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Short Communication

Reciprocal Symmetry and its Relation to Einstein's Postulate, Lorentz Transformation and Discreteness

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Abstract

Objectivity demands that kinematics should be independent of the quantities (velocities or their reciprocals) we define. This demand is translated as Reciprocal Symmetry (RS), which we have defined. We have shown that RS gives an upper bound velocity, the mathematical analogue of Einstein's postulate, and Lorentz transformation. RS also requires discreteness. RS promises to bridge the gap between relativity and quantum mechanics.

Keywords: Galilean velocity; Lorentz-algebraic velocity; Reciprocal symmetry; Reciprocal velocity; Reciprocal symmetric distance; Discrete time; Discreteness.

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1. Introduction

Velocity v is defined as the distance covered per unit time; $v = x/t$. We might as well define slowness v' as the reciprocal of v; $v'=1/v = t/x$. Physics is independent of the quantities we define. Therefore, objectivity demands that motion described in terms of slowness should be as valid as the description in terms of velocities. We postulate:

Postulate: Kinematics is symmetric under reciprocal inversion of velocities.

The postulate demands that description of motion should remain invariant under inversion $v \to 1/v$ and vice-versa. Relative velocity or distance covered should be independent of the reciprocity of velocity. We shall see that relativistic relative velocity (Lorentz transformation) is reciprocal symmetric.

Reciprocal symmetry requires that velocity have an upper bound (like Einstein's postulate). The reciprocal (slowness) has a lower bound. Slowness must form a group; difference between two slownesses is slowness. Therefore, the difference must be bounded below. Therefore, slowness must be discrete [1]. This relates reciprocal

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symmetry to discreteness. Since discreteness is in the heart of quantum mechanics, we expect to find a relation between relativity and quantum mechanics through reciprocal symmetry

2. Reciprocal Symmetry

Definition 1. A function *f* will be called reciprocal symmetric if it remains invariant when one (or more) of the arguments is replaced by its reciprocal, i.e.

$$f(1/a,m) = f(a,m) \tag{2.1}$$

Definition 2. A function *f* will be called ant-symmetric (or reciprocal ant-symmetric) if it changes sign when one of the arguments is replaced by its reciprocal, i.e.

$$f(1/a,m) = -f(a,m) \tag{2.2}$$

If

$$f(1/a,m) = f(a,m), \text{ when } m \text{ is an even integer} \tag{2.3}$$

$$f(1/a,m) = -f(a,m), \text{ when } m \text{ is an odd integer} \tag{2.4}$$

The two above relations may be combined into one

$$f(1/a,m) = \exp(i.m.\pi).f(a,m), \text{ when } m \text{ is an integer} \tag{2.5}$$

3. Reciprocal Symmetry and Upper Bound

Definition: Let $\oplus$ represent a reciprocal symmetry sum so that

$$c/u \oplus (\text{-}c/v) = u/c \oplus (\text{-}v/c) \tag{3.1}$$

Theorem:

$$u/c \oplus 1 = 1 \tag{3.2}$$

Proof:

Let

$$c/u \oplus (\text{-}c/v) = u/c \oplus (\text{-}v/c) = w/c \tag{3.3}$$

Therefore,

$$\{c/u \oplus (\text{-}c/v)\} \oplus (c/v) = w/c \oplus c/v \tag{3.4}$$

Assuming $\oplus$ is associative, $(\text{-}c/v) \oplus (c/v) = 0$ and $c/u \oplus 0 = c/u$, (3.4) gives

$$c/u = w/c \oplus c/v \tag{3.5}$$

Similarly looking at the other part of (3.3), we have

$$u/c = w/c \oplus v/c \tag{3.6}$$

Comparison between (3.5) and (3.6) shows

$$w/c \oplus v/c = \frac{1}{w/c \oplus c/v} \tag{3.7}$$

$v = c$ *in* (3.7) gives

$$w/c \oplus 1 = \frac{1}{w/c \oplus 1} \tag{3.8}$$

Therefore,

$$w/c \oplus 1 = 1 \tag{3.9}$$

(3.9) is Einstein's postulate [1] if we assume that c is the speed of light.

4. Lorentz Transformation

A representation of $\oplus$, which fulfills (3.1), is

$$c/u \oplus (\text{-}c/v) = u/c \oplus (\text{-}v/c) = \frac{u/c - v/c}{1 - u.v/c^2} \tag{4.1}$$

This is Lorentz transformation [2] for motion in one space dimension.

5. Galilean and Reciprocal Symmetric Velocities

To study symmetry properties of velocity we shall transform Galilean velocities to a form where symmetry properties come to surface. Consider the identity

$$V = \frac{c}{2} \ln \frac{1 + \tanh(V/c)}{1 - \tanh(V/c)} \tag{5.1}$$

We define [2] $v/c = \tanh(V/c)$ and call v the Lorentz algebraic (or reciprocal symmetric) velocity corresponding to Galilean velocity V. We may write

$$V = \frac{c}{2} \ln \frac{1 + v/c}{1 - v/c} \tag{5.2}$$

Using (5.2), the Galilean relative velocity, $W = U - V$, is

$$W = U - V = \frac{c}{2} \ln \frac{1 + u/c}{1 - u/c} \frac{1 - v/c}{1 + v/c} = \frac{c}{2} \ln \frac{1 + \{u/c \oplus (-v/c)\}}{1 - \{u/c \oplus (-v/c)\}} = \frac{c}{2} \ln \frac{1 + w/c}{1 - w/c} \tag{5.3}$$

Therefore, W corresponds to $w/c = u/c \oplus (-v/c)$ as given by (4.1).

6. Lorentz-Algebraic Distance

To study symmetry properties if distance, we define, as in (5.2), reciprocal symmetric distance x corresponding to (familiar) Galilean distance X by

$$X = \frac{L}{2}\ln\frac{1+x/L}{1-x/L} \tag{6.1}$$

L is a constant distance (corresponding to c of Eq. (5.2)).

Let $X = Vt = VT(t/T)$ be the distance traveled in time *t*. *T* is quantum of time. Using (5.2)

$$X = VT(t/T) = \frac{cT}{2}\ln\left(\frac{1+v/c}{1-v/c}\right)^{t/T} = \frac{cT}{2}\ln\left(\frac{1+\{(t/T)\otimes(v/c)\}}{1-\{(t/T)\otimes(v/c)\}}\right) \tag{6.2}$$

where

$$(t/T)\otimes(v/c) = \frac{\left(\frac{1+v/c}{1-v/c}\right)^{t/T} - 1}{\left(\frac{1+v/c}{1-v/c}\right)^{t/T} + 1} \tag{6.3}$$

(6.1) and (6.2) will agree if

$$L = cT \text{ and } x/L = (t/T)\otimes(v/c) \tag{6.4}$$

In the limit $c \to \infty$

$$x = L\{(t/T)\otimes(v/c)\} \xrightarrow[c\to\infty]{} L\frac{vt}{cT} = vt \tag{6.5}$$

7. Reciprocal Symmetric Multiplication

The rule of multiplication (6.3) has the following symmetry properties under reciprocal inversion $v/c \to c/v$

$$(t/T)\otimes(c/v) = \frac{(-1)^{t/T}\left(\frac{1+v/c}{1-v/c}\right)^{t/T} - 1}{(-1)^{t/T}\left(\frac{1+v/c}{1-v/c}\right)^{t/T} + 1} \tag{7.1}$$

For *t/T* = even integer

$$(t/T)\otimes(c/v) = (t/T)\otimes(v/c) \tag{7.2}$$

For *t/T* = odd integer

$$(t/T)\otimes(c/v) = \frac{1}{(t/T)\otimes(v/c)} \tag{7.3}$$

Relation (6.3) may be called “Lorentz multiplication” as it is consistent with (4.1) in the sense that it is distributive with respect to Lorentz-Einstein addition

$$\left(t/T\right)\otimes\{(u/c)\oplus\left(v/c\right)\}=\{(t/T)\otimes(u/c)\}\oplus\{(t/T)\otimes(v/c)\} \tag{7.4}$$

8. Reciprocal Symmetric Distance and Discrete Time

To show the dependence of x of (6.4) on v/c, we write it as x(v/c). Using (7.2) and (7.3) we can write

$$x=\frac{1}{2}\left\{x\left(v/c\right)+x\left(c/v\right)\right\} \quad \text{for } t/T = \text{even integer} \tag{8.1}$$

$$x=\frac{1}{2}\left\{x\left(v/c\right)+\frac{1}{x(c/v)}\right\} \quad \text{for } t/T = \text{odd integer} \tag{8.2}$$

Let us consider the reciprocation operation

$$x\oplus\left(i\tan\left(\frac{\pi.t}{2T}\right)\right)=x \quad \text{for } t/T = \text{even integer} \tag{8.3}$$

$$x\oplus\left(i\tan\left(\frac{\pi.t}{2T}\right)\right)=\frac{1}{x} \quad \text{for } t/T = \text{odd integer} \tag{8.4}$$

Using the above relations (8.1) and (8.1) may be written in the compact form

$$x=\frac{1}{2}\left(x\left(v/c\right)\oplus\left\{x\left(c/v\right)\oplus\left(i\tan\left(\frac{\pi.t}{2T}\right)\right)\right\}\right) \quad \text{for } t/T = \text{integer} \tag{8.5}$$

(8.3) gives the distance traveled by a body in time t. A part of the contribution comes from velocity *v/c*. The other part comes from reciprocal velocity *c/v*. The time of travel must be an integral multiple of quantum of time *T*. As seen in (6.5), in the limit $c \to \infty$, we get the classical expression $x \to vt$.

9. Conclusion

It is shown that reciprocal symmetry yields Einstein's postulate and Lorentz transformation. The symmetry also gives discreteness. Reciprocal symmetry is, therefore, the ground where relativity and quantum mechanics may meet.

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