

## Symbol Error Performance Analysis and Comparison between Alamouti and MRRC Diversity Combining Scheme for Wireless Communication

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### Abstract

The symbol error performance of a wireless communication system is analyzed using diversity combining scheme in Rayleigh fading channels. The performance of the system is compared using Alamouti scheme and maximal-ratio receiver combining (MRRC) scheme. The performance results of the system are evaluated by numerical computation and by simulation using MATLAB. A detailed analysis and exact (closed-form) expressions of the probability of error of the wireless communication systems in Rayleigh fading channels are provided for both M-ary phase-shift keying (M-PSK) and M-ary quadrature amplitude modulation (M-QAM) schemes. Monte Carlo simulation shows exact match with the theoretical results.

*Keywords:* Alamouti scheme; M-PSK; M-QAM; MRRC; Symbol error probability (SEP).

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## 1. Introduction

Diversity techniques have almost exclusively been applied to base stations to improve their reception quality. This is because it is more economical to add equipment to base stations rather than the remote units. For this reason, transmit diversity schemes are very attractive. Receiver diversity is a form of space diversity, where there are multiple antennas at the receiver. Numerous research works have been carried out during the last few years on the performance evaluation of wireless communication system using diversity technique. Wireless Communications and Networking including coding, different multiplexing and multiple accesses, forms of diversity, generations of communications, WiMAX and many topics related to wireless communications are well discussed [1-4]. Discussions on diversity technique, multiple accesses and various

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information on wireless communication system can also be found in the Wikipedia [5]. A simple two-branch transmit diversity scheme is presented in ref. [6]. Using two transmit antennas and one receive antenna the scheme provides the same diversity order as maximal-ratio receiver combining (MRRC) with one transmit antenna, and two receive antennas. It is also shown that the scheme may easily be generalized to two transmit antennas and M receive antennas to provide a diversity order of 2M. Based on Alamouti's work, Tarokh *et al.* [7] proposed orthogonal space-time block codes (OSTBCs) combining the orthogonal coding method and the simple diversity technique. Tarokh [8] studies the encoding and decoding algorithms for various codes and provides simulation results demonstrating their performance. A squaring method to simplify the decoding of orthogonal space-time block codes in a wire-less system with an arbitrary number of transmit and receive antennas is reported in ref. [9].

In this paper, we present the theoretical analysis to evaluate the symbol error performance of a wireless communication system using Alamouti-scheme and MRRC diversity combining scheme. Closed-form expressions of signal-to-noise-ratio (SNR) are analytically derived for both Alamouti scheme and MRRC scheme. Using these SNR equations Exact expressions of symbol error probability (SEP) are derived analytically for up to 2Tx-2Rx transmit diversity scheme and as well as 4 branch receiver diversity scheme for binary phase-shift keying (BPSK), quadrature phase shift-keying ( QPSK) and M-ary quadrature amplitude modulation (M-QAM) schemes. The symbol error performance results are numerically evaluated and simulated using MATLAB software.

## 2. Transmit Diversity Scheme

### 2.1. Two-branch transmit diversity with one receiver

The combiner builds the following combined signal that is sent to the maximum likelihood detector. We can write as [6]

$$\hat{x}_0 = \left[ |h_0|^2 + |h_1|^2 \right] x_0 + h_0^* n_0 + h_1 n_1^* \tag{1}$$

The signal power

$$P_x = \left[ |h_0|^2 + |h_1|^2 \right]^2 E \{ |x|^2 \} \tag{2}$$

Assume  $\frac{E_s}{n_T} = \frac{1}{2} E \{ |x_1|^2 \} = \dots = \frac{1}{2} E \{ |x_B|^2 \} = \frac{\sigma_x^2}{2}$ , where  $E \{ \cdot \}$  denotes expectation,  $\sigma_x^2$  is the average symbol power and  $n_T$  means number of transmitter. The signal-to-noise-ratio (SNR) becomes

$$SNR = \frac{P_x}{P_w} = \frac{\left[ |h_0|^2 + |h_1|^2 \right] \sigma_x^2}{2\sigma_w^2} \tag{3}$$

where,  $\sigma_w^2$  is the noise power.

## 2.2. Two-branch transmit diversity with two receiver

The received signal can then be expressed as [6]

$$\begin{aligned}
 r_0 &= h_0 x_0 + h_1 x_1 + n_0 \\
 r_1 &= -h_0 x_1^* + h_1 x_0^* + n_1 \\
 r_2 &= h_2 x_0 + h_3 x_1 + n_2 \\
 r_3 &= -h_2 x_1^* + h_3 x_0^* + n_3
 \end{aligned} \tag{4}$$

where  $n_0$ ,  $n_1$ ,  $n_2$  and  $n_3$  are complex random variables representing receiver thermal noise and interference. The combiner builds the following signal that is sent to the maximum likelihood detector:

$$\hat{x}_0 = h_0^* r_0 + h_1 r_1^* + h_2^* r_2 + h_3 r_3^* \tag{5}$$

Substitution of Eq. (4) in Eq. (5) gives

$$\hat{x}_0 = \left[ |h_0|^2 + |h_1|^2 + |h_2|^2 + |h_3|^2 \right] x_0 + h_0^* n_0 + h_1 n_1^* + h_2^* n_2 + h_3 n_3^* \tag{6}$$

The signal-to-noise-ratio (SNR) becomes

$$SNR = \frac{P_X}{P_W} = \frac{\left[ |h_0|^2 + |h_1|^2 + |h_2|^2 + |h_3|^2 \right] \sigma_x^2}{2\sigma_w^2} \tag{7}$$

## 3. Receiver Diversity: MRRC Scheme

### 3.1. One transmitter – two receivers

#### *Two branch maximal-ratio receive combining (MRRC) scheme*

The receiver combining scheme for two-branch MRRC is as follows:

$$\hat{x}_0 = h_0^* r_0 + h_1^* r_1 = |h_0|^2 + |h_1|^2 + h_0^* n_0 + h_1^* n_1 \tag{8}$$

The signal-to-noise-ratio (SNR) becomes

$$SNR = \frac{P_X}{P_W} = \frac{\left[ |h_0|^2 + |h_1|^2 \right] \sigma_x^2}{\sigma_w^2} \tag{9}$$

### 3.2. One transmitter – four receivers

#### *Four branch maximal-ratio receive combining (MRRC) scheme*

The resulting received baseband signals are

$$r_1 = hx_1 + n_1 \tag{10}$$

The receiver combining scheme for the four-branch MRRC is as follows:

$$\hat{x}_0 = h_0^*r_0 + h_1^*r_1 = |h_0|^2 + |h_1|^2 + |h_2|^2 + |h_3|^2 + h_0^*n_0 + h_1^*n_1 + h_2^*n_2 + h_3^*n_3 \tag{11}$$

The signal-to-noise-ratio (SNR) becomes

$$SNR = \frac{P_x}{P_w} = \frac{\left[ |h_0|^2 + |h_1|^2 + |h_2|^2 + |h_3|^2 \right] \sigma_x^2}{\sigma_w^2} \tag{12}$$

#### 4. Probability of Error Analysis

##### 4.1. One transmitter - one receiver

From (3) the instantaneous SNR per symbol  $\eta$  for single input single output (SISO) [10] case becomes,

$$\eta = \Delta |h|^2, \quad \text{where } \Delta = \frac{\sigma_x^2}{\sigma_w^2}$$

Let  $v = |h|^2$  which has a central chi-square distribution with  $2n_T n_R$  freedom. So, the PDF [11] of V becomes  $p_v(v) = e^{-v}$ . The PDF of  $\eta$  becomes

$$P_\eta(\eta) = \frac{1}{\Delta} \exp\left(\frac{-\eta}{\Delta}\right) \tag{13}$$

Let  $P_e(\eta)$  be the bit or symbol error probability for a given modulation scheme (M-PSK or M-QAM) at the instantaneous SNR value  $\eta$ . Then the average probability of error  $P_e^{SISO}$  for the SISO system can be calculate as

$$P_e^{SISO} = \int_0^\infty P_e(\eta) p_\eta(\eta) d\eta \tag{14}$$

##### 4.1.1 M-ary PSK modulation

The symbol- error probability  $P_e(\eta)$  for M-ary PSK modulation can be given as [12]

$$P_e(\eta) = \frac{1}{\pi} \int_0^\beta \exp\left(\frac{-\eta a}{\sin^2 \theta}\right) d\theta \tag{15}$$

where,  $a = \sin^2(\pi/M)$  and  $\beta = \pi - \pi/M$ . Substitution of Eq. (13) and Eq. (15) in (14) gives

$$P_e^{SISO} = \frac{1}{\pi\Delta} \int_0^{\beta} \int_0^{\infty} \exp\left[-\left(\frac{a}{\sin^2 \theta} + \frac{1}{\Delta}\right)\eta\right] d\eta d\theta = \frac{\beta}{\pi} - \frac{1}{\pi} \sqrt{\frac{b_1}{b_1+1}} \tan^{-1} \left\{ \sqrt{\frac{b_1+1}{b_1}} \tan \beta \right\} \quad (16)$$

where,  $b_1 = \Delta a = \Delta \sin^2(\pi/M)$

#### 4.1.2. *M-QAM modulation*

The symbol-error probability  $P_e(\eta)$  for square M-QAM modulation can be given as [12]

$$P_e(\eta) = \frac{4q}{\pi} \int_0^{\pi/2} \exp\left(\frac{-\eta g}{\sin^2 \theta}\right) d\theta - \frac{4q^2}{\pi} \int_0^{\pi/4} \exp\left(\frac{-\eta g}{\sin^2 \theta}\right) d\theta \quad (17)$$

where,  $q = 1 - 1/\sqrt{M}$  and  $g = \frac{3}{2}(M-1)$ . The average symbol-error probability  $P_e^{SISO}$  can be determined by evaluating the integral in (14), where  $p_\eta(\eta)$  is given by (15). Let's define the function  $\psi_1(\Delta, a, \beta)$  as

$$\psi_1(\Delta, a, \beta) = \frac{1}{\pi\Delta} \int_0^{\beta} \int_0^{\infty} \exp\left[-\left(\frac{a}{\sin^2 \theta} + \frac{1}{\Delta}\right)\eta\right] d\eta d\theta \quad (18)$$

$\psi_1(\Delta, a, \beta)$  is given by the right-hand-side expression in (16), where  $b_1 = \Delta a$ . Using  $\psi_1(\Delta, a, \beta)$  the average symbol-error probability for M-QAM modulation can be given as

$$P_e^{SISO} = 4q\psi_1(\Delta, g, \pi/2) - 4q^2\psi_1(\Delta, g, \pi/4) \quad (19)$$

where,

$$\psi_1(\Delta, g, \pi/2) = \frac{1}{2} \left(1 - \sqrt{\frac{c}{c+1}}\right) \quad (20)$$

$$\psi_1(\Delta, g, \pi/4) = \frac{1}{4} - \frac{1}{\pi} \sqrt{\frac{c}{c+1}} \tan^{-1} \left(\sqrt{\frac{c+1}{c}}\right) \quad (21)$$

and  $c = g\Delta$ .

#### 4.2. *Two transmitters and one receiver*

From (3) the instantaneous SNR per symbol  $\lambda$  becomes,

$$\lambda = \frac{\Delta}{2} \left[ |h_0|^2 + |h_1|^2 \right], \quad \text{where } \Delta = \frac{\sigma_s^2}{\sigma_w^2}. \quad \text{The PDF of } \lambda \text{ becomes}$$

$$P_\lambda(\lambda) = \frac{4\lambda}{\Delta^2} \exp\left(-\frac{2\lambda}{\Delta}\right) \quad (22)$$

The average probability of error  $P_e^{2Tx-1Rx}$  for the system can be calculated as

$$P_e^{2T_s-1R_s} = \int_0^\infty P_e(\lambda) p_\lambda(\lambda) d\lambda \tag{23}$$

4.2.1. M-ary PSK modulation

The symbol-error probability  $P_e(\lambda)$  for M-ary PSK modulation can be given as

$$P_e(\lambda) = \frac{1}{\pi} \int_0^\beta \exp\left(\frac{-\lambda a}{\sin^2 \theta}\right) d\theta \tag{24}$$

where,  $a = \sin^2(\pi/M)$  and  $\beta = \pi - \pi/M$ . Substitution of (22) and (24) in (23) and by performing the integral the equation becomes

$$P_e^{2T_s-1R_s} = \frac{\beta}{\pi} + \frac{1}{\pi(b_1+2)} \left[ \frac{b_1 \sin \beta \cos \beta}{(b_1+2 \sin^2 \beta)} - (b_1+3) \sqrt{\frac{b_1}{b_1+2}} \tan^{-1} \left\{ \sqrt{\frac{b_1+2}{b_1}} \tan \beta \right\} \right] \tag{25}$$

4.2.2. M-QAM modulation

The symbol-error probability  $P_e(\lambda)$  for square M-QAM modulation can be given as

$$P_e(\lambda) = \frac{4q}{\pi} \int_0^{\pi/2} \exp\left(\frac{-\lambda g}{\sin^2 \theta}\right) d\theta - \frac{4q^2}{\pi} \int_0^{\pi/4} \exp\left(\frac{-\lambda g}{\sin^2 \theta}\right) d\theta \tag{26}$$

Let us define the function  $\psi_2(\Delta, a, \beta)$  as

$$\psi_2(\Delta, a, \beta) = \frac{1}{\pi \Delta^2} \int_0^\beta \int_0^\infty \exp\left[-\left(\frac{a}{\sin^2 \theta} + \frac{1}{\Delta}\right) \lambda\right] d\lambda d\theta$$

$\psi_2(\Delta, a, \beta)$  is given by the right-hand-side expression in (25). The average symbol-error probability for M-QAM modulation can be given as

$$P_e^{2T_s-1R_s} = 4q\psi_2(\Delta, g, \pi/2) - 4q^2\psi_2(\Delta, g, \pi/4) \tag{27}$$

where,

$$\psi_2(\Delta, g, \pi/2) = \frac{1}{2} \left( 1 - \frac{c+3}{c+2} \sqrt{\frac{c}{c+2}} \right) \tag{28}$$

$$\psi_2(\Delta, g, \pi/4) = \frac{1}{4} + \frac{1}{\pi(c+2)} \left\{ \frac{c}{2(c+1)} - (c+3) \sqrt{\frac{c}{c+1}} \tan^{-1} \left( \sqrt{\frac{c+2}{c}} \right) \right\} \tag{29}$$

and  $c = g\Delta$ .

4.3. One transmitter and two receivers

The PDF of instantaneous SNR per symbol  $\lambda_1$  becomes

$$P_{\lambda_1}(\lambda_1) = \frac{\lambda_1}{\Delta^2} \exp\left(\frac{-\lambda_1}{\Delta}\right) \tag{30}$$

The average probability of error  $P_e^{1T_s-2R_s}$  for the system can be calculated as

$$P_e^{1T_x-2R_x} = \int_0^\infty P_e(\lambda_1) p_{\lambda_1}(\lambda_1) d\lambda_1 \quad (31)$$

#### 4.3.1. *M*-ary PSK modulation

The symbol-error probability  $P_e(\lambda_1)$  for *M*-ary PSK modulation can be given as

$$P_e(\lambda_1) = \frac{1}{\pi} \int_0^\beta \exp\left(\frac{-\lambda_1 a}{\sin^2 \theta}\right) d\theta \quad (32)$$

Substitution of (30) and (32) in (31) gives and by performing the integral the equation becomes

$$P_e^{1T_x-2R_x} = \frac{\beta}{\pi} + \frac{1}{2\pi(b_1+1)} \left[ \frac{b_1 \sin \beta \cos \beta}{(b_1 + \sin^2 \beta)} - (2b_1 + 3) \sqrt{\frac{b_1}{b_1+1}} \tan^{-1} \left\{ \sqrt{\frac{b_1+1}{b_1}} \tan \beta \right\} \right] \quad (33)$$

#### 4.3.2. *M*-QAM modulation

The symbol-error probability  $P_e(\lambda_1)$  for square *M*-QAM modulation can be given as

$$P_e(\lambda_1) = \frac{4q}{\pi} \int_0^{\pi/2} \exp\left(\frac{-\lambda_1 g}{\sin^2 \theta}\right) d\theta - \frac{4q^2}{\pi} \int_0^{\pi/4} \exp\left(\frac{-\lambda_1 g}{\sin^2 \theta}\right) d\theta \quad (34)$$

$\psi_2(\Delta, a, \beta)$  is given by the right-hand-side expression in Eq. (33). The average symbol-error probability for *M*-QAM modulation can be given as

$$P_e^{2T_x-1R_x} = 4q\psi_2(\Delta, g, \pi/2) - 4q^2\psi_2(\Delta, g, \pi/4) \quad (35)$$

where,

$$\begin{aligned} \psi_2(\Delta, g, \pi/2) &= \frac{1}{2} \left( 1 - \frac{(2c+3)}{2(c+1)} \sqrt{\frac{c}{c+1}} \right) \\ \psi_2(\Delta, g, \pi/4) &= \frac{1}{4} + \frac{1}{8(c+1)} \left\{ \frac{c}{(2c+1)} - (2c+3) \sqrt{\frac{c}{c+1}} \tan^{-1} \left( \sqrt{\frac{c+1}{c}} \right) \right\} \end{aligned} \quad (36)$$

### 4.4. *One transmitter - four receivers*

From (12) the instantaneous SNR per symbol  $\varepsilon_1$  becomes,

$$\varepsilon_1 = \Delta \left[ |h_0|^2 + |h_1|^2 + |h_2|^2 + |h_3|^2 \right], \quad \text{where } \Delta = \frac{\sigma_x^2}{\sigma_w^2}. \quad \text{The PDF of } \varepsilon_1 \text{ becomes}$$

$$P_e(\varepsilon_1) = \frac{\varepsilon_1^3}{6\Delta^4} \exp\left(\frac{-\varepsilon_1}{\Delta}\right) \quad (37)$$

The average probability of error  $P_e^{1T_x-4R_x}$  for the system can be calculated as

$$P_e^{4T_x-1R_x} = \int_0^\infty P_e(\varepsilon_1) p_\varepsilon(\varepsilon_1) d\varepsilon_1 \tag{38}$$

4.4.1. M-ary PSK modulation

The symbol-error probability  $P_e(\varepsilon_1)$  for M-ary PSK modulation can be written as

$$P_e(\varepsilon_1) = \frac{1}{\pi} \int_0^\beta \exp\left(\frac{-\varepsilon_1 a}{\sin^2 \theta}\right) d\theta \tag{39}$$

Substitution of (37) and (39) in (38) and by performing the integral [13] the equation becomes

$$P_e^{4T_x-1R_x} = \frac{1}{\pi} \left[ \beta + \frac{1}{(b_1+1)^3} + \left\{ \frac{\left(\frac{29}{16}b_1 + \frac{13}{8}b_1^2 + \frac{1}{2}b_1^3\right) \sin \theta \cos \theta}{(b_1 + \sin^2 \theta)} + \frac{b_1^3 \tan \theta \cos^6 \theta}{6(b_1 + \sin^2 \theta)^3} \right\} + \left\{ \frac{\left(-\frac{19}{24}b_1^2 - \frac{1}{4}b_1^3\right) \tan \theta \cos^4 \theta}{(b_1 + \sin^2 \theta)^2} \right\} + \left(-\frac{35b_1}{16} - \frac{35b_1^2}{8} - \frac{7}{2}b_1^3 - b_1^4\right) \left\{ \frac{1}{\sqrt{b_1(b_1+1)}} \tan^{-1} \left( \sqrt{\frac{b_1+1}{b_1}} \tan \theta \right) \right\} \right] \tag{40}$$

4.4.2. M-QAM modulation

The symbol-error probability  $P_e(\varepsilon_1)$  for square M-QAM modulation can be given as

$$P_e(\varepsilon_1) = \frac{4q}{\pi} \int_0^{\pi/2} \exp\left(\frac{-\varepsilon_1 g}{\sin^2 \theta}\right) d\theta - \frac{4q^2}{\pi} \int_0^{\pi/4} \exp\left(\frac{-\varepsilon_1 g}{\sin^2 \theta}\right) d\theta \tag{41}$$

The average symbol-error probability  $P_e^{1T_x-4R_x}$  can be determined by evaluating the integral in Eq. (38), where  $p_{\varepsilon_1}(\varepsilon_1)$  is given by Eq.(37). From Eq. (40) we can write for

$$\beta = \frac{\pi}{2}, \quad P_{e,\beta=\frac{\pi}{2}}^{1T_x-4R_x} = \left[ \frac{1}{2} - \frac{\left(\frac{35c}{16} + \frac{35c^2}{8} + \frac{7}{2}c^3 + c^4\right)}{2(c+1)^3} \frac{1}{\sqrt{c(c+1)}} \right] \tag{42}$$

and for  $\beta = \frac{\pi}{4}$



$$P_{e, \beta = \frac{\pi}{4}}^{3T_x - 1R_x} = \frac{1}{\pi} \left[ \frac{\pi}{4} + \frac{1}{(c+1)^3} + \left\{ \frac{\left( \frac{29}{16}c + \frac{13}{8}c^2 + \frac{1}{8}c^3 \right)}{(2c+1)} \right\} + \frac{c^3}{6(2c+1)^3} \right. \\ \left. + \left\{ \frac{\left( -\frac{19}{24}c^2 - \frac{1}{4}c^3 \right)}{12(2c+1)^2} \right\} + \left( -\frac{35c}{16} - \frac{35c^2}{8} - \frac{7}{2}c^3 - c^4 \right) \right. \\ \left. + \left\{ \frac{1}{\sqrt{c(c+2)}} \tan^{-1} \left( \sqrt{\frac{c+2}{c}} \right) \right\} \right] \quad (43)$$

Using Eq. (42) and Eq. (43) the average symbol-error probability for M-QAM modulation can be given as

$$P_e^{1T_x - 4R_x} = 4q \times P_{e, \beta = \frac{\pi}{2}}^{1T_x - 4R_x} - 4q^2 P_{e, \beta = \frac{\pi}{4}}^{1T_x - 4R_x} \quad (44)$$

where  $c = \Delta g$  and  $g = \frac{3}{2}(M-1)$

#### 4.5. Two transmitters-two receivers

The PDF of instantaneous SNR per symbol  $\varepsilon_3$  becomes

$$P_e(\varepsilon_3) = \frac{8}{3\Delta^4} \varepsilon_3^3 \exp\left(\frac{-2\varepsilon_3}{\Delta}\right) \quad (45)$$

The average probability of error  $P_e^{2T_x - 2R_x}$  for the system can be calculate as

$$P_e^{2T_x - 2R_x} = \int_0^\infty P_e(\varepsilon_3) p_e(\varepsilon_3) d\varepsilon_3 \quad (46)$$

##### 4.5.1. M-ary PSK modulation

The symbol-error probability  $P_e(\varepsilon_3)$  for M-ary PSK modulation can be given as

$$P_e(\varepsilon_3) = \frac{1}{\pi} \int_0^\beta \exp\left(\frac{-\varepsilon_3 a}{\sin^2 \theta}\right) d\theta \quad (47)$$

Substitution of (45) and (47) in (46) and performing the integral [13] the equation becomes

$$P_e^{2T_x-2R_x} = \frac{1}{\pi} \left[ \beta + \frac{1}{(b_1+2)^3} \left[ \left\{ \frac{(29b_1+13b_1^2+2b^3) \sin \theta \cos \theta}{2(b_1+2 \sin^2 \theta)} \right\} + \frac{4b_1^3 \tan \theta \cos^6 \theta}{3(b_1+2 \sin^2 \theta)^3} \right. \right. \quad (48)$$

$$\left. \left. + \left\{ \frac{(-19b_1^2-3b_1^3) \tan \theta \cos^4 \theta}{3(b_1+2 \sin^2 \theta)^2} \right\} + \left( -\frac{35b_1}{2} - \frac{35b_1^2}{2} - 7b_1^3 - b_1^4 \right) \right] \right.$$

$$\left. \left[ \left\{ \frac{1}{\sqrt{b_1(b_1+2)}} \tan^{-1} \left( \sqrt{\frac{b_1+2}{b_1}} \tan \theta \right) \right\} \right] \right]$$

4.5.2. M-QAM modulation

The symbol-error probability  $P_e(\varepsilon_3)$  for square M-QAM modulation can be given as

$$P_e(\varepsilon_3) = \frac{4q}{\pi} \int_0^{\pi/2} \exp\left(\frac{-\varepsilon_3 g}{\sin^2 \theta}\right) d\theta - \frac{4q^2}{\pi} \int_0^{\pi/4} \exp\left(\frac{-\varepsilon_3 g}{\sin^2 \theta}\right) d\theta \quad (58)$$

The average symbol-error probability  $P_e^{2T_x-2R_x}$  can be determined by evaluating the integral in Eq. (46), where  $p_e(\varepsilon_3)$  is given by Eq. (47). From Eq. (48) we can write for

$$\beta = \frac{\pi}{2}, \quad P_{e,\beta=\frac{\pi}{2}}^{2T_x-2R_x} = \left[ \frac{1}{2} - \frac{\left(\frac{35b_1}{2} + \frac{35b_1^2}{2} + 7b_1^3 + b_1^4\right)}{2(b_1+2)^3} \frac{1}{\sqrt{b_1(b_1+2)}} \right] \quad (49)$$

and for  $\beta = \frac{\pi}{4}$

$$P_{e,\beta=\frac{\pi}{4}}^{2T_x-2R_x} = \frac{1}{\pi} \left[ \frac{\pi}{4} + \frac{1}{(c+2)^3} \left[ \left\{ \frac{(29b_1+13b_1^2+2b^3)}{4(c+1)} \right\} + \frac{c^3}{6(c+1)^3} \right. \right. \quad (50)$$

$$\left. \left. + \left\{ \frac{(-19b_1^2-3b_1^3)}{12(c+1)^2} \right\} + \left( -\frac{35c}{2} - \frac{35c}{2} - 7c - c \right) \right] \right.$$

$$\left. \left[ \left\{ \frac{1}{\sqrt{c(c+2)}} \tan^{-1} \left( \sqrt{\frac{c+2}{c}} \right) \right\} \right] \right]$$

Using Eq. (49) and Eq. (50) the average symbol-error probability for M-QAM modulation can be written as

$$P_e^{2T_x-2R_x} = 4q \times P_{e,\beta=\frac{\pi}{2}}^{2T_x-2R_x} - 4q^2 P_{e,\beta=\frac{\pi}{4}}^{2T_x-2R_x} \quad (51)$$

4. Performance Results

Following the analytical approach we compute the symbol error rate performance of wireless communication system using Alamouti scheme and MRRC scheme. The symbol

error rates using two transmitter-one receiver antenna and two transmitter-two receiver antenna for BPSK and QPSK modulation depicted in Figs. 1 and 2, respectively.

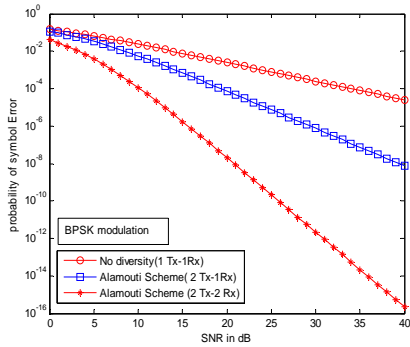


Fig. 1. Symbol Error probability of Alamouti scheme (2Tx-1Rx) and (2Tx-2Rx) for BPSK modulation in Rayleigh fading. (Analytical Derivation)

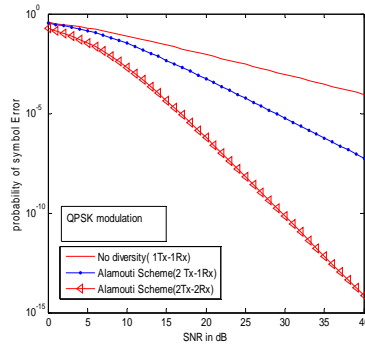


Fig. 2. Symbol Error probability of Alamouti Scheme (2Tx-1Rx) and (2Tx-2Rx) for QPSK modulation in Rayleigh fading (Analytical derivation).

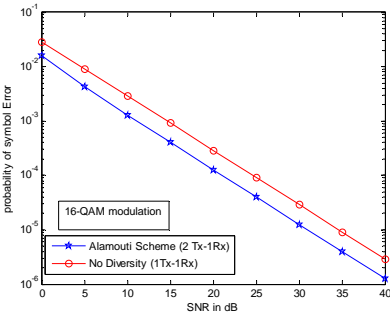


Fig. 3. Symbol error probability of Alamouti scheme (1Tx-1Rx) and (2Tx-1Rx) for 16-QAM modulation in Rayleigh fading (analytical derivation).

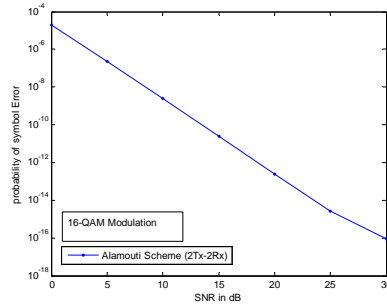


Fig. 4. Symbol error probability of Alamouti scheme (2Tx-2Rx) for 16-QAM modulation in Rayleigh fading (analytical derivation).

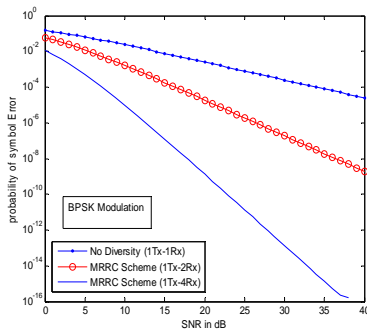


Fig. 5. Symbol error probability of MRRC scheme (1Tx-2Rx) and (1Tx-4Rx) for BPSK modulation in Rayleigh fading (Analytical derivation).

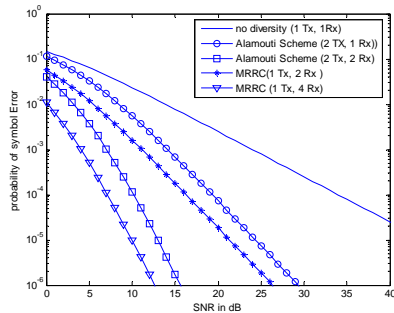


Fig. 6. Comparison of symbol error performance between maximum ratio receive combining scheme (MRRC) and Alamouti-STBC scheme for BPSK modulation in Rayleigh fading (analytical derivation).

It is evident from Fig. 1 and Fig. 2 that significant improvement of the SEP performance is achieved by applying diversity scheme and as the number of transmitting antenna increases probability of error decreases. The symbol error probability of Alamouti scheme using one and two transmit antenna with single receive antenna for 16-QAM modulation in Rayleigh fading is shown in Fig. 3. Fig. 4 shows the symbol error probability for two transmitter- two receiver for 16-QAM modulation. The symbol error probability of MRRC scheme using one transmitter-two receiver and one transmitter-four receiving antenna for BPSK modulation in Rayleigh fading shown in Fig. 5. The comparison between Alamouti scheme and MRRC scheme is shown in Fig. 6. The maximum ratio receive combining scheme (MRRC) technique that was used for Fig. 6 employed one transmit antenna and two receive antennas. It is evident from the Fig. 6 that adding another transmit-antenna and employing temporal diversity leads to a significant boost in symbol error performance. The Alamouti scheme gradually outperforms the MRRC technique as the SNR values increase. The MRRC technique outperforms the Alamouti scheme with just one receive antenna. The graph shows about a 3-dB penalty in error rate performance. Alamouti explains this penalty by noting a constraint that he imposes on his code; namely, he has each transmit antenna radiating half of the energy that the one antenna for the MRRC scheme radiates. The Alamouti scheme is practical because it allows a boost in performance through the addition of receiver antennas. The Monte Carlo simulation of symbol error rate performance of Alamouti coded system for BPSK modulation in Rayleigh fading for 2Tx-1Rx and 2Tx-2Rx are depicted in Figs. 7 and 8, respectively. It is seen from these figures that, Monte Carlo simulation showed an exact match with the theoretical results.

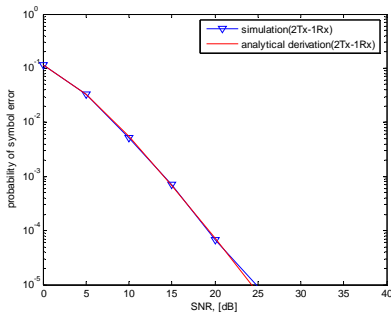


Fig. 7. Symbol error performance comparison of Alamouti scheme(2Tx-1Rx), monte carlo simulation result and analytical derivation for BPSK modulation in Rayleigh fading

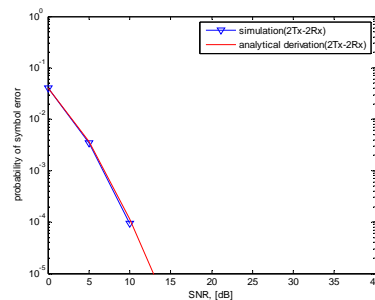


Fig. 8. Symbol error performance comparison of Alamouti scheme(2Tx-2Rx), monte carlo simulation result and analytical derivation for BPSK modulation in Rayleigh fading

## 5. Conclusion

We investigate the probability of error of single-input-single-output (SISO) system and Alamouti scheme using 2 Tx-1Rx, 2Tx-2Rx and MRRC scheme using 1Tx-2Rx, 1Tx-4Rx

in Rayleigh fading channels. A detailed analysis for finding the symbol error probability of the wireless communication systems in Rayleigh fading channels is provided. Exact closed-form expressions are derived for average symbol-error-probability (SEP) for both M-PSK and M-QAM modulation schemes. The numerical results show exact match with Monte Carlo simulation.

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