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Unsteady MHD Flow of a Dusty Visco-elastic Fluid between Parallel Plates with Exponentially Decaying Pressure Gradient in an Inclined Magnetic Field

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Abstract

The present paper deals with the unsteady laminar flow of an incompressible, electrically conducting dusty visco-elastic fluid between two parallel stationary plates. The flow is caused by an exponentially decaying pressure gradient. A uniform magnetic field is applied on the lower plate at different inclinations. We observe that the motions of the fluid and dust particles are affected by the variation of some significant physical parameters of the visco-elastic fluid. Mass concentration number, time-relaxation parameter, visco-elastic parameter, intensity of the applied magnetic field and time are some of indispensable physical parameters of fluid flow. The governing equations of motion have been solved by analytical method and the results have been discussed with the help of graphs. The velocity is observed to be symmetrical with the centre of the channel of fluid flow as well as of dust particles. The velocity of the fluid particles and that of the dust particles go on decreasing with an increase in the values of mass concentration number, magnetic field intensity, visco-elastic parameter and time whereas the velocity profiles of fluid and dust particles are observed to be increasing with an increase in the time- relaxation parameter.

Keywords: Visco-elastic dusty fluid; MHD flow; Exponential pressure gradient; Relaxation parameter; Mass concentration number.

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1. Introduction

The flow of visco-elastic fluid has attracted the attention of a large number of eminent scholars due to its wide applications in different transport phenomena. The visco-elastic fluid often embedded with spherical non-conducting dust particles as impurities

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are used in industries. The influence of dust particles on this fluid has its importance in many applications such as extrusion of plastics in the manufacture of Nylon, purification of crude oil in petroleum industry, pulp oil, paper industries, textile industry etc. Many authors have already studied the flow of dusty viscous fluid such as Kuiry and Bahadur [1] studied MHD flow of viscous fluid between two parallel porous plates with heat transfer in an inclined magnetic field and some of the important studies in the domain of dusty viscous fluid with elastic properties have been carried out by many researchers. A large variety of industrial products are under the visco-elastic behavior of the fluid containing dust particles as impurities. This type of fluid is known as Rivlin-Erickson second order fluids. Saffman [2] has studied the stability of laminar flow of a dusty gas. Singh and Ram [3] have studied the unsteady flow of an electrically conducting dusty viscous liquid through a channel. Varshney [4] has studied the flow of a dusty Rivlin-Erickson Fluid through a channel. Madhura and Kalpana [5] have studied the thermal effect on unsteady flow of a dusty visco-elastic fluid between two parallel plates under different pressure gradients. Attia and Ewis [6] have studied the unsteady MHD Couette flow with heat transfer of a visco-elastic fluid under exponentially decaying pressure gradient. Saxena and Sharma [7] have studied unsteady flow of an electrically conducting dusty visco-elastic fluid between two parallel plates. Ajadi [8] has studied analytical solutions of unsteady oscillatory particulate visco-elastic fluid between two parallel walls. Abel [9] has studied the heat transfer in a visco-elastic boundary layer flow over a stretching sheet with viscous dissipation and non-uniform heat source. Sivaraj and Rushi [10] have studied the unsteady MHD dusty visco-elastic fluid Couette flow in an irregular channel with varying mass diffusion. Venkateswarlui and SatyaNarayana [11] have studied the MHD visco-elastic fluid flow over a continuously moving vertical surface with chemical reaction.

In the present work, it has been attempted to study the flow behavior of a dusty and visco-elastic fluid flow through a channel bounded by two stationary parallel plates under the action of an exponentially decaying pressure gradient in the presence of a magnetic force applied at different inclinations. The fluid and dust particles are initially at rest. We observe the changes in the velocity of the both the fluid particles and dust particles due to various values of visco-elastic and time-relaxation parameters, mass concentration number and time. The influence of the magnetic field intensity applied at different inclinations at a fixed time has been depicted graphically.

2. Mathematical Formulation of the Problem

The present problem considers the dust particles be spherical in shape and uniformly distributed. No chemical reaction, mass transfer and radiation are there between the fluid and dust particles. Within a particle, there is uniform temperature and no interaction between particles is considered. The flow is assumed to be fully developed and the buoyancy force is absent. The number density of the dust particles is assumed to be constant in the fluid. We consider an unsteady laminar flow of an incompressible, electrically conducting visco-elastic fluid with uniform distribution of dust particles between two infinite parallel plates separated by a distance '2*h*' and the axis of the channel formed by the plates is taken from the origin. Let the x-axis be chosen along the plates and the y-axis normal to the plates. The flow is due to the time dependent pressure gradient. The motion of dusty visco-elastic fluid between the plates whose equations are $y = -h$ and $y = h$ studied under the influence of an exponentially decaying pressure gradient with an inclined uniform magnetic field applied on the lower plate. Both the fluid and the dust particles are supposed to be static at the beginning. The hydromagnetic governing partial differential equations of the flow are given below

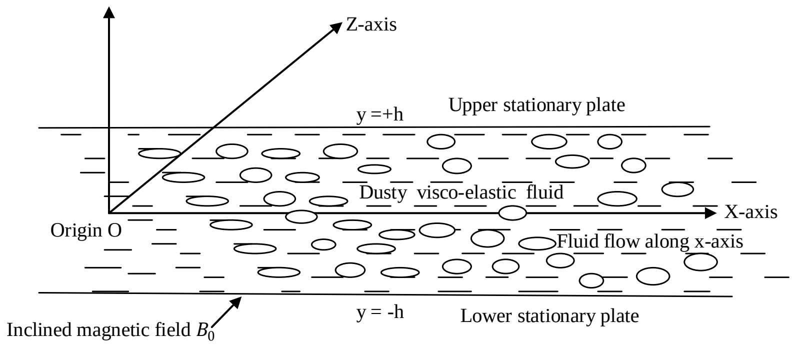

Fig. 1. Model of the problem.

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \left(\xi - \beta\frac{\partial}{\partial t}\right)\frac{\partial^2 u}{\partial y^2} + \frac{kN_0}{\rho}(v - u) - \sigma\frac{{B_0}^2}{\rho}u \qquad (1)$$

$$\frac{\partial p}{\partial y} = 0 \qquad (2)$$

and

$$m\frac{\partial v}{\partial t} = k(u - v) \qquad (3)$$

where $u(y, t)$ is the fluid velocity, $v(y, t)$ is the velocity of the dust particles, k is the Stoke's resistance co-efficient, ξ is the kinematic co-efficient of viscosity of the fluid, β is the kinematic co-efficient of visco-elasticity of the fluid, m is the mass per unit volume of the dust particles, σ is the electrical conductivity of the fluid, ρ is the density of the fluid, N_0 is the constant for density of the dust particles, B_0 is the magnitude of the magnetic induction B, μ is the coefficient of viscosity of the fluid, t is the time, p is the fluid pressure. The equation (2) indicates that the fluid pressure is independent of y.

Let us take the decaying pressure gradient as $(-\frac{\partial p}{\partial x}) = a_0 e^{-\lambda t}$, where $\lambda > 0$ and both the plates are at rest. The fluid motion is due to the pressure gradient. The equations (1) – (3) are to be solved subject to the initial and boundary conditions mentioned below;

$$u = 0, v = 0, when\ t = 0 \text{ and } for\ t > 0,\ u = 0, v = 0 \text{ at } y = \pm h \quad (4)$$

Substituting the non-dimensional quantities:

$$x^* = \frac{x}{h},\ y^* = \frac{y}{h},\ t^* = \frac{\xi t}{h^2},\ u^* = \frac{uh}{\xi},\ v^* = \frac{vh}{\xi},\ p^* = \frac{ph^2}{\rho\xi^2},\ \lambda^* = \frac{\lambda h^2}{\xi} \quad (5)$$

into the equation (1) to (3), we get

$$\frac{\partial u^*}{\partial t^*} = -\frac{1}{\rho}\frac{\partial p^*}{\partial x^*} + \left(1 - \varphi\frac{\partial}{\partial t^*}\right)\frac{\partial^2 u^*}{\partial y^{*2}} + \frac{l}{\omega}(v^* - u^*) - M^2 u^* \quad (6)$$

$$\omega\frac{\partial v^*}{\partial t^*} = v^* - u^*, \quad (7)$$

where $l = \frac{mN_0}{\rho}$ (Mass concentration); $\varphi = -\frac{\beta}{h^2}$ (Visco-elastic parameter); $\omega = \frac{m\xi}{Kh^2}$(Time-relaxation parameter); $M = H_a sin\alpha;$ $H_a = hB_0\sqrt{\frac{\sigma}{\mu}}$ (Hartman number).

Initial and boundary conditions are at $t^* = 0$, $u^* = 0, v^* = 0$ and for $t^* > 0,\ u^* = 0, v^* = 0$ at $y^* = \pm 1$.

Now dropping the stars and substituting the pressure gradient $-\frac{\partial p}{\partial x} = a_0 e^{-\lambda t}$, where $a_0 > 0$ is a constant and $\lambda > 0$ into the equations (6) and (7), we get

$$\frac{\partial u}{\partial t} = a_0 e^{-\lambda t} + \left(1 - \varphi\frac{\partial}{\partial t}\right)\frac{\partial^2 u}{\partial y^2} + \frac{l}{\omega}(v - u) - M^2 u \quad (8)$$

and

$$\omega\frac{\partial v}{\partial t} = (u - v) \quad (9)$$

Initial and boundary condition reduce to

t= 0, $u = 0, v = 0$

and

$t > 0,\ u = 0, v = 0$ at $y = \pm 1$.

The equations (8) and (9) are solved subject to the constraints (10) by assuming $u = \phi(y)e^{-\lambda t}$ and $v = \psi(y)e^{-\lambda t}$. Eliminating v from the equation (8) with the help of the equation (9), we get

$$\frac{d^2\phi}{dy^2} - \left[\frac{\lambda^2\omega - \lambda(1 + l + \omega M^2) + M^2}{\lambda\varphi - \lambda\omega + 1 - \lambda^2\omega\varphi}\right]\phi(y) = \frac{a_0(\omega\lambda - 1)}{\lambda\varphi - \lambda\omega + 1 - \lambda^2\omega\varphi} \quad (11)$$

This equation (11) has been solved subject to the boundary conditions (10) by taking some permissible arbitrary fixed values of the parameters involved in the equation. The solution is used to determine the velocity of the fluid as well as the velocity of dust particles.

3. Results and Discussion

We obtain $\Phi(y)$ and utilizing it in the equation (11), we find the velocity of the fluid particles $u = (y, t)$ for different values of t. This value is used in the equation (9) to obtain the velocity of dust particles $v = (y, t)$ in the following cases.

In Figs. 2 and 3, taking $\lambda = 0.5, l = 0.5, M = 1, \omega = 0.8, \varphi = 0.5, a_0 = 1$, we observe that the velocity of fluid particles and that of dust particles go on decreasing as the time t increases.

In Figs. 4 and 5, the velocity of fluid and that of the dust particles are represented graphically against the various values of visco-elastic parameter φ for $\lambda = 0.5, l = 0.5, M = 1, \omega = 0.9, t = 1, a_0 = 1$, we observe that the velocity of fluid particles and that of dust particles decrease with an increase of the visco-elastic parameter.

In Figs. 6 and 7, the impact of the relaxation parameter ω on the motion of the visco-elastic dusty fluid is studied for the parameters $\lambda = 0.5, l = 0.5, M = 1, t = 1, \varphi = 0.5, a_0 = 1$. We observe that the velocity of fluid particles and that of the dust particles increase along with an increase of the relaxation parameter ω.

In the Figs. 8 and 9, the effect of magnetic inclination α on the motion of visco-elastic fluid is shown for the parameters $\lambda = 0.5, l = 0.5, \omega = 0.9, t = 1, \varphi = 0.5, a_0 = 1$. The velocity of the fluid particles and that of the dust particles go on decreasing when the inclination of the applied magnetic field increases.

In the Figs. 10 and 11, the effect of mass concentration l on the motion of visco-elastic fluid is shown for $\lambda = 0.5, M = 1, t = 1, \omega = 0.9, \varphi = 0.5, a_0 = 1$.The velocity of the fluid particles and that of the dust particles go on decreasing when mass number increases.

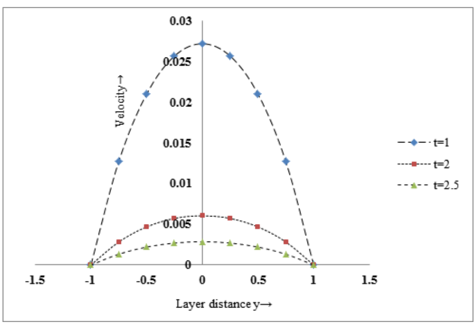

Fig. 2. Velocity profile of fluid particles for the time parameter t.

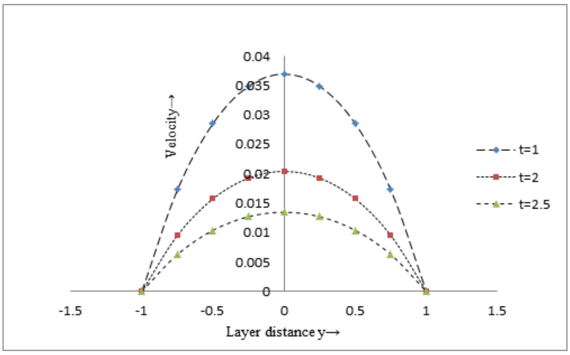

Fig. 3. Velocity profile of dust particles for the time parameter t.

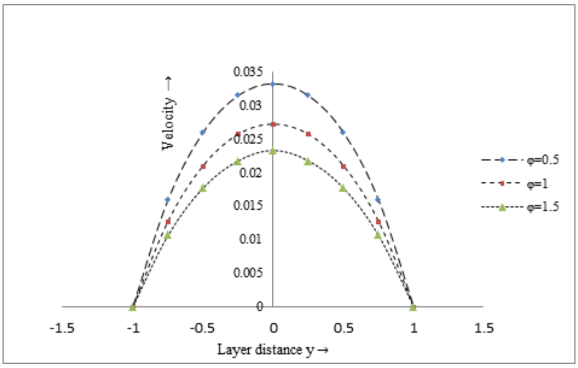

Fig. 4. Velocity profile of fluid particles for visco-elastic parameter $\boldsymbol{\varphi}$.

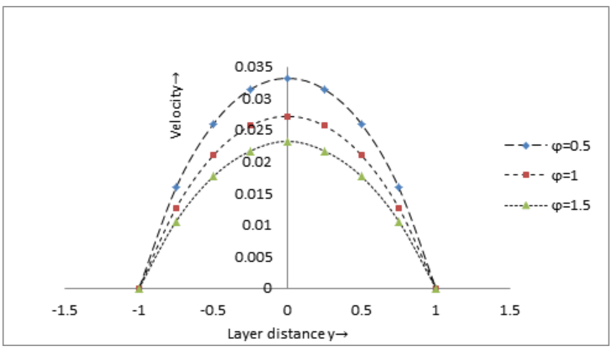

Fig. 5. Velocity profile of dust particles for visco-elastic parameter $\boldsymbol{\varphi}$.

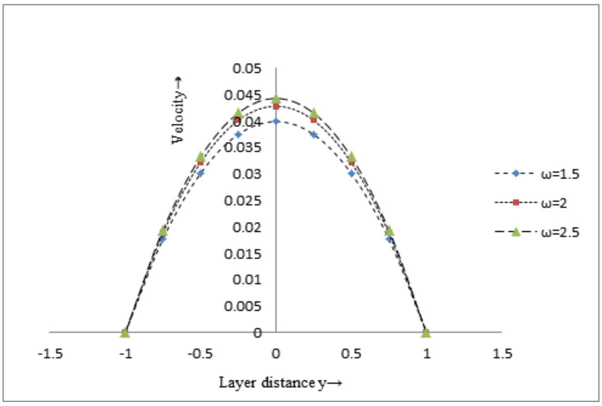

Fig. 6. Velocity profile of fluid particles for time-relaxation parameter ω.

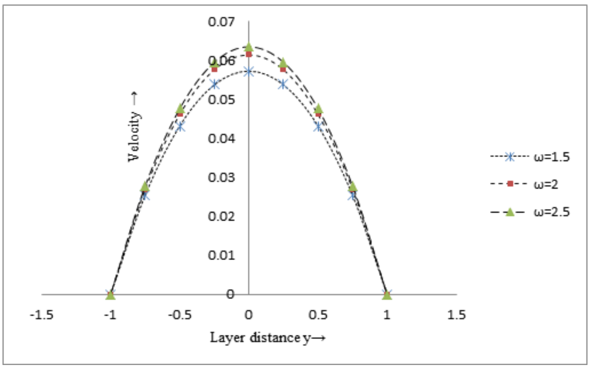

Fig. 7. Velocity profile of dust particles for time-relaxation parameter ω.

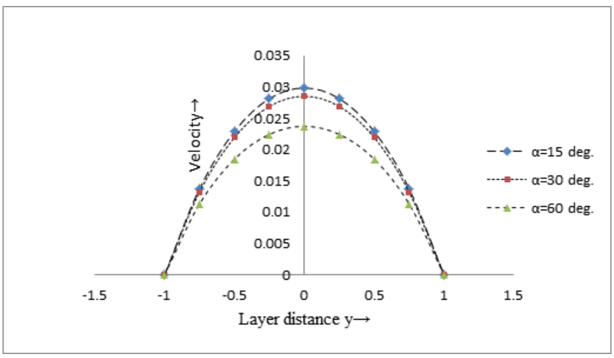

Fig. 8. Velocity profile of fluid particles at different inclinations of magnetic field.

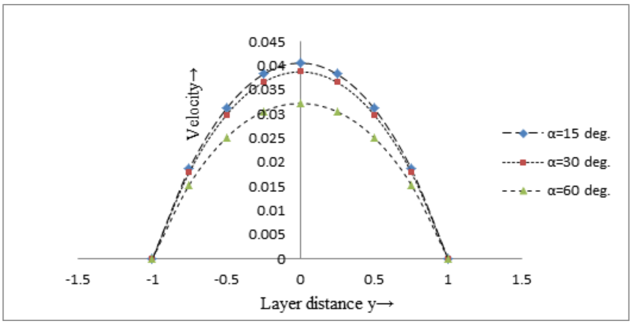

Fig. 9. Velocity profile of dust particles at different inclinations of magnetic field.

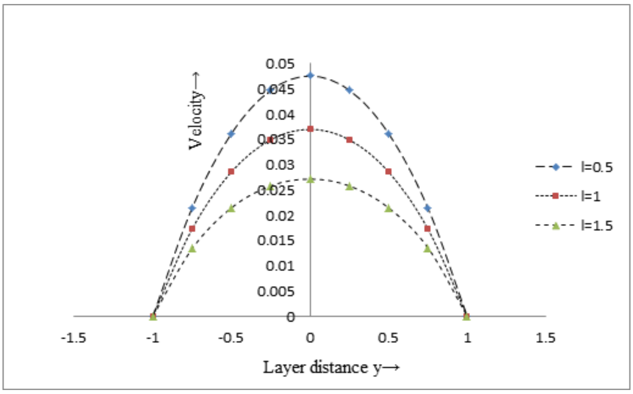

Fig. 10. Velocity profile of fluid particles for mass concentration l.

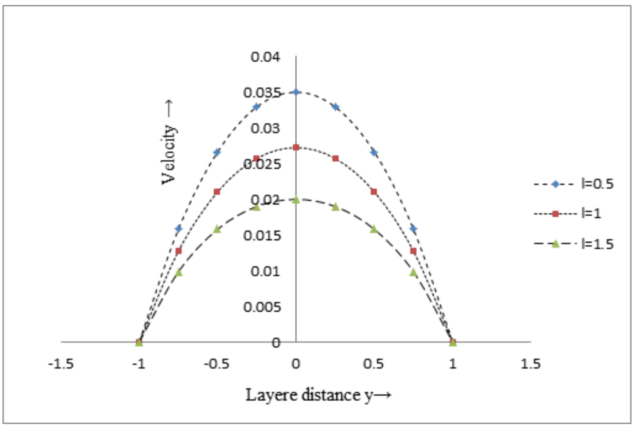

Fig.11. Velocity profile of dust particles for mass concentration l.

4. Conclusion

In the unsteady laminar MHD flow of an incompressible, electrically conducting dusty visco-elastic fluid between two parallel stationary plates under the effect of an exponentially decaying pressure gradient, some significant parameters of the fluid are observed to be effective in the determination of velocity profiles of both fluid and dust. The fluid has viscous as well as elastic properties and so the visco-elastic parameter plays pivotal role in modifying the velocity profiles of the fluid. Mass concentration and time-relaxation parameters are also indispensible in depicting the velocity profiles. Apart from this, the velocity profiles of fluid particles and those of dust particles are seen to be dependent upon time. The velocity of fluid particles and that of dust particles go on decreasing along with an increase in the parameters such as time t, mass concentration, visco-elastic parameter, and magnetic field intensity. On the contrary the velocity of fluid particles and also that of the dust particles are observed to be increasing along with an increase of relaxation parameter.

References

1. D. R.Kuiry and Surya Bahadur, J. Sci. Res. **7(3)**, 21, (2015).
   http://dx.doi.org/10.3329/jsr.v7i3.22574
2. P. G. Saffman, J. Fluid Mech. **13(1)**, 1962, 120.
   http://dx.doi.org/10.1017/S0022112062000555
3. C. B. Singh and P. C. Ram, Ind. J. Pure Appl. Math. **8(9)**, 1022, (1977).
4. O. P. Varshney, Ph.D Thesis. Agra University, Agra, India (1983).
5. K. R. Madhura and G. Kalpana, Int. J. Engg. Technol. **2(2)**, 88 (2013).
   http://dx.doi.org/10.14741/ijcet/spl.2.2014.16
6. H. A. Attia and K. M. Ewis, Tamkang J. Sci. Engg. **13(4**), 359 (2010).
7. S. Saxena and G. C. Sharma, Ind. J. Pure Appl. Math. **18(12)**, 1131 (1987).
8. S. O. Ajadi, Int. J. Non-linear Sci. **9**, 131 (2010).
9. M. S.Abel, Int. J. Heat Mass Trans. **50**, 960 (2007).
   http://dx.doi.org/10.1016/j.ijheatmasstransfer.2006.08.010
10. R. Sivaraj and B. R. Kumar, Int. J. Heat Mass Trans. **55**, 3076 (2012).
    http://dx.doi.org/10.1016/j.ijheatmasstransfer.2012.01.049
11. B. Venkateswarlui and P. V. SatyaNarayana, Walailak J. Sci. Technol. **12(9)**, 775 (2015).