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Some Features of Intuitionistic L- R_1 Spaces

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Abstract

In this paper, we have introduced four notions of R_1 space in intuitionistic L-topological spaces and established some implications among them. We have also proved that all of these definitions satisfy "good extension" and "hereditary" property. Finally, it has been shown that all concepts are preserved under one-one, onto and continuous mapping.

Keywords: Intuitionistic L-fuzzy set; Intuitionistic L-fuzzy open set; Intuitionistic L-fuzzy point; Intuitionistic L-topology.

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1. Introduction

The idea of fuzzy sets and L-fuzzy sets were initially introduced by Zadeh [1] in 1965 and Goguen [2] in 1967 respectively. In 1984, Atanassove [3] defined the concept of intuitionistic fuzzy sets (which take into account both the degree of membership and non membership subject to the condition that their sum does not exceed 1) and many works by the same author and his colleagues appeared in the literature [4-6]. Later, this concept was generalized to 'intuitionistic L-fuzzy sets' by Atanassov and Stoeva [7]. Coker [8-10] first defines intuitionistic fuzzy topological spaces and some of its properties which is in the sense of C. L. Chang [11]. After then, many fuzzy topologists [12-18] work in separation axioms of fuzzy topological spaces and intuitionistic fuzzy topological spaces, especially Ahmed *et al.* [19] defines some types of R_1 spaces in intuitionistic fuzzy topological spaces and Islam *et al.* [20] defines some types of T_2 spaces in intuitionistic L- topological spaces. In this paper, we define some new notions of L-R_1 spaces using intuitionistic L-fuzzy sets and investigate the property of L-R_1 spaces.

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2. Notation and Preliminaries

Through this paper, X will be a nonempty set, $\emptyset$ be the empty set, L be a complete distributive lattice with 0 and 1. A, B, … be intuitionistic L-fuzzy sets, t be the intuitionistic topology, τ be the intuitionistic L-topology, $I =$ [0, 1], and the functions $\mu_A: X \to L$ and $\gamma_A: X \to L$ denote the degree of membership (namely $\mu_A(x)$) and the degree of none membership (namely $\gamma_A(x)$).

Now we recall some basic definitions and known results in intuitionistic L-fuzzy sets and intuitionistic L-topological spaces.

Definition 2.1. [1] Let X be a non-empty set and $I =$ [0, 1]. A fuzzy set in X is a function $u: X \to I$ which assigns to each element $x \,\varepsilon\, X$, a degree of membership $u(x) \,\varepsilon\, I$.

Definition 2.2. [21] Let $f: X \to Y$ be a function and u be fuzzy set in X. Then the image $f(u)$ is a fuzzy set in Y which membership function is defined by
$(f(u))(y) = \{sup\,(u\,(x)) | f(x) = y\}$ if $f^{-1}(y) \neq \emptyset, x \in X$
$(f(u))(y) = 0$ if $f^{-1}\,(y) = \emptyset, x \in X$.

Definition 2.3. [12] Let P be a property of a topological space and FP its fuzzy topological analogue. Then FP is called a '*good extension'* of P if and only if the statement "(X, T) has P if and only if $((X, \omega(T))$ has FP" holds good for every topological space(X, T).

Definition 2.4. [2] Let X be a non-empty set and L be a complete distributivelattice with 0 and 1. An L-fuzzy set in X is a function $\alpha: X \to L$ which assigns to each element $x \in X$, a degree of membership, $\alpha(x) \in L$.

Remark 2.5. [20] Throughout this paper we consider the complete distributive lattice $L = \{0, 0.1, 0.2, \dots, 1\}$ and from above definitions we show that every L-fuzzy set is also a fuzzy set but converse is not true in general.

Example 2.5.1. [20] Let $X = \{a, b, c\}$ and $L = \{0, 0.1, 0.2, \dots, 1\}$. A function $\alpha: X \to L$ is defined by $\alpha(a) = 0.2, \alpha(b) = 0.5, \alpha(c) = 0$ which is L-fuzzy set and also a fuzzy set.

Example 2.5.2. [20] Let $X = \{a, b, c\}$ and $I = [0, 1]$. A function $u: X \to I$ is defined by $u(a) = 0.25, u(b) = 0.55, u(c) = 0$ which is fuzzy set but not an L-fuzzy set because $0.25, 0.55 \notin L$.

Definition 2.6. [7] Let X be a non-empty set and L be a complete distributive lattice with 0 and 1. An intuitionistic L-fuzzy set (ILFS for short) A in X is an object having the form $A = \{(x, \mu_A(x), \gamma_A(x)): x \in X\}$. Where the functions $\mu_A: X \to L$ and $\gamma_A: X \to L$ denote the degree of membership (namely $\mu_A(x)$) and the degree of none membership (namely

$\gamma_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \le \mu_A(x) + \gamma_A(x) \le 1$ for each $x \in X$.

Let $L(X)$ denote the set of all intuitionistic L-fuzzy set in X. Obviously every L-fuzzy set $\mu_A(x)$ in X is an intuitionistic L-fuzzy set of the form $(\mu_A, 1 - \mu_A)$.
Throughout this paper we use the simpler notation $A = (\mu_A, \gamma_A)$ instead of $A = \{(x, \mu_A(x), \gamma_A(x)): x \in X\}$.

Definition 2.7. [7] Let $A = (\mu_A, \gamma_A)$ and $B = (\mu_B, \gamma_B)$ be intuitionistic L-fuzzy sets in X. Then

(1) $A \subseteq B$ if and only if $\mu_A \le \mu_B$ and $\gamma_A \ge \gamma_B$
(2) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$
(3) $A^c = (\gamma_A, \mu_A)$
(4) $A \cap B = (\mu_A \cap \mu_B; \gamma_A \cup \gamma_B)$
(5) $A \cup B = (\mu_A \cup \mu_B; \gamma_A \cap \gamma_B)$
(6) $0_\sim = (0^\sim, 1^\sim)$ and $1_\sim = (1^\sim, 0^\sim)$.

Let f be a map from a set X to a set Y. Let $A = (\mu_A, \gamma_A)$ be an ILFS of X and $B = (\mu_B, \gamma_B)$ be an ILFS of Y. Then $f^{-1}(B)$ is an ILFS of X defined by $f^{-1}(B) = (f^{-1}(\mu_B), f^{-1}(\gamma_B))$ and $f(A)$is an ILFS of Y defined by $f(A) = \big(f(\mu_A), 1 - f(1 - \gamma_A)\big)$.

Definition 2.8. [10] An intuitionistic topology (IT for short) on a nonempty set X is a family t of IS's in X satisfies the following axioms:

(i) $\emptyset_\sim, X_\sim \in t$.
(ii) If $G_1, G_2 \in t$ then $G_1 \cap G_2 \in t$.
(iii) If $G_i \in t$ for each $i \in \Lambda$ then $\cup_{i \in \Lambda} G_i \in t$.

Then the pair (X, t) is called an intuitionistic topological space (ITS for short) and the members of t are called intuitionistic open sets (IOS for short).

Definition 2.9. [19] An ITS (X, t) is called intuitionistic R_1-space (I-R_1 space) if for all $x, y \in X, x \ne y$ whenever $\exists\ C = (C_1, C_2) \in t$ with $(x \in C_1, y \in C_2)$ or $(y \in C_1, x \in C_2)$ then $\exists\ A = (A_1, A_2), B = (B_1, B_2) \in t$ such that $x \in A_1, x \notin A_2;\ y \in B_1, y \notin B_2$ and $A \cap B = \emptyset_\sim$.

Theorem 2.10. [19] Let (X, t) be an intuitionistic topological space and let $T = \{1_A: A \in T\}, 1_{(A_1,\ A_2)} = \big(1_{A_1}, 1_{A_2}\big)$, then (X, T) is the corresponding intuitionistic fuzzy topological space of $(X,\ t)$.

Definition 2.11. [20] Let $p, q \in L = \{0, 0.1, 0.2, \ldots, 1\}$ and $p + q \le 1$. An intuitionistic L-fuzzy point (ILFP for short) $x_{(p,q)}$of X is an ILFS of X defined by

$$x_{(p,q)}(y) = \begin{cases} (p,q)\ if\ y = x, \\ (0,1) if\ y \ne x \end{cases}$$

In this case, x is called the support of $x_{(p,q)}$ and p and q are called the value and none value of $x_{(p,q)}$, respectively. The set of all ILFP of X we denoted it by $S(X)$.
An ILFP $x_{(p,q)}$ is said to belong to an ILFS $A = (\mu_A, \gamma_A)$ of X denoted by $x_{(p,q)} \in A$, if and only if $p \leq \mu_A(x)$ and $q \geq \gamma_A(x)$ but $x_{(p,q)} \notin A$ if and only if $p \geq \mu_A(x)$ and $q \leq \gamma_A(x)$.

Definition 2.12. [20] If A is an ILFS and $x_{(p,q)}$ is an ILFP then the intersection between ILFS and ILFP is defined as $x_{(p,q)} \cap A = \left(p \cap \mu_A(x); q \cup \gamma_A(x)\right)$.

Definition 2.13. [20] An intuitionistic L-topology (ILT for short) on X is a family τ of ILFSs in X which satisfies the following conditions:

(i) $0_\sim, 1_\sim \in \tau$.

(ii) If $A_1, A_2 \in \tau$ then $A_1 \cap A_2 \in \tau$.

(iii) If $A_i \in \tau$ for each $i \in \Lambda$ then $\cup_{i\in\Lambda} A_i \in \tau$.

Then the pair (X, τ) is called an intuitionistic L-topological space (ILTS for short) and the members of τ are called intuitionistic L-fuzzy open sets (ILFOS for short). An intuitionistic L-fuzzy set B is called an intuitionistic L-fuzzy closed set (ILFC for short) if $1 - B \in \tau$.

Definition 2.14. [20] Let (X, τ) and (Y, s) be two ILTSs. Then a map $f: X \to Y$ is said to be

(i) Continuous if $f^{-1}(B)$ is an ILFOS of X for each ILFOS B of Y, or equivalently, $f^{-1}(B)$ is an ILFCS of X for each ILFCS B of Y,

(ii) Open if $f(A)$ is an ILFOS of Y for each ILFOS A of X,

(iii) Closed if $f(A)$ is an ILFCS of Y for each ILFCS A of X,

(iv) A homeomorphism if f is bijective, continuous and open.

3. Definition and Properties of Intuitionistic Lattice Fuzzy R_1 Spaces

In this section, we give four definitions and establish two theorems of R_1 spaces in intuitionistic L-topological spaces.

Definition 3.1. An ILTS (X, τ) is called

(a) $IL - R_1(i)$ if for all $x, y \in X, x \neq y$ whenever $\exists\ C = (\mu_C, \gamma_C) \in \tau$ with $\mu_C(x) \neq \mu_C(y), \gamma_C(x) \neq \gamma_C(y)$ then $\exists\ A = (\mu_A, \gamma_A), B = (\mu_B, \gamma_B) \in \tau$ such that $\mu_A(x) = 1 = \mu_B(y)$ and $A \cap B = 0_\sim$.

(b) $IL - R_1(ii)$ if for any pair of distinct ILFP $x_{(p,q)}, y_{(r,s)} \in S(X)$ whenever $\exists\ C = (\mu_C, \gamma_C) \in \tau$ with $\mu_C(x) \neq \mu_C(y), \gamma_C(x) \neq \gamma_C(y)$then $\exists A = (\mu_A, \gamma_A), B = (\mu_B, \gamma_B) \in \tau$ such that $x_{(p,q)} \in A, y_{(r,s)} \in B$ and $A \cap B = (0_\sim, \alpha_\sim)$ where $\alpha \in L - \{0\}$.

(c) $IL - R_1(iii)$ if for all $x, y \in X, x \neq y$ whenever $\exists\ C = (\mu_C, \gamma_C) \in \tau$ with $\mu_C(x) \neq \mu_C(y), \gamma_C(x) \neq \gamma_C(y)$then $\exists\ A = (\mu_A, \gamma_A), B = (\mu_B, \gamma_B) \in \tau$ such that $\mu_A(x) > 0, \mu_B(y) > 0$ and $A \cap B = 0_\sim$.

(d) $IL - R_1(iv)$ if for all $x, y \in X, x \neq y$ whenever $\exists\ C = (\mu_C, \gamma_C) \in \tau$ with $\mu_C(x) \neq \mu_C(y), \gamma_C(x) \neq \gamma_C(y)$then $\exists\ A = (\mu_A, \gamma_A), B = (\mu_B, \gamma_B) \in \tau$ such that $\mu_A(x) = 1 = \mu_B(y), \gamma_A(x) = 0 = \gamma_B(y)$ and $A \subseteq B^c$ where B^c is the complement of B.

Theorem 3.2. Let (X, τ) be an ILTS. Then we have the following implications:

$$\begin{tikzcd}
& IL - R_1(iii) & \\
IL - R_1(i) \arrow[ur] \arrow[dr] & & IL - R_1(ii) \arrow[ul] \arrow[dl] \\
& IL - R_1(iv) \arrow[uu] &
\end{tikzcd}$$

Fig. 1. Implications among the $IL - R_1$ properties.

Proof: $\boldsymbol{IL - R_1(i) \Longrightarrow IL - R_1(iii)}$**and** $\boldsymbol{IL - R_1(i) \Longrightarrow IL - R_1(iv)}$: Suppose (X, τ) is an $IL - R_1(i)$. Then we have by definition, for all $x, y \in X, x \neq y$ whenever $\exists\, C = (\mu_C, \gamma_C) \in \tau$ with $\mu_C(x) \neq \mu_C(y), \gamma_C(x) \neq \gamma_C(y)$ then $\exists\, A = (\mu_A, \gamma_A), B = (\mu_B, \gamma_B) \in \tau$ such that $\mu_A(x) = 1 = \mu_B(y)$ and $A \cap B = 0_\sim$.

$$(1) \qquad \begin{cases} \text{whenever } \exists\, C = (\mu_C, \gamma_C) \in \tau \text{ with } \mu_C(x) \neq \mu_C(y), \gamma_C(x) \neq \gamma_C(y) \\ \text{then } \exists\, A = (\mu_A, \gamma_A), B = (\mu_B, \gamma_B) \in \tau \text{ such that} \\ \mu_A(x) > 0, \mu_B(y) > 0 \text{ and } A \cap B = 0_\sim. \end{cases}$$

$$(2) \qquad \begin{cases} \text{whenever } \exists\, C = (\mu_C, \gamma_C) \in \tau \text{ with } \mu_C(x) \neq \mu_C(y), \gamma_C(x) \neq \gamma_C(y) \\ \text{then } \exists\, A = (\mu_A, \gamma_A), B = (\mu_B, \gamma_B) \in \tau \text{ such that} \\ \mu_A(x) = 1 = \mu_B(y); \gamma_A(x) = 0 = \gamma_B(y) \text{ and } A \subseteq B^c. \end{cases}$$

From (1) and (2) we see that $IL - R_1(i) \Longrightarrow IL - R_1(iii)$and $IL - R_1(i) \Longrightarrow IL - R_1(iv)$.

$\boldsymbol{IL - R_1(ii) \Longrightarrow IL - R_1(iii)}$ **and** $\boldsymbol{IL - R_1(ii) \Longrightarrow IL - R_1(iv)}$: Suppose $(X, \tau)$is an $IL - R_1(ii)$. Then we have by definition, if for any pair of distinct ILFP $x_{(p,q)}$, $y_{(r,s)} \in S(X)$ whenever $\exists\, C = (\mu_C, \gamma_C) \in \tau$ with $\mu_C(x) \neq \mu_C(y), \gamma_C(x) \neq \gamma_C(y)$then $\exists A = (\mu_A, \gamma_A), B = (\mu_B, \gamma_B) \in \tau$ such that $x_{(p,q)} \in A, y_{(r,s)} \in B$ and $A \cap B = (0_\sim, \alpha_\sim)$ where $\alpha \in L - \{0\}$.

$$(3) \qquad \begin{cases} \text{whenever } \exists\, C = (\mu_C, \gamma_C) \in \tau \text{ with } \mu_C(x) \neq \mu_C(y), \gamma_C(x) \neq \gamma_C(y \\ \text{then } \exists\, A = (\mu_A, \gamma_A), B = (\mu_B, \gamma_B) \in \tau \text{ such that} \\ p \leq \mu_A(x), q \geq \gamma_A(x); r \leq \mu_B(y), s \geq \gamma_B(y) \text{ and } A \cap B = (0_\sim, \alpha_\sim). \end{cases}$$

Since $p, q, r, s \in L = \{0, 0.1, 0.2, \ldots, 1\}$ and $\alpha \in L - \{0\}$, we have from (3)

$$(4) \qquad \begin{cases} \text{whenever } \exists\, C = (\mu_C, \gamma_C) \in \tau \text{ with } \mu_C(x) \neq \mu_C(y), \gamma_C(x) \neq \gamma_C(y \\ \text{then } \exists\, A = (\mu_A, \gamma_A), B = (\mu_B, \gamma_B) \in \tau \text{ such that} \\ \mu_A(x) > 0, \mu_B(y) > 0 \text{ and } A \cap B = 0_\sim. \end{cases}$$

$$(5) \qquad \begin{cases} \text{whenever } \exists\, C = (\mu_C, \gamma_C) \in \tau \text{ with } \mu_C(x) \neq \mu_C(y), \gamma_C(x) \neq \gamma_C(y \\ \text{then } \exists\, A = (\mu_A, \gamma_A), B = (\mu_B, \gamma_B) \in \tau \text{ such that} \\ \mu_A(x) = 1 = \mu_B(y); \gamma_A(x) = 0 = \gamma_B(y) \text{ and } A \subseteq B^c. \end{cases}$$

From (4) and (5) which shows that $IL - R_1(ii) \Longrightarrow IL - R_1(iii)$ and $IL - R_1(ii) \Longrightarrow IL - R_1(iv)$.

$\boldsymbol{IL - R_1(iv) \Longrightarrow IL - R_1(iii)}$: Suppose (X, τ) is an $IL - R_1(iv)$. Then we have by definition, if for all $x, y \in X, x \neq y$ whenever $\exists\, C = (\mu_C, \gamma_C) \in \tau$ with $\mu_C(x) \neq \mu_C(y), \gamma_C(x) \neq \gamma_C(y)$then $\exists\, A = (\mu_A, \gamma_A), B = (\mu_B, \gamma_B) \in \tau$ such that $\mu_A(x) = 1 = \mu_B(y), \gamma_A(x) = 0 = \gamma_B(y)$ and $A \subseteq B^c$ where B^c is the complement of B.

$$(6) \qquad \begin{cases} \text{whenever } \exists\, C = (\mu_C, \gamma_C) \in \tau \text{ with } \mu_C(x) \neq \mu_C(y), \gamma_C(x) \neq \gamma_C(y \\ \text{then } \exists\, A = (\mu_A, \gamma_A), B = (\mu_B, \gamma_B) \in \tau \text{ such that} \\ \mu_A(x) > 0, \mu_B(y) > 0 \text{ and } A \cap B = 0_{\sim}. \end{cases}$$

This is $IL - R_1(iii) \Longrightarrow IL - R_1(iv)$.

None of the reverse implications is true in general which can be seen from the following counter examples:

Example 3.2.1. Let $X = \{x, y\}$, $L = \{0, 0.1, 0.2, \dots, 1\}$ and τ be an ILT on X generated by $\{A, B, C\}$ where $A = \{(x, 0.5, 0), (y, 0, 0.5)\}, B = \{(x, 0, 0.4), (y, 0.4, 0)\}$ and $C = \{(x, 0, 0.3), (y, 0.3, 0)\}$. Hence we see that (X, τ) is an $IL - R_1(iii)$ but not $IL - R_1(i)$ and $IL - R_1(ii)$.

Example 3.2.2. Let $X = \{x, y\}, L = \{0, 0.1, 0.2, \dots, 1\}$ and τ be an ILT on X generated by $\{A, B, C\}$ where $A = \{(x, 1, 0), (y, 0, 1)\}, B = \{(x, 0, 1), (y, 1, 0)\}$ and $C = \{(x, 0, 0.3), (y, 0.3, 0)\}$. Hence we see that (X, τ) is an $IL - R_1(iv)$ but not $IL - R_1(i)$ and $IL - R_1(ii)$.

Example 3.2.3. Let $X = \{x, y\}, L = \{0, 0.1, 0.2, \dots, 1\}$ and τ be an ILT on X generated by $\{A, B, C\}$ where $A = \{(x, 0.4, 0), (y, 0, 0.3)\}, B = \{(x, 0, 0.5), (y, 0.6, 0)\}$ and $C = \{(x, 0, 0.3), (y, 0.3, 0)\}$. Hence we see that (X, τ) is an $IL - R_1(iii)$ but not $IL - R_1(iv)$.

Theorem 3.3. Let (X, τ) be an ILTS and (X, t) be an ITS. Then we have the following implications:

$$\begin{tikzcd} & \boldsymbol{IL - R_1(iii)} & \\ \boldsymbol{IL - R_1(i)} \arrow[r, leftrightarrow] & \boldsymbol{I - R_1} \arrow[u] \arrow[r] \arrow[d] & \boldsymbol{IL - R_1(ii)} \\ & \boldsymbol{IL - R_1(iv)} & \end{tikzcd}$$

Fig. 2. Implications among the $I - R_1$ and $IL - R_1$ spaces.

Proof: Suppose (X, t) is $I - R_1$. We shall prove that (X, τ) is $IL - R_1(i)$. Let $x, y \in X, x \neq y$. Since (X, t) is $I - R_1$,whenever $\exists\, C = (C_1, C_2) \in t$ with $x \in C_1, y \in C_2$ or $x \in C_2, y \in C_1$ then $\exists\, A = (A_1, A_2), B = (B_1, B_2) \in t$ such that $x \in A_1 x \notin A_2, y \in B_1, y \notin B_2$and $A \cap B = \emptyset_{\sim}$. Implies that $1_{A_1}(x) = 1, 1_{A_2}(x) = 0; 1_{B_1}(y) = 1, 1_{B_2}(y) = 0$and $A \cap B = 0_{\sim}$. Let $1_{A_1} = \mu_A, 1_{A_2} = \gamma_A$ and $1_{B_1} = \mu_B, 1_{B_2} = \gamma_B$. Then $\mu_A(x) = 1, \gamma_A(x) = 0; \mu_B(y) = 1, \gamma_B(y) = 0$ and $A \cap B = 0_{\sim}$. Again since $x \in C_1, y \in C_2$ or $x \in C_2, y \in C_1$,we have $1_{C_1}(x) = 1, 1_{C_2}(x) = 0; 1_{C_1}(y) = 0, 1_{C_2}(y) = 1$. Let $1_{C_1} = \mu_C, 1_{C_2} = \gamma_C$. Therefore $\mu_C(x) = 1, \gamma_C(x) = 0; \mu_C(y) = 0, \gamma_C(y) = 1$. Therefore whenever $\exists\, C = (\mu_C, \gamma_C) \in \tau$ with $\mu_C(x) \neq \mu_C(y), \gamma_C(x) \neq \gamma_C(y)$ then $\exists\, A = (\mu_A, \gamma_A), B = (\mu_B, \gamma_B) \in \tau$ such that $\mu_A(x) = 1 = \mu_B(y)$ and $A \cap B = 0_{\sim}$which is $IL - R_1(i)$.

Conversely suppose that (X, τ) is $IL - R_1(i)$. We prove that (X, t) is $I - R_1$. Since (X, τ) is $IL - R_1(i)$, we have by definition, for all $x, y \in X, x \neq y$, whenever $\exists\, C = (\mu_C, \gamma_C) \in \tau$ with $\mu_C(x) \neq \mu_C(y), \gamma_C(x) \neq \gamma_C(y)$ then $\exists\, A = (\mu_A, \gamma_A), B = (\mu_B, \gamma_B) \in \tau$

such that $\mu_A(x) = 1 = \mu_B(y)$ and $A \cap B = 0_\sim$. Let $C_1 = \mu_C^{-1}\{1\}, C_2 = \gamma_C^{-1}\{1\}$. Then $x \in C_1, y \in C_1$ or $x \in C_2, y \in C_1$. Again let $A_1 = \mu_A^{-1}\{1\}, A_2 = \gamma_A^{-1}\{1\}$ and $B_1 = \mu_B^{-1}\{1\}, B_2 = \gamma_B^{-1}\{1\}$. Then we have $x \in A_1, x \notin A_2, y \in B_1, y \notin B_2$. Hence we have whenever $\exists\, C = (C_1, C_2) \in t$ with $x \in C_1, y \in C_2$ or $x \in C_2, y \in C_1$ then $\exists\, A = (A_1, A_2), B = (B_1, B_2) \in t$ such that $x \in A_1, x \notin A_2, y \in B_1, y \notin B_2$ and $A \cap B = \emptyset_\sim$which is $I - R_1$. Therefore $I - R_1. \Leftrightarrow IL - R_1(i)$. Furthermore it can be shown that $I - R_1 \Longrightarrow IL - R_1(ii), I - R_1 \Longrightarrow IL - R_1(iii)$and $I - R_1 \Longrightarrow IL - R_1(iv)$.

None of the reverse implications is true in general which can be seen from the following counter examples:

Example 3.3.1. Let $X = \{x, y\}, L = \{0, 0.1, 0.2, \ldots, 1\}, x_{(0.3,0.4)}, y_{(0.3,0.4)} \in S(X)$ and τ be an ILT on X generated by $\{A, B, C\}$ where $A = \{(x, 0.5, 0), (y, 0, 0.5)\}, B = \{(x, 0, 0.5), (y, 0.5, 0)\}$ and $C = \{(x, 0, 0.3), (y, 0.3, 0)\}$. Hence we see that (X, τ) is an $IL - R_1(ii)$ but not $I - R_1$.

Example 3.3.2. Let $X = \{x, y\}, L = \{0, 0.1, 0.2, \ldots, 1\}$ and τ be an ILT on X generated by $\{A, B, C\}$ where $A = \{(x, 0.5, 0), (y, 0, 1)\}, B = \{(x, 0, 1), (y, 0.4, 0)\}$ and $C = \{(x, 0, 0.3), (y, 0.3, 0)\}$. Hence we see that (X, τ) is an $IL - R_1(iii)$ but not $I - R_1$.

Example 3.3.3. Let $X = \{x, y\}, L = \{0, 0.1, 0.2, \ldots, 1\}$ and τ be an ILT on X generated by $\{A, B, C\}$ where $A = \{(x, 1, 0), (y, 0, 1)\}, B = \{(x, 0, 1), (y, 1, 0)\}$ and $C = \{(x, 0, 0.3), (y, 0.3, 0)\}$. Hence we see that (X, τ) is an $IL - R_1(iv)$ but not $I - R_1$.

4. Hereditary Properties of ILF –R_1(j) Concepts, Where (j = i, ii, iii, iv.)

In this section, we define subspace and mapping in intuitionistic L-R_1 spaces and some of their related theorem.

Definition 4.1. Let (X, τ) be an ILTS and $A \subseteq X$. we define $\tau_A = \{u|A : u \in \tau\}$ the subspace ILT's on A induced by τ. Then (A, τ_A) is called the subspace of (X, τ) with the underlying set A. An IL-topological property 'P' is called hereditary if each subspace of an IL-topological space with property 'P' also has property 'P'.

Theorem 4.2. Let (X, τ) be an ILTS, $U \subseteq X$ and $\tau_U = \{A|U : A \in \tau\}$. Then

(a) (X, τ) is $IL - R_1(i) \Longrightarrow (U, \tau_U)$ is $IL - R_1(i)$.
(b) (X, τ) is $IL - R_1(ii) \Longrightarrow (U, \tau_U)$ is $IL - R_1(ii)$.
(c) (X, τ) is $IL - R_1(iii) \Longrightarrow (U, \tau_U)$ is $IL - R_1(iii)$.
(d) (X, τ) is $IL - R_1(iv) \Longrightarrow (U, \tau_U)$ is $IL - R_1(iv)$.

Proof: We prove only (a). Suppose (X, τ) is $IL - R_1(i)$, we prove that (U, τ_U) is $IL - R_1(i)$. Let $x, y \in U, x \neq y$. Then $x, y \in X, x \neq y$ as $U \subseteq X$. Since (X, τ) is $IL - R_1(i)$, we have for all $x, y \in X, x \neq y$, whenever $\exists\, C = (\mu_C, \gamma_C) \in \tau$ with $\mu_C(x) \neq \mu_C(y), \gamma_C(x) \neq \gamma_C(y)$ then $\exists\, A = (\mu_A, \gamma_A), B = (\mu_B, \gamma_B) \in \tau$ such that $\mu_A(x) = 1 = \mu_B(y)$ and $A \cap B =$

$0_{\sim}$. For $U \subseteq X$, we find whenever $\exists\, C|U = (\mu_{C|U}, \gamma_{C|U}) \in \tau|U$ with $\mu_{C|U}(x) \neq \mu_{C|U}(y), \gamma_{C|U}(x) \neq \gamma_{C|U}(y)$then $\exists\, A|U = (\mu_{A|U}, \gamma_{A|U}), B|U = (\mu_{B|U}, \gamma_{B|U}) \in \tau_U$ such that $\mu_{A|U}(x) = 1 = \mu_{B|U}(y)$ and $A|U \cap B|U = (A \cap B)|U = 0_{\sim}\ as\ A \cap B = 0_{\sim}$. Hence (U, τ_U) is $IL - R_1(i)$. Similarly (b), (c), (d) can be proved.

We observe here that ILF-R_1(j), (j = i, ii, iii, iv) concepts are preserved under continuous, one-one and open maps**.**

Theorem 4.3. Let (X, τ) and (Y, s) be two ILTS, $f: (X, \tau) \to (Y, s)$ be one-one, onto and continuous map. Then

(a) (X, τ) is $IL - R_1(i) \Leftrightarrow (Y, s)$ is $IL - R_1(i)$
(b) (X, τ) is $IL - R_1(ii) \Leftrightarrow (Y, s)$ is $IL - R_1(ii)$
(c) (X, τ) is $IL - R_1(iii) \Leftrightarrow (Y, s)$ is $IL - R_1(iii)$
(d) (X, τ) is $IL - R_1(iv) \Leftrightarrow (Y, s)$ is $IL - R_1(iv)$

Proof: We prove only (a). Suppose (X, τ) is $IL - R_1(i)$, we prove that (Y, s) is $IL - R_1(i)$. Let $y_1, y_2 \in Y$ with $y_1 \neq y_2$. Since f is onto, $\exists\, x_1, x_2 \in X$, such that $f(x_1) = y_1, f(x_2) = y_2$ and $f(x_1) \neq f(x_2)$ as f is one-one. Also,(X, τ) is $IL - R_1(i)$, we have for all $x_1, x_2 \in X, x_1 \neq x_2$ whenever $\exists\, C = (\mu_C, \gamma_C) \in \tau$ with $\mu_C(x_1) \neq \mu_C(x_2), \gamma_C(x_1) \neq \gamma_C(x_2)$ then $\exists\, A = (\mu_A, \gamma_A), B = (\mu_B, \gamma_B) \in \tau$ such that $\mu_A(x_1) = 1 = \mu_B(x_2)$ and $A \cap B = 0_{\sim}$. Now we have if $\exists$ $f(C) = (f(\mu_C), 1 - f(1 - \gamma_C)) \in s$ with$f(\mu_C)(x_1) \neq f(\mu_C)(x_2)$ and$1 - f(1 - \gamma_C)(x_1) \neq 1 - f(1 - \gamma_C)(x_2)$ then $\exists\, f(A) = (f(\mu_A), 1 - f(1 - \gamma_A)), f(B) = (f(\mu_B), 1 - f(1 - \gamma_B)) \in s$ such that $f(\mu_A)(y_1) = \{\sup \mu_A(x_1): f(x_1) = y_1\} = 1, f(\mu_B)(y_2) = \{\sup \mu_B(x_2): f(x_2) = y_2\} = 1$ and $f(A) \cap f(B) = f(A \cap B) = 0_{\sim}$as $A \cap B = 0_{\sim}$. Hence (Y, s) is $IL - R_1(i)$.

Conversely suppose that (Y, s) is $IL - R_1(i)$. We proved that (X, τ) is $IL - R_1(i)$. Let $x_1, x_2 \in X$ with $x_1 \neq x_2 \Longrightarrow f(x_1) \neq f(x_2)$ as f is one-one. Put $f(x_1) = y_1$ and $f(x_2) = y_2$ then $y_1 \neq y_2$. Since (Y, s) is $IL - R_1(i)$, we have whenever $\exists\, C = (\mu_C, \gamma_C) \in s$ with $\mu_C(y_1) \neq \mu_C(y_2)$; $\gamma_C(y_1) \neq \gamma_C(y_2)$ then $\exists\, A = (\mu_A, \gamma_A), B = (\mu_B, \gamma_B) \in s$ such that $\mu_A(y_1) = 1 = \mu_B(y_2)$ and $A \cap B = 0_{\sim}$. Since $f: (X, \tau) \to (Y, s)$, we have $f^{-1}(C) = (f^{-1}(\mu_C), f^{-1}(\gamma_C)) \in \tau$ with $f^{-1}\mu_C(x_1) \neq f^{-1}\mu_C(x_2)$; $f^{-1}\gamma_C(x_1) \neq f^{-1}\gamma_C(x_2)$ then $\exists$ $f^{-1}(A) = (f^{-1}(\mu_A), f^{-1}(\gamma_A)), f^{-1}(B) = (f^{-1}(\mu_B), f^{-1}(\gamma_B)) \in \tau$ such that $f^{-1}\mu_A(x_1) = \mu_A f(x_1) = \mu_A(y_1) = 1, f^{-1}\mu_B(x_2) = \mu_B f(x_2) = \mu_B(y_2) = 1$ and $f^{-1}(A) \cap f^{-1}(B) = f^{-1}(A \cap B) = 0_{\sim}$ as $A \cap B = 0_{\sim}$. Hence (X, τ) is also $IL - R_1(i)$. Similarly, (b), (c) and (d) can be proved.

5. Conclusion

In this paper, our notions *IL - R_1(j), j* = i, ii, iii, iv are satisfied "good extension" property, so defined notions are well-defined. Again we showed that our notions fulfilled "hereditary" property. Further it is clear that all notions are preserved under one-one, onto and continuous mapping, so these notions are topological property.

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