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Short Communication

The Semi-splitting Block Graph of a Graph

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Abstract

We present here characterizations of graphs whose semi-splitting block graphs are planar, outer planar. Also we characterize graphs whose semi-splitting block graphs are planar and outer planar in terms of forbidden sub graphs.

Keywords: Semi-splitting; Block; Planar; Outer planar; Forbidden.

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1. Introduction

All graphs considered here are finite, undirected and without loops or multiple lines. We use the terminology of [1]. The open-neighbourhood $N(u)$ of a point u in $V(G)$ is the set of points adjacent to u. $N(u) = [v/uv \in E(G)]$.

For each point v_i of G, we take a new point u_i and the resulting set of points is denoted by $V_1(G)$.

We introduce the concept of the semi-splitting block graph $S_B(G)$ of a graph G. The semi-splitting block graph $S_B(G)$ of a graph G is defined as the graph having point set $V(G) \cup V_1(G) \cup b(G)$ with two points are adjacent if they correspond to a adjacent points of G or one corresponds to a point v_i of $V_1(G)$ and the other to a point w_j of G and w_j is in $N(v_i)$ or one corresponds to a point u_i of $V(G)$ and the other to a point b_i of $b(G)$, where $b(G)$ is the set of blocks of G.

The splitting graph $S(G)$ of a graph G is defined as the graph having point set $V(G) \cup V_1(G)$ with two points are adjacent if they correspond to a adjacent points of G or

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one corresponds to a point v_i of $V_1(G)$ and the other to a point w_j of G and w_j is in $N(v_i)$. This concept was introduced by Sampathkumar and Walikar [2]. A graph G, the semi-splitting block graph $S_B(G)$ and the splitting graph $S(G)$ are shown in Fig. 1.

In 1975, Kulli [3] introduced the idea of a minimally nonouterplanar graph. The inner point number $i(G)$ of a planar graph G is the minimum possible number of points not belonging to the boundary of the exterior region in any embedding of G in the plane.

Obviously G is outer planar if and only if $i(G)=0$. A graph G is minimally nonouterplanar if $i(G)=1$, and G is n-minimally ($n\geq 2$) nonouterplanar if $i(G)=n$.

We make use of the following results to prove our main results.

Theorem A [4]**.** The splitting graph $S(G)$ of a graph G is planar if and only if every block of G is an even cycle or a line or a triangle.

Theorem B [4]**.** If G has a cycle of odd length $p\geq 5$ then $S(G)$ is nonplanar.

Theorem C [4]**.** The splitting graph $S(G)$ of a graph G is outer planar if and only if every component of G is a path or a triangle.

2. Main Results

We prove that semi-splitting block graph $S_B(G)$ of a graph G is nonplanar.

Theorem 1. If G is a cycle with $p\geq 4$ points, then $S_B(G)$ is nonplanar.

Proof. Suppose G is a cycle with $p\geq 5$ points. Suppose p is odd. Then by Theorem B, $S(G)$ is non planar. Then $S_B(G)$ is also nonplanar, since $S(G)$ is a sub graph of $S_B(G)$.

Suppose p is even and suppose G is a cycle with four points. In an optimal drawing of $S_B(G)$ we observe that it has a sub graph homeomorphic from $K_{3,3}$. Hence $S_B(G)$ is nonplanar.

The cycle C_4 is the least cycle with even length and $S_B(C_4)$ is nonplanar, $S_B(C_n)$, $n\geq 6$ is also nonplanar. Therefore $S_B(G)$ is nonplanar when G is a cycle of even length. Hence the theorem. □

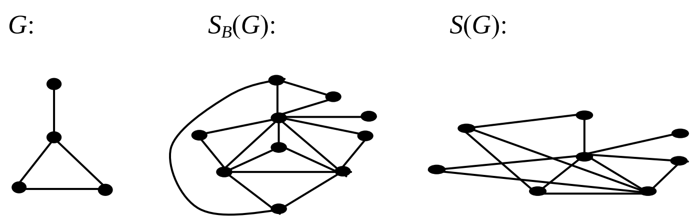

Fig. 1

We now present a characterization of graphs whose semi-splitting block graphs are planar.

Theorem 2. The semi-splitting block graph $S_B(G)$ of a graph G is planar if and only if every block of G is a line or a triangle or a triangle together one line is adjoined to some point.

Proof. Suppose $S_B(G)$ is planar. Then obviously $S(G)$ is planar. Therefore by Theorem A, every block of G is either an even cycle or a line or a triangle. But by Theorem 1, G has no cycle with $p \geq 4$ points and a triangle together one line is adjoined to some point then as shown in Fig. 1. Hence every block of G is either a line or a triangle or a triangle together one line is adjoined to some point.

Conversely, suppose every block of G is either a line or a triangle or a triangle together one line is adjoined to some point. Then it is easy to see that $S_B(G)$ is planar. □

We now establish a characterization of the semi-splitting block graph $S_B(G)$ of a graph G is outer planar.

Theorem 3. The semi-splitting block graph $S_B(G)$ is outer planar if and only if G is either $K_{1,2}$ or K_2.

Proof. Suppose $S_B(G)$ is outer planar. Assume a graph G with $p \geq 3$ points. We consider the following cases.

Case 1. Suppose G is a cycle with $p>3$ points. Then by Theorem 1, $S_B(G)$ is nonplanar, a contradiction.

Case 2. Suppose G is a cycle with $p=3$ points. Assume G is C_3. Clearly $i(S_B(C_3))=1$ see Fig. 2, again a contradiction. Then G is $K_{1,2}$ *or* K_2.

Case 3. Suppose G is P_n, $n \geq 3$. Then clearly $i(S_B(G)) \geq 2$, a contradiction.

Case 4. Suppose G is $K_{1,n}$, $n \geq 3$.Then by Theorem 3, a contradiction. Then G is $K_{1,2}$ or K_2.

Conversely, suppose G is K_2 or $K_{1,2}$. Then it is easy to see that, $S_B(G)$ is outer planar. This completes the proof of the theorem. □

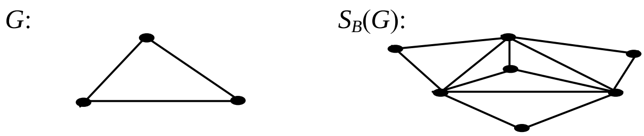

Fig. 2

We now present characterizations of graphs whose semi-splitting block graphs are planar and outer planar in terms of forbidden sub graphs.

Theorem 4. The semi-splitting block graph $S_B(G)$ of a graph G is planar if and only if G has no sub graph homeomorphic to C_4 or K_{4-x} or G_1 (see Fig. 3).

Proof. Suppose $S_B(G)$ is planar. Then by Theorem 2, G must be a triangle or every block of G is a line.

Suppose G is a triangle, then G has no sub graph homeomorphic to C_4 or K_{4-x} or G_1-.Suppose every block of G is a line. Then G does not contain any cycle. Hence G has no sub graph homeomorphic to C_4 or K_{4-x} or G_1.

Conversely, suppose that G contains no sub graph homeomorphic to C_4 or K_{4-x} or G_1. Then every block of G is either a line or a triangle. Then by Theorem 2, $S_B(G)$ is planar. Hence the proof the theorem. □

Theorem 5. The semi-splitting block graph $S_B(G)$ of a graph G is outer planar if and only if G has no sub graph homeomorphic to $K_{1,3}$ or P_4 or C_3.

Proof. Suppose $S_B(G)$ be outer planar. Then by Theorem 4, G is either $K_{1,2}$ or K_2. Hence G has no sub graph homeomorphic to $K_{1,3}$ or P_4 or C_3.

Conversely, suppose G has no sub graph homeomorphic to $K_{1,3}$ or P_4 or C_3.Then G must be $K_{1,2}$ *or* K_2.

□

G_1:

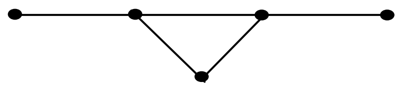

Fig. 3

3. Conclusions

We present here characterizations of graphs whose semi-splitting block graphs are planar, outer planar. Also we characterize graphs whose semi-splitting block graphs are planar and outer planar in terms of forbidden sub graphs.

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