in (10) to get $w\beta[D(x, y, z)\alpha v + x\alpha D(v, y, z)] = [D(x, y, z)\alpha v + x\alpha D(v, y, z)]\beta w$ and thus, from Lemma 3.4 and the hypothesis, it follows that $D(x, y, z)\beta w\alpha v + D(v, y, z)\alpha w\beta x = D(x, y, z)\alpha v\beta w + D(v, y, z)\beta x\alpha w$ which means that

$$D(x, y, z)\beta[w, v]_\alpha = D(v, y, z)\beta[x, w]_\alpha \text{ for all } v, w, x, y, z \in R,\ \alpha, \beta \in \Gamma. \tag{11}$$

Setting $\mathrm{d}(u)$ in place of v in (11) and using $\mathrm{d}(x) \in C(R)$ for all $x \in R$, by the hypothesis, we obtain

$$D(\mathrm{d}(u), y, z)\beta[x, w]_\alpha = 0 \text{ for all } u, w, x, y, z \in R,\ \alpha, \beta \in \Gamma. \tag{12}$$

The substitution $v\alpha x$ for x in (12) yields that $D(\mathrm{d}(u), y, z)\beta v\alpha[x, w]_\alpha = 0$ for all $u, v, w, x, y, z \in R$, $\alpha, \beta \in \Gamma$. Since R is prime, we obtain either $D(\mathrm{d}(u), y, z) = 0$ or $[x, w]_\alpha = 0$ for all $u, w, x, y, z \in R$, $\alpha \in \Gamma$.

Assume that

$$D(\mathrm{d}(u), y, z) = 0 \text{ for all } u, y, z \in R. \tag{13}$$

Let us take $u + x$ instead of u in (13). Then we obtain

$$\begin{aligned} 0 = D(\mathrm{d}(u + x), y, z) &= D(\mathrm{d}(u) + \mathrm{d}(x) + 3D(u, u, x) + 3D(u, x, x), y, z) \\ &= 3D(D(u, u, x), y, z) + 3D(D(u, x, x), y, z), \end{aligned}$$

that is,

$$D(D(u, u, x), y, z) + D(D(u, x, x), y, z) = 0 \text{ for all } v, w, x, y \in R. \tag{14}$$

Setting $u = -u$ in (14) and then comparing the result with (14), we see that

$$D(D(u, u, x), y, z) = 0 \text{ for all } u, x, y, z \in R. \tag{15}$$

Substituting $u\lambda x$ for x in (15) and employing (13) give the relation $d(u)\lambda D(x, y, z) + D(u, y, z)\lambda D(u, u, x) = 0$ and so it follows from the hypothesis that

$$d(u)\lambda D(x, y, z) + D(u, u, x)\lambda D(u, y, z) = 0 \text{ for all } u, x, y, z \in R,\ \lambda \in \Gamma. \tag{16}$$

We put $u = y = x$ in (16) to obtain,

$d(x)\lambda D(x, x, w) = 0$ for all $w, x \in R$, $\lambda \in \Gamma$. (17)

Taking $w\lambda x$ in substitute for w in (17) yields $d(x)\lambda w\lambda d(x) = 0$, for all $\lambda \in \Gamma$, and so the primeness of R implies that $d(x) = 0$ for all $x \in R$. Hence, by Lemma 3.2, we have $D = 0$ which is a contradiction. So R is a commutative Γ-ring. This proves the theorem.

Theorem 3.6. Let R be a 3!-torsion free prime Γ-near-ring. Suppose that there exists a nonzero permuting tri-derivation $D : R \times R \times R \to R$ such that $d(x)$, $d(x) + d(x) \in C(D(u, v, w))$ for all $u, v, w, x \in R$, where d is the trace of D. Then R is a commutative Γ-ring.

Proof. Assume that

$d(x)$, $d(x) + d(x) \in C(D(u, v, w))$ for all $u, v, w, x \in R$. (18)

From (18), we get

$D(u + t, v, w)\alpha(d(x) + d(x))$
$= (d(x) + d(x))\alpha D(u + t, v, w)$
$= (d(x) + d(x))\alpha[D(u, v, w) + D(t, v, w)]$
$= (d(x) + d(x))\alpha D(u, v, w) + (d(x) + d(x))\alpha D(t, v, w)$
$= d(x)\alpha D(u, v, w) + d(x)\alpha D(u, v, w) + d(x)\alpha D(t, v, w) + d(x)\alpha D(t, v, w)$
$= d(x)\alpha[D(u, v, w) + D(u, v, w) + D(t, v, w) + D(t, v, w)]$
$= [D(u, v, w) + D(u, v, w) + D(t, v, w) + D(t, v, w)]\alpha d(x)$ for all $t, u, v, w, x \in R$, $\alpha \in \Gamma$, (19)

and

$D(u + t, v, w)\alpha(d(x) + d(x))$
$= D(u + t, v, w)\alpha d(x) + D(u + t, v, w)\alpha d(x)$
$= [D(u, v, w) + D(t, v, w)]\alpha d(x) + [D(u, v, w) + D(t, v, w)]\alpha d(x)$
$= [D(u, v, w) + D(t, v, w) + D(u, v, w) + D(t, v, w)]\alpha d(x)$ for all $t, u, v, w, x \in R$, $\alpha \in \Gamma$. (20)

Comparing (19) and (20), we obtain $D((u, t), v, w)\alpha d(x) = 0$ for all $t, u, v, w, x \in R$, $\alpha \in \Gamma$. Hence it follows from Lemma 3.3 that

$D((u, t), v, w) = 0$ for all $t, u, v, w \in R$. (21)

We substitute $u\beta z$ for u and $u\beta t$ for t in (21) to get

$0 = D(u\beta(z, t), v, w) = D(u, v, w)\beta(z, t) + u\beta D((z, t), v, w) = D(u, v, w)\beta(z, t)$, $\beta \in \Gamma$.

That is,

$D(u, v, w)\beta(z, t) = 0$ for all $t, u, v, w, z \in R$, $\beta \in \Gamma$. (22)

Letting $z = s\delta z$ in (22) and comparing the results (22) we obtain,

$D(u, v, w)\beta s\delta(z, t) = 0$ for all $s, t, u, v, w, z \in R$, $\beta, \delta \in \Gamma$. (23)

Since $D \neq 0$, we conclude, from (23) and the primeness of R, that $(z, t) = 0$ is fulfilled for all $t, z \in R$. Therefore $(R, +)$ is abelian.

By the hypothesis, we know that

$[d(x), D(u, v, w)]_{\alpha} = 0$ for all $u, v, w, x \in R$, $\alpha \in \Gamma$. (24)

Hence if we let $x = x + y$ in (24) and since $d(x + y) = d(x) + 2D(x, x, y) + D(x, y, y) + D(x, x, y) + 2D(x, y, y) + d(y)$, then we deduce from (24) that $3[D(x, x, y), D(u, v, w)]_{\alpha} + 3[D(x, y, y), D(u, v, w)]_{\alpha} = 0$ for all $u, v, w, x, y \in R$, $\alpha \in \Gamma$.

Since R is 3-torsion-free, we obtain,

$[D(x, x, y), D(u, v, w)]_{\alpha} + [D(x, y, y), D(u, v, w)]_{\alpha} = 0$ for all $u, v, w, x, y \in R$, $\alpha \in \Gamma$. (25)

Setting $y = -y$ in (25) and comparing the result with (25), we obtain

$[D(x, y, y), D(u, v, w)]_{\alpha} = 0$ for all $u, v, w, x, y \in R$, $\alpha \in \Gamma$. (26)

Replacing y by $y + z$ in (26) and using (26), we have $[D(x, y, z), D(u, v, w)]_{\alpha} = 0$, $\alpha \in \Gamma$, since D is permuting, i.e.,

$D(x, y, z)\alpha D(u, v, w) = D(u, v, w)\alpha D(x, y, z)$ for all $u, v, w, x, y, z \in R$, $\alpha \in \Gamma$. (27)

Taking $u\beta t$ instead of u in (27), we obtain,

$D(u, v ,w)\beta t\alpha D(x, y, z) - D(x, y, z)\alpha D(u, v, w)\beta t + u\beta D(t, v, w)\alpha D(x, y, z)$
$- D(x, y, z)\beta u\alpha D(t, v, w) = 0$ for all $t, u, v, w, x, y, z \in R$, $\alpha, \beta \in \Gamma$. (28)

Substituting $d(u)$ for u in (28) and then utilizing the hypothesis and (27), we get

$D(\mathrm{d}(u), v, w)\beta[t, D(x, y, z)]_{\alpha} = 0$ for all $t, u, v, w, x, y, z \in R$, $\alpha, \beta \in \Gamma$. (29)

Let us write in (29) $w\delta s$ instead of w. Then we have $D(d(u), v, w)\delta s\beta[t, D(x, y, z)]_{\alpha} = 0$ for all $s, t, u, v, w, x, y, z \in R$, $\alpha, \beta, \delta \in \Gamma$. Since R is prime, we arrive at either $D(d(u), v, w) = 0$ or $[t, D(x, y, z)]_{\alpha} = 0$ for all $t, u, v, w, x, y, z \in R$, $\alpha \in \Gamma$. As in the proof of Theorem 3.5, the case when $D(d(u), v, w) = 0$ holds for all $u, v, w \in R$ leads to the contradiction. Consequently, we arrive at $[t, D(x, y, z)]_{\alpha} = 0$ for all $t, x, y, z \in R$,$\alpha \in \Gamma$, i.e, $D(x, y, z) \in C(R)$ for all $x, y, z \in R$. Therefore, Theorem 3.5 yields that R is a commutative Γ-ring which completes the proof.

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