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A New Method of Construction of E-optimal Generalized Group Divisible Designs (GGDD)

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Abstract

In this article, we develop a new method of construction of E-optimal generalized group divisible designs through group testing designs.

Keywords: Balanced Incomplete Block Design (BIBD); Group Divisible (GD); Generalized Group Divisible Design (GDD); E-optimality.

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1. Introduction

The E-optimality criterion was introduced by Ehrenfeld [1]. Block designs are used for experiments where it is important to eliminate heterogeneity in one direction. Those considered here have v treatments arranged in b blocks containing k experimental units each. The structure of any such block design is determined by its $v \times b$ incidence matrix whose (i, j) the entry gives the number of times treatment i occurs in block j. Jacroux [2, 3] studied the construction of E-optimal block designs unequal number of replicates and some E-optimal designs for the one way elimination of heterogeneity. Jacroux [3] developed the method of construction of E-optimal generalized group divisible design (GGDD). Thannippara *et al.* [4] investigated the E-optimality of hypercubic designs. Thannippara *et al.* [5] developed a new method of λ singly linked block designs through non-adaptive hypergeometric group testing designs for identifying at most two defectives for $v^* = 0(\text{mod } 6)$ and $v^* = 2(\text{mod } 6)$. Thannippara *et al.* [6] developed E-optimal

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generalized group divisible designs from hypercubic designs with parameters $v = t^m$ ($t = 2$ or 3), $\lambda_2 = \lambda_1 + 1$, $k = 2$ or 3.

2. Preliminary Results

2.1 Definitions

Balanced Incomplete Block Designs (BIBD): An incomplete block design with v treatments distributed over b blocks, each of size k, where k is less than v such that each treatment occurs in r blocks, no treatment occurs more than once in a block and each pair of treatment occurs together in λ blocks, is called a balanced incomplete block design. The symbols $v, b, k, r,$ and λ are the parameters of the design.

Group Testing Designs (GTD): The group testing design d can be obtained by dualizing the dual design d^* of any known design satisfying the conditions stated below.

1. $B_i^* \cup B_j^* = B_k^* \cup B_l^*$, $i, j, k, l = 1, 2, \cdots, n$, $(i, j) \neq (k, l)$ where B_i^* denotes the number of the test in d in which the i-th item is tested.

2. In d^* any pair of treatments can appear at most once.

3. $n \leq \left[v^*(v^* + 1)/6 \right]$ where $[x]$ denotes the greatest integer contained in x.

This definition is due to Weideman and Raghavarao [7].

Group Divisible Design (GDD): In a group divisible design $v = mn$ treatments are divided into m groups of size n each such that any two treatments from the first group are first associates and two treatments from different groups are second associates. The association scheme can be displayed by arranging the treatment numbers in a rectangular arrangement of m rows and n columns where each row of n treatments gives a group. Evidently $n_1 = n - 1$ and $n_2 = n(m-1)$. The secondary parameters are

$$p_{jk}^1 = \begin{bmatrix} n-2 & 0 \\ 0 & n(m-1) \end{bmatrix} \text{ and } p_{jk}^2 = \begin{bmatrix} 0 & n-1 \\ n-1 & n(m-2) \end{bmatrix}.$$

Generalized Group Divisible Design with s Groups (GGDD): Let $d(v, b, k)$ be any block design having v treatments arranged in b blocks of size k. Then d is called a generalized group divisible design with s groups if the subscripts corresponding to the treatments in d can be divided into s mutually disjoint sets $V_1, V_2, \cdots, V_s$ of size $v_1, v_2, \cdots, v_s$ such that

1. for $i = 1, 2, \cdots, s$ and for all $a \in V_i$, $r_{da} = \lambda_{daa} = r_i$ where r_i is a constant.

2. for $i, j = 1, 2, \cdots, s$ and for all $a \in V_i$, $b \in V_j$, $a \neq b$, $\lambda_{ab} = \lambda_{ij}$ where λ_{ij} is a constant (the entries of $N_d N'_d$ are denoted by λ_{dij}. This definition is due to Jacroux [3].

E-optimality: A design $d(v,b,k)$ is said to be E-optimal if it maximizes $\gamma = \min\{\gamma_i\}$ where $\gamma_1, \gamma_2, \cdots, \gamma_v$ are the eigen values of the C-matrix of the design.

Lemma 2.1. *Suppose that* $\bar{d} \in D(v,b,k)$ *is a BIBD and* w *is an integer satisfying* $v/k^2 \leq w \leq (v-1)/k$. *If* d *is the design obtained from* $\bar{d}$ *by deleting* w *mutually disjoint blocks, then* d *is E-optimal in* $D(v,\bar{b},k)$ *where* $\bar{b} = b - w$.

This Lemma may be found in Jacroux [3].

Theorem 2.1. *A BIBD can be obtained from GTD*(d) *for* $v^* \equiv 0(\bmod 6)$ *by adding a single block.*

Proof. If d is an RGD, $N_d N'_d$ has all of its diagonal elements are equal and off diagonal elements differing by at most one and its association matrix has the additional property that all of its off diagonal elements are equal, then d is called a BIBD (See Thannippara *et al.* [5]).

3. Method of Construction

In the present investigation we construct E-optimal GGDD from GTD for $v^* \equiv 0(\bmod 6)$. The method of construction of GGDD is based on Theorem 2.1 and Lemma 2.1. This can be illustrated by the following example (obtained from Weideman and Raghavarao [7] and Ghosh and Thannippara [8]) which is a GTD with blocks:

$$B_1 = \{1,4,5\};\ B_2 = \{1,6,7\};\ B_3 = \{2,4,6\}$$
$$B_4 = \{2,5,7\};\ B_5 = \{3,4,7\};\ B_6 = \{3,5,6\}.$$

This is a GTD for $v^* \equiv 0(\bmod 6)$. In this GTD, by adding a single block $\{1,2,3\}$, we obtain a BIBD with parameters $v = 7, b = 7$ and $k = 3$ (by theorem 2.1). $d(7,7,3)$ contains one disjoint block, that is, $7/9 \leq w \leq 6/3$ or $0 \leq w \leq 2$. An E-optimal $GGDD(7,6,3)$ obtained from a d by deleting a single block (see Lemma 2.1).

An upper bound for efficiency (E) of optimal $GGDD(7,6,3)$*:* An upper bound for efficiency of $d(v,b,k)$ are defined in Raghavarao [9] as $E \leq (vr-b)/r(v-1)$, where $r = bk/v$. Using this we get the upper for E for the design $GGDD(7,6,3)$ as $E \leq 0.77$.

Table 1 below shows the blocks of E-optimal $GGDD(7,6,3)$.

Table 1

1	1	2	2	3	3
4	6	4	5	4	5
6	7	6	7	7	6

Some remarks on E-optimality designs for $v = 1(\text{mod}\,6)$ *and* $v = 3(\text{mod}\,6)$.

Jacroux [3] developed the method of E-optimal generalized group divisible designs from BIBD. This is evident from Lemma 2.1. In this section we point out some important features of E-optimal GGDD obtained from $v \equiv 1(\text{mod}\,6)$ and $v \equiv 3(\text{mod}\,6)$ balanced incomplete block designs.

Table 2 below shows the parameters of E-optimal GGDD obtained from $v \equiv 1(\text{mod}\,6)$.

Table 2

v	b	k	No. of Disjoint Blocks (d')	$v/k^2 \le w \le (v-1)/k.$	Parameters of E-optimal GGDD Obtained from Lemma 2.1
7	7	3	1	$0 \le w \le 2$	$d(7,6,3)$, $d(7,5,3)$
13	26	3	4	$1 \le w \le 4$	$d(7,25,3)$, $d(7,24,3)$
19	57	3	5	$2 \le w \le 6$	$d(19,55,3), d(19,54,3), d(19,53,3)$ $d(19,52,3), d(19,51,3)$
25	100	3	8	$2 \le w \le 8$	$d(25,98,3), d(25,97,3), d(25,96,3)$ $d(25,95,3), d(25,94,30, d(25,93,3)$ $d(25,92,3)$

Table 3 below shows the parameters of E-optimal GGDD that are obtained from $v = 3(\text{mod}\,6)$ BIBD.

Table 3

v	b	k	No. of Disjoint Blocks (d')	$v/k^2 \le w \le (v-1)/k.$	Parameters of E-optimal GGDD Obtained from Lemma 2.1
9	12	3	3	$1 \le w \le 2.66$	$d(9,11,3)$, $d(9,10,3)$
15	35	3	5	$1 \le w \le 4.6$	$d(15,34,3)$, $d(15,33,3)$ $d(15,32,3)$, $d(15,31,6)$
15	70	3	7	$1 \le w \le 6.6$	$d(21,69,3), d(21,68,3), d(21,67,3)$ $d(21,66,3), d(21,65,3), d(21,61,3)$

3. Concluding Remarks

From Table 2 we see that total number of E-optimal GGDD obtained from $v \equiv 1(\text{mod}\,6)$ BIBD is equal to the number of disjoint blocks d' or $d'-1$ or $d'+1$. But from Table 3 we see that the total number of E-optimal GGDD obtained from $v \equiv 3(\mathrm{mod}\,6)$ BIBD is equal to the number of disjoint blocks-1 i.e. $d' - 1$.

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