Publications

Available Online

J. Sci. Res. **8** (1), 13-19 (2016)

JOURNAL OF SCIENTIFIC RESEARCH

www.banglajol.info/index.php/JSR

On Vietoris Soft Topology I

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Received 20 May 2015, accepted in final revised form 19 September 2015

Abstract

In this article, we define the hyperspace of soft closed sets of a soft topological space $(F_A, \tilde{\tau})$. In addition, we define the Vietoris soft topology, $\tilde{\tau}_V$, by determining the soft base of this topology which has the form $\langle F_{H_1}, F_{H_2}, \dots, F_{H_n} \rangle$, where $F_{H_1}, F_{H_2}, \dots, F_{H_n}$ are soft open sets in $(F_A, \tilde{\tau})$. Some properties of this topology are also investigated. The impact of introducing the Vietoris soft topology is to enable us to understand many properties of the structure of soft topologies corresponding to it.

Keywords: Soft set; Soft open set; Soft hyperspace; Vietoris soft topology.

doi: http://dx.doi.org/10.3329/jsr.v8i1.23440 J. Sci. Res. **8** (1), 13-19 (2016)

1. Introduction

The comparison between the number of the articles, researches and applications of the theory of soft set and its young age which introduced in 1999 by Molodtsov [1] reveals the significance of such a mathematical tool for dealing with uncertainty and vagueness. The appearance of this theory was welcomed by mathematicians by investigating potential properties as well as introducing and generalising many concepts that certain complicated problems can be solved by building models depending on.

Shabir [2] introduced the soft topological spaces which are defined over an initial universe with a fixed set of parameters and studied some basic notions of soft topological spaces such as soft open, soft closed sets, soft subspace, soft closure, soft neighbourhood of a point and soft separation axioms. This work followed by many researches that dealt with various generalizations of this certain types of open sets.

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There is a comprehensive survey covers the development of soft set theory and its applications as reported elsewhere [3]. The literature of studying hyperspaces in topological space has a rich history which started almost ten decades ago when Vietoris [4] introduced one hyperspace called finite or the Vietoris hyperspace. After that many other hyperspaces were introduced and investigated their structures. The significance of hyperspaces theory is represented by investigating their properties which can help to understand the structures of topological spaces corresponding to them. Many types of hyperspaces were investigated by E. Michael [5].

This article adds a new type of hyperspaces completely depends on soft open and soft closed sets in a soft topological space. We recall the definition of the Vietoris hyperspace by using both open and closed soft sets. Then we investigate some properties of this soft hyperspace. All definitions which stated are defined over an initial universe with a fixed set of parameters.

2. Preliminaries

Let $\boldsymbol{U}$ be an initial universe set and E be the set of all possible parameters with respect to $\boldsymbol{U}$ and $\mathcal{P}(\boldsymbol{U})$ represents the power set of $\boldsymbol{U}$. The parameters are often attributes or properties of the objects in the initial universe.

Definition 2.1: [1] A soft set $\boldsymbol{F_A}$ on the universe set $\boldsymbol{U}$ is denoted by the set of ordered pairs:
$\boldsymbol{F_A} = \{(\boldsymbol{x}, \boldsymbol{f_A}(\boldsymbol{x})) : \boldsymbol{x} \in \boldsymbol{E}, \boldsymbol{f_A}(\boldsymbol{x}) \in \mathcal{P}(\boldsymbol{U})\}$, where $\boldsymbol{f_A}: \boldsymbol{E} \to \mathcal{P}(\boldsymbol{U})$ such that $\boldsymbol{f_A}(\boldsymbol{x}) = \emptyset$ $\boldsymbol{if}$ $\boldsymbol{x} \notin \boldsymbol{A}$. $\boldsymbol{f_A}$ is called an approximate function of the soft set $\boldsymbol{F_A}$. The value of $\boldsymbol{f_A}$ may be an arbitrary.

Example 2.2: Suppose that there are eight cars in the universe $\boldsymbol{U} = \{\boldsymbol{c_1}, \boldsymbol{c_2}, \dots, \boldsymbol{c_8}\}$ and let $\boldsymbol{E} = \{\boldsymbol{x_1}, \boldsymbol{x_2}, \boldsymbol{x_3}, \boldsymbol{x_4}, \boldsymbol{x_5}\}$ is the set of decision parameters such that $\boldsymbol{x_1} =$ new, $\boldsymbol{x_2} =$ expensive, $\boldsymbol{x_3} =$ high-tech, $\boldsymbol{x_4} =$ model, $\boldsymbol{x_5} =$ interior design. Consider the map $\boldsymbol{f_A} \equiv$ cars(attributes), so for example $\boldsymbol{f_A}(\boldsymbol{x_3})$ means "cars(high-tech)". Thus the functional value of $\boldsymbol{f_A}(\boldsymbol{x_3})$ is the set $\{\boldsymbol{c} \in \boldsymbol{U} : \boldsymbol{c}$ is high-tech$\}$. Now let $\boldsymbol{A} = \{\boldsymbol{x_2}, \boldsymbol{x_3}, \boldsymbol{x_5}\}$ and $\boldsymbol{f_A}(\boldsymbol{x_2}) = \{\boldsymbol{c_2}, \boldsymbol{c_6}\}$, $\boldsymbol{f_A}(\boldsymbol{x_3}) = \{\boldsymbol{c_1}, \boldsymbol{c_3}, \boldsymbol{c_4}\}$, $\boldsymbol{f_A}(\boldsymbol{x_5}) = \{\boldsymbol{c_1}, \boldsymbol{c_7}, \boldsymbol{c_8}\}$. Then the soft set $\boldsymbol{F_A} = \{(\boldsymbol{x_2}, \{\boldsymbol{c_2}, \boldsymbol{c_6}\}), (\boldsymbol{x_3}, \{\boldsymbol{c_1}, \boldsymbol{c_3}, \boldsymbol{c_4}\}), (\boldsymbol{x_5}, \{\boldsymbol{c_1}, \boldsymbol{c_7}, \boldsymbol{c_8}\})\}$.

Definition 2.3: [1] Let $\boldsymbol{F_A}$ be a soft set, if $\boldsymbol{f_A}(\boldsymbol{x}) = \boldsymbol{U}$ for all $\boldsymbol{x} \in \boldsymbol{A}$, then $\boldsymbol{F_A}$ is called an A-universe soft set and is denoted by $\boldsymbol{F_{\tilde{A}}}$.
If $\boldsymbol{A} = \boldsymbol{E}$, then $\boldsymbol{F_{\tilde{E}}}$ is called a universe soft set.

Example 2.4: Let $\boldsymbol{U} = \{\boldsymbol{u_1}, \boldsymbol{u_2}, \boldsymbol{u_3}\}$, $\boldsymbol{E} = \{\boldsymbol{x_1}, \boldsymbol{x_2}, \boldsymbol{x_3}\}$ and $\boldsymbol{A} = \{\boldsymbol{x_1}, \boldsymbol{x_2}\}$, then $\boldsymbol{F_{\tilde{A}}} = \{(\boldsymbol{x_1}, \boldsymbol{U}), (\boldsymbol{x_2}, \boldsymbol{U})\}$ and $\boldsymbol{F_{\tilde{E}}} = \{(\boldsymbol{x_1}, \boldsymbol{U}), (\boldsymbol{x_2}, \boldsymbol{U}), (\boldsymbol{x_3}, \boldsymbol{U})\}$.

Definition 2.5: [1] Let $\boldsymbol{F_A}$ be a soft set, if $\boldsymbol{f_A}(\boldsymbol{x}) = \emptyset$ for all $\boldsymbol{x} \in \boldsymbol{E}$, then $\boldsymbol{F_A}$ is called the empty soft set and denoted by $\boldsymbol{F_{\Phi}}$.

Example 2.6: Let $U = \{u_1, u_2, , u_3, u_4, u_5\}$ and $E = \{x_1, x_2, x_3\}$, then $F_{\Phi} = \{(x_1, \emptyset),$ $(x_2, \emptyset), (x_3, \emptyset)\}$.

Definition 2.7: [1] Let F_A and F_B be soft sets. Then F_A is a soft subset of F_B if $f_A(x) \subseteq f_B(x)$ for all $x \in E$ and is denoted by $F_A \tilde{\subseteq} F_B$.

Example 2.8: Let $U = \{u_1, u_2, u_3, u_4, u_5\}$, $E = \{x_1, x_2, x_3, x_4, x_5\}$, $A = \{x_1, x_4\}$ and $B = \{x_4\}$. Suppose $F_A = \{(x_1, \{u_1, u_5\}), (x_4, \{u_2, u_3, u_4\})\}$ and $F_B = \{(x_4, \{u_2, u_3\})\}$,, then $F_B \tilde{\subseteq} F_A$.

Definition 2.9: [1] Let F_A and F_B be soft sets, then F_A and F_B are soft equal if $f_A(x) = f_B(x)$ for all $x \in E$.

Definition 2.10: [1] Let F_A and F_B be soft sets, then the union of F_A and F_B (denoted by $F_A \tilde{\cup} F_B$) is defined by $F_A \tilde{\cup} F_B = f_A(x) \cup f_B(x)$ for all $x \in A \cup B$.

Example 2.11: Let $U = \{u_1, u_2, , u_3\}$, $E = \{x_1, x_2, x_3\}$, $A = \{x_1, x_2\}$, $B = \{x_3\}$, $F_A = \{(x_1, \{u_1, u_2\}), (x_2, \{u_2\})\}$ and $F_B = \{(x_2, \{u_1, u_3\}), (x_3, \{u_3\})\}$. Then $F_A \tilde{\cup} F_B = \{(x_1, \{u_1, u_2\}), (x_2, U), (x_3, \{u_3\})\}$.

Definition 2.12: [1] Let F_A and F_B be soft sets, then the intersection of F_A and F_B (denoted by $F_A \tilde{\cap} F_B$) is defined by $F_A \tilde{\cap} F_B = f_A(x) \cap f_B(x)$ for all $x \in A \cap B$.

Example 2.13: Let $U = \{u_1, u_2, , u_3, u_4\}$, $E = \{x_1, x_2, x_3, x_4\}$, $A = \{x_1, x_4\}$, $B = \{x_1, x_3\}$, $F_A = \{(x_1, \{u_1\}), (x_4, \{u_2, u_3, u_4\})\}$ and $F_B = \{(x_1, \{u_1, u_3\}), (x_3, \{u_3\})\}$. Then $F_A \tilde{\cap} F_B = \{(x_1, \{u_1\})\}$.

Definition 2.14: [1] Let F_A be a soft set, $\alpha = (x, \{u\})$ is a nonempty soft element of F_A, denoted by $\alpha \tilde{\in} F_A$ if there exists $x \in E$ and $u \in f_A(x)$.
Notice that the singleton set of a soft point is denoted by F_α.

Example 2.15: Let $U = \{u_1, u_2, , u_3\}$, $E = \{x_1, x_2, x_3\}$, $A = \{x_2, x_3\}$ and let $F_A = \{(x_2, \{u_2, u_3\}), (x_3, \{u_1, u_2\})\}$, then the following are nonempty elements in F_A:
$\alpha_1 = (x_2, \{u_2\}) \tilde{\in} F_A$; since $u_2 \in f_A(x_2) = \{u_2, u_3\}$.
$\alpha_2 = (x_2, \{u_3\}) \tilde{\in} F_A$; since $u_3 \in f_A(x_2) = \{u_2, u_3\}$.
$\alpha_3 = (x_3, \{u_1\}) \tilde{\in} F_A$; since $u_1 \in f_A(x_3) = \{u_1, u_2\}$.
$\alpha_4 = (x_3, \{u_2\}) \tilde{\in} F_A$; since $u_2 \in f_A(x_3) = \{u_1, u_2\}$.

Definition 2.16: [1] Let F_A be a soft set, the soft complement of F_A (denoted by $F_A^{\tilde{c}}$) is defined by the approximate function $F_A^{\tilde{c}} = f_A^c(x)$, where $f_A^c(x) = U - f_A(x)$ for all $x \in A$.

Example 2.17: Let $U = \{u_1, u_2, , u_3\}$, $E = \{x_1, x_2, x_3\}$, $A = \{x_2, x_3\}$ and let $F_A = \{(x_1, \{u_2\}), (x_3, \{u_1, u_3\})\}$. Then $F_A^{\tilde{c}} = \{(x_1, \{u_1, u_3\}), (x_3, \{u_2\})\}$.

Definition 2.18: [2] Let F_A be a soft set, a soft topology on F_A, denoted by $\tilde{\tau}$, is a collection of soft subsets of F_A satisfying the following conditions:

(1) $F_\Phi, F_A \in \tilde{\tau}$.

(2) If $\{F_{E_i} \tilde{\subseteq} F_A : i \in I \subseteq \mathbb{N}\} \subseteq \tilde{\tau}$, then $\tilde{\bigcup}_{i\in I} F_{H_i} \in \tilde{\tau}$.

(3) If $\{F_{E_i} \tilde{\subseteq} F_A : 1 \le i \le n,\ n \in \mathbb{N}\} \subseteq \tilde{\tau}$, then $\tilde{\bigcap}_{i=1}^{n} F_{H_i} \in \tilde{\tau}$.

Then $\tilde{\tau}$ is called a soft topology and the pair $(F_A, \tilde{\tau})$ is called a soft topological space.

Example 2.19: Let $U = \{u_1, u_2, u_3\}$, $E = \{x_1, x_2, x_3\}$, $A = \{x_1, x_2\}$, then $(F_A, \tilde{\tau}) = \{F_\Phi, F_{A_1}, F_{A_2}, F_{A_3}, F_A\}$ is a soft topological space, where $F_{A_1} = \{(x_1, \{u_2\})\}$, $F_{A_2} = \{(x_1, \{u_2\}), (x_2, \{u_2\})\}$, $F_{A_3} = \{(x_1, \{u_1, u_2\}), (x_2, \{u_2\})\}$.

Definition 2.20: [2] Let $(F_A, \tilde{\tau})$ be a soft topological space and $F_B \tilde{\subseteq} F_A$. Then F_B is said to be a soft closed if $F_B^{\tilde{c}}$ is a soft open set.

Example 2.21: Consider the previous example, then $F_B = \{(x_1, \{u_1, u_3\}), (x_2, \{u_1, u_3\})\}$ is a soft closed set.

Definition 2.22: [2] Let $(F_A, \tilde{\tau})$ be a soft topological space and $\alpha \tilde{\in} F_A$, if there is a soft open set F_B such that $\alpha \tilde{\in} F_B$ then F_B is called a soft open neighbourhood (or soft neighbourhood) of α .

The set of all neighbourhood of α is called the family of soft neighbourhood of α and denoted by $\mathcal{V}(\alpha) = \{F_B : F_B \in \tilde{\tau}\ and\ \alpha \tilde{\in} F_B\}$.

Definition 2.23: A soft point is called a soft isolated point α if and only if F_α is soft open.

Definition 2.24: [6] Let $(F_A, \tilde{\tau})$ be a soft topological space. Then a family $\tilde{\beta} \subseteq \tilde{\tau}$ is called a soft base for $(F_A, \tilde{\tau})$ if for every soft open set $F_H \neq F_\phi$, there exist $F_{B_i} \in \tilde{\beta}$, $i \in I$, such that $F_H = \tilde{\bigcup}\{F_{B_i} : i \in I\}$.

Definition 2.25: [2] A soft topological space $(F_A, \tilde{\tau})$ is called $\tilde{T}_0$ if for each $\alpha_1, \alpha_2 \tilde{\in} F_A$ with $\alpha_1 \neq \alpha_2$ there exist soft open sets $F_{H_1} \in \mathcal{V}(\alpha_1)$ and $F_{H_2} \in \mathcal{V}(\alpha_2)$ such that $\alpha_2 \tilde{\notin} F_{H_1}$ **or** $\alpha_1 \tilde{\notin} F_{H_2}$.

Definition 2.26: [2] A soft topological space $(F_A, \tilde{\tau})$ is called $\tilde{T}_1$ if for each $\alpha_1, \alpha_2 \tilde{\in} F_A$ with $\alpha_1 \neq \alpha_2$ there exist soft open sets $F_{H_1} \in \mathcal{V}(\alpha_1)$ and $F_{H_2} \in \mathcal{V}(\alpha_2)$ such that $\alpha_2 \tilde{\notin} F_{H_1}$ and $\alpha_1 \tilde{\notin} F_{H_2}$.

Definition 2.27: A sequence of soft sets $\{F_{H_n}\}_{n=1}^{\infty}$ is eventually contained in a soft set F_K if there is an integer $N \in \mathbb{N}$ such that $F_{H_n} \tilde{\subseteq} F_K$ for each $n \ge N$.

If a sequence $\{F_{H_n}\}_{n=1}^{\infty}$ of soft sets in a soft topological space contained eventually in each soft neighbourhood of a soft set F_K then this sequence converges to F_K , this soft convergence is denoted by $F_{H_n} \simeq F_K$.

Definition 2.28: A sequence $\{F_{H_n}\}_{n=1}^{\infty}$ is called an increasing sequence if $F_{H_n} \tilde{\subseteq} F_{H_{n+1}}$ for each $n \in \mathbb{N}$.

3. The Vietoris soft hyperspace

Definition 3.1: The soft hyperspace of the soft topological space F_A is the following collection $\widetilde{CL}(F_A) = \{F_K \tilde{\subseteq} F_A : F_K$ is a soft closed in $F_A\}$.

Definition 3.2: Let $(F_A, \tilde{\tau})$ be a T_1 soft topological space, then the Vietoris soft topology, denoted by $\tilde{\tau}_V$, has the following base:

$\langle F_{H_1}, F_{H_2}, \dots, F_{H_n} \rangle = \{F_K \tilde{\in} \widetilde{CL}(F_A): F_K \tilde{\subseteq} \widetilde{\bigcup_{i=1}^{n}} F_{H_i} \, and \, F_{H_i} \tilde{\cap} F_K \neq F_{\Phi} \, for \, each \, i\}$

where $F_{H_1}, F_{H_2}, \dots, F_{H_n}$ are soft open sets in $(F_A, \tilde{\tau})$.

Example 3.3: Let $U = \{u_1, u_2\}$, $E = A = \{x_1, x_2\}$, then $(F_A, \tilde{\tau}) = \{F_{\Phi}, F_{A_1}, F_{A_2}, F_{A_3}, F_A\}$ is a soft topological space, where $F_{A_1} = \{(x_1, \{u_1\})\}$, $F_{A_2} = \{(x_2, \{u_2\})\}$, $F_{A_3} = \{(x_1, \{u_1\}), (x_2, \{u_2\})\}$. Then $\widetilde{CL}(F_A) = \{F_A, F_{B_1}, F_{B_2}, F_{B_3}\}$ where $F_{B_1} = \{(x_1, \{u_2\})\}$, $F_{B_2} = \{(x_2, \{u_1\})\}$ and $F_{B_3} = \{(x_1, \{u_2\}), (x_2, \{u_1\})\}$. So for example, $\langle F_{A_1}, F_A \rangle = \{F_A\}$ is a soft open basic set in the soft Vietoris topology defined on F_A.

Proposition 3.4: (1) $\tilde{\tau}_V$ is a $\tilde{T}_0$. (2) $\tilde{\tau}_V$is a $\tilde{T}_1$.

Proof: (1) Let $F_K, F_H \tilde{\in} \widetilde{CL}(F_A)$ such that $F_K \neq F_H$. So without losing generality, there exists $\alpha \tilde{\in} F_K \setminus F_H$. Now $\langle F_A \setminus F_{\alpha} \rangle$ is an open basic neighbourhood of F_H and does not contain F_K . ■

(2) Let $F_K, F_H \tilde{\in} \widetilde{CL}(F_A)$ such that $F_K \neq F_H$ so without losing generality, there exists $\alpha \tilde{\in} F_K \setminus F_H$. Now $\langle F_A \setminus F_{\alpha} \rangle$ is an open basic neighbourhood of F_H and does not contains F_K and $\langle F_A \setminus F_H \rangle$ is a basic open neighbourhood of F_K and does not contain F_H. ■

Lemma 3.5: $\langle F_{H_1}, F_{H_2}, \dots, F_{H_n} \rangle = F_{\Phi}$ if there exists at least one $F_{H_c} = F_{\Phi}$.

Proof: Suppose $F_{H_c} = F_{\Phi}$ for some $c \in \{1, 2, \dots, n\}$, then

$$\langle F_{H_1}, F_{H_2}, \dots, F_{H_n} \rangle = \left\{ F_K \tilde{\in} \widetilde{CL}(F_A): F_K \tilde{\subseteq} \widetilde{\bigcup_{i=1}^{n}} F_{H_i} and \, F_{H_i} \tilde{\cap} F_K \neq F_{\Phi} \, for \, i \in \{1, 2, \dots, n\} \setminus \{c\} \right\} and \, F_{H_c} \tilde{\cap} F_K = F_{\Phi} \} = F_{\Phi} . \blacksquare$$

Proposition 3.6: Let $(F_A, \tilde{\tau})$ be a $\tilde{T}_1$ soft topological space, then $\langle F_{H_1}, F_{H_2}, \dots, F_{H_n} \rangle \tilde{\subseteq} \langle F_{K_1}, F_{K_2}, \dots, F_{K_m} \rangle$ if and only if $\tilde{\bigcup}_{i=1}^{n} F_{H_i} \tilde{\subseteq} \tilde{\bigcup}_{j=1}^{m} F_{K_j}$ and for every F_{K_j} there exists F_{H_i} such that $F_{H_i} \tilde{\subseteq} F_{K_j}$.

Proof: $(\Rightarrow)$Let $\alpha \tilde{\in} \tilde{\bigcup}_{i=1}^{n} F_{H_i}$ and suppose $\alpha \tilde{\notin} \tilde{\bigcup}_{j=1}^{m} F_{K_j}$. Now pick $\gamma_i \tilde{\in} F_{H_i}$ for each $i = 1, 2, \dots, n$. Thus each F_{γ_i} is soft closed subset of F_A. Hence $(\tilde{\bigcup}_{i=1}^{n} F_{\gamma_i})\tilde{\cup} F_\alpha \tilde{\in} \tilde{CL}(F_A)$. It is clear that $(\tilde{\bigcup}_{i=1}^{n} F_{\gamma_i})\tilde{\cup} F_\alpha \tilde{\in} \langle F_{H_1}, F_{H_2}, \dots, F_{H_n} \rangle$. But $(\tilde{\bigcup}_{i=1}^{n} F_{\gamma_i})\tilde{\cup} F_\alpha \tilde{\notin} \langle F_{K_1}, F_{K_2}, \dots, F_{K_m} \rangle$ (since $\alpha \tilde{\notin} \tilde{\bigcup}_{j=1}^{m} F_{K_j}$) so we get a contradiction. Therefore $\alpha \tilde{\in} \tilde{\bigcup}_{j=1}^{m} F_{K_j}$.

On the other hand, suppose there is $F_{K_j^*}$ such that $F_{H_i} \tilde{\nsubseteq} F_{K_j^*}$ for each $i = 1, 2, \dots, n$. Let $\beta_i \tilde{\in} F_{H_i} \setminus F_{K_j^*}$ for each $i = 1, 2, \dots, n$. Clear that $\tilde{\bigcup}_{i=1}^{n} F_{\beta_i} \tilde{\in} \langle F_{H_1}, F_{H_2}, \dots, F_{H_n} \rangle$ but $\tilde{\bigcup}_{i=1}^{n} F_{\beta_i} \tilde{\notin} \langle F_{K_1}, F_{K_2}, \dots, F_{K_m} \rangle$, hence we get a contradiction.

$(\Leftarrow)$ Suppose that $F_M \tilde{\in} \langle F_{H_1}, F_{H_2}, \dots, F_{H_n} \rangle$, so $F_M \tilde{\subseteq} \tilde{\bigcup}_{i=1}^{n} F_{H_i}$ and $F_M \tilde{\cap} F_{H_i} \neq F_\Phi$ for each *i*. But $\tilde{\bigcup}_{i=1}^{n} F_{H_i} \tilde{\subseteq} \tilde{\bigcup}_{j=1}^{m} F_{K_j}$ hence $F_M \tilde{\subseteq} \tilde{\bigcup}_{i=1}^{n} F_{H_i}$. Now since $F_M \tilde{\cap} F_{H_i} \neq F_\Phi$ for each *i*, and for each F_{K_j} there exist F_{H_i} such that $F_{H_i} \tilde{\subseteq} F_{K_j}$. Hence $F_\Phi \neq F_M \tilde{\cap} F_{H_i} \tilde{\subseteq} F_M \tilde{\cap} F_{K_j}$ for each $j = 1, 2, \dots, m$. Therefore $F_M \tilde{\in} \langle F_{K_1}, F_{K_2}, \dots, F_{K_m} \rangle$. ■

Proposition 3.7: Let $(F_A, \tilde{\tau})$ be a $\tilde{T}_1$ soft topological space, then $\tilde{\tau}_V$ has no soft isolated point if and only if F_A has no soft isolated point.

Proof: $(\Rightarrow)$ Supoose F_A has a soft isolated point, say α, so F_α is a soft open in F_A. Moreover F_α is soft closed subset of F_A (since F_A is $\tilde{T}_1$). Now, $\langle F_\alpha \rangle = \{ F_K \tilde{\in} \tilde{CL}(F_A) : F_K \tilde{\subseteq} F_\alpha \} = \{F_\alpha\}$. This is a contradiction of being the Vietoris soft topology has no isolated point.

$(\Leftarrow)$ Suppose the Vietoris soft topology has a soft isolated point, say F_M, so $\{F_M\}$ is a soft open set in $\tilde{CL}(F_A)$. Thus there exist $F_{H_1}, F_{H_2}, \dots, F_{H_n}$ which are soft open in F_A such that $\{F_M\} = \langle F_{H_1}, F_{H_2}, \dots, F_{H_n} \rangle$. Hence $F_M \tilde{\subseteq} \tilde{\bigcup}_{i=1}^{n} F_{H_i}$ and $F_M \tilde{\cap} F_{H_i} \neq F_\Phi$ for each i. Now, for each *i* let $\alpha_i \tilde{\in} F_M \tilde{\cap} F_{H_i}$

Fix $F_{\alpha_{i_*}}$ for some i_*. So $F_{\alpha_{i_*}}$ soft closed subset of F_A. Now, $\{F_M\}$ is open in the Vietoris topology, so $F_{\alpha_{i_*}}$ is a soft interior point of $\{F_M\}$. Thus there exists a neighbourhood $\langle F_{K_1}, F_{K_2}, \dots, F_{K_m} \rangle$ of $F_{\alpha_{i_*}}$ such that $\langle F_{K_1}, F_{K_2}, \dots, F_{K_m} \rangle \tilde{\subseteq} \langle F_{H_1}, F_{H_2}, \dots, F_{H_n} \rangle$. Being $F_{\alpha_{i_*}} \tilde{\subseteq} \tilde{\bigcup}_{j=1}^{m} F_{K_j}$ and $F_{\alpha_{i_*}} \tilde{\cap} F_{K_j} \neq F_\Phi$ means that $F_{\alpha_{i_*}} \tilde{\cap} F_{K_j} = F_{\alpha_{i_*}}$. Hence $F_{\alpha_{i_*}}$ is a soft open in F_A which is a contradiction of being F_A has no soft isolated point. ■

Proposition 3.8: Let $(F_A, \tilde{\tau})$ be a soft topological space and let $\{F_{K_n}\}_{n=1}^{\infty}$ be any increasing sequence of soft closed sets in the Vietoris soft topology, then $F_{k_n} \rightrightarrows \tilde{\bigcup}_{n=1}^{\infty} F_{K_n}$.

Proof: Let F_L be an arbitrary neighbourhood of $\tilde{\bigcup}_{n=1}^{\infty} F_{K_n}$ in $\tilde{\tau}_V$. Then there exist $F_{H_1}, F_{H_2}, \dots, F_{H_n}$ soft open in F_A such that $\tilde{\bigcup}_{n=1}^{\infty} F_{K_n} \tilde{\in} \langle F_{H_1}, F_{H_2}, \dots, F_{H_m} \rangle \tilde{\subseteq} F_L$. Now for every $j \in \{1, 2, \dots, m\}$, there exist $\alpha_j \tilde{\in} (\tilde{\bigcup}_{n=1}^{\infty} F_{K_n})\tilde{\cap} F_{H_j}$. Thus there exist $F_{K_{n_j}}$ such that $\alpha_j \tilde{\in} F_{K_{n_j}}$. Set $N = max\{n_j : j = 1, 2, \dots, m\}$. Then for all $j, \alpha_j \tilde{\in} F_{K_N}$ and hence for all j, $F_{K_n} \tilde{\cap} F_{H_j} \neq F_{\Phi}$ for all $n \geq N$. Consequently $F_{K_n} \tilde{\in} \langle F_{H_1}, F_{H_2}, \dots, F_{H_n} \rangle$ for all $n \geq N$. ■

6. Conclusion

This article introduced a hyperspace by depending on the theory of sot closed subsets of a soft topological space. Then this hyperspace is topologised by a certain topology namely, the soft Vietoris topology. Many properties of the new topology are discussed.

Acknowledgment

The author would like to thank the Iraqi ministry of higher education and scientific research for its support.

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