

Volume Charge Density in Mixed Number Lorentz Transformation

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Abstract

We know that charge density is changed when it is observed from a moving frame of reference due to the length contraction. In this paper the transformation formula for the volume charge density in mixed number Lorentz transformation has been derived and the changes of the volume charge density of moving system in terms of rest system in mixed number Lorentz transformations at different angles and velocities have been shown graphically.

Keywords: Lorentz Transformation (LT); Volume charge density (VCD); Mixed Number Lorentz Transformation (MNLT).

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1. Introduction

Lorentz Transformation is the relation between two coordinate frames where one is in uniform motion with respect to the other. In special relativity the line of motion is placed in the x-axis. Here the interest is to observe the situation when the line of motion does not coexist with any of the coordinate axes. Charge density means how much charge is present in a given length, area or volume. Therefore, volume charge density tells specifically about how much electric charge is present in a given volume [9]. Rafiq and Alam [1] have derived the transformation equation of surface charge density in mixed number Lorentz Transformation. Bhuiyan *et al.* [2] have derived the transformation equation of surface charge density in different types of Lorentz transformation. In this case a cube is used to find out the formula of volume charge density in mixed number Lorentz transformation. In Lorentz transformation length is contracted, so the volume charge density will be different in different types of Lorentz transformations. In this paper the transformation equation for volume charge density in mixed number Lorentz transformation has been derived.

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1.1. Special Lorentz Transformation

Let S and S' be two frames of references, where S is at rest and S' is moving along x -axis with velocity \mathbf{v} with respect to S frame. The space and time coordinates of S and S' are (x, y, z, t) and (x', y', z', t') respectively. The relation between the coordinates of S and S' which is called the special Lorentz Transformation [1-3,9], can be written as

$$x' = \gamma(x - vt) \quad (1)$$

$$y' = y \quad (2)$$

$$z' = z \quad (3)$$

$$t' = \gamma(t - vx) \quad (4)$$

where, $\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$ and $c = 1$

The inverse Lorentz Transformations [1-3,9] can be written as

$$x = \gamma(x' + vt') \quad (5)$$

$$y = y' \quad (6)$$

$$z = z' \quad (7)$$

$$t = \gamma(t' + vx') \quad (8)$$

where, $\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$ and $c = 1$

1.2. Mixed Number Lorentz Transformation

Mixed number is the extension of complex number [10]. Mixed number Lorentz transformation can be formed using mixed number algebra. Let us consider the velocity \mathbf{v} of S' frame with respect to S frame is not along the x -axis i.e the velocity \mathbf{v} has three components v_x, v_y and v_z . Consider in this case \mathbf{z} and \mathbf{z}' be the space part in S and S' . Then we can write [1-3,9]

$$\mathbf{Z}' = \gamma[\mathbf{Z} - t\mathbf{v} + i(\mathbf{Z} \times \mathbf{v})] \quad (9)$$

$$t' = \gamma(t - \mathbf{Z} \cdot \mathbf{v}) \quad (10)$$

Similarly we can write

$$\mathbf{Z} = \gamma[\mathbf{Z}' + t'\mathbf{v} + i(\mathbf{Z}' \times \mathbf{v})] \quad (11)$$

$$t = \gamma(t' + \mathbf{Z}' \cdot \mathbf{v}) \quad (12)$$

Eqs. (9), (10), (11) and (12) are the mixed number Lorentz transformation.

2. Volume Charge Density

Volume charge density is the amount of electric charge per unit volume. It is denoted by ρ .

$$\therefore \rho = \frac{q}{V} \quad (13)$$

It is measured by coulombs per cubic meter (C/m^3). We know that the charge on the electron and proton is the minimum, called the elementary charge $e (= 1.6 \times 10^{-19} \text{ coul.})$. The electric charge is distinct which may be determined by counting the number of elementary charged particles. It is known that the total number of elementary charges cannot depend on the state of the motion of the observer therefore we may conclude that

the electric charge is relativistically invariant. Based on this conclusion we have derived the transformation equation for the volume charge density ρ [4-6,9].

3. Transformation of Volume Charge Density in Mixed Number Lorentz Transformation

Let two inertial systems S and S' where the frame S' is moving with uniform velocity \mathbf{v} relative to the rest system S in arbitrary direction as shown in fig.1. Thus velocity \mathbf{v} has three components v_x, v_y, v_z . Let us consider a stationary cube of uniform charge density $+\rho \text{ coul}/m^3$ at rest in system S' having one edge parallel to x -axis and let the length of the side of the cube is L_0 . The observer in system S will observe that the cube is moving with velocity \mathbf{v} which is parallel to x -axis.

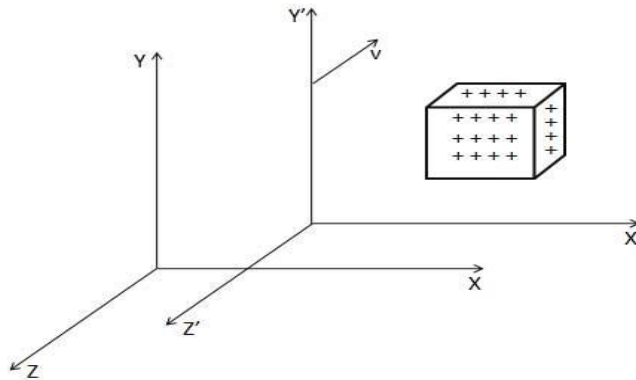


Fig. 1. The system S' is moving with velocity \mathbf{v} relative to the system S .

If L_0 is the length of the cube in S , then the length contraction in the moving frame for the mixed number Lorentz transformation can be written as [8]

$$\begin{aligned}
 \mathbf{L}_0 &= \gamma(\mathbf{L} - i\mathbf{L} \times \mathbf{v}) \\
 \text{or, } L_0^2 &= \gamma^2[L^2 - Li(\mathbf{L} \times \mathbf{v}) - i(\mathbf{L} \times \mathbf{v}).\mathbf{L} + i(\mathbf{L} \times \mathbf{v})i(\mathbf{L} \times \mathbf{v})] \\
 \text{or, } L_0^2 &= \gamma^2[L^2 + i^2(L^2v^2 - L^2v^2\cos^2\theta)] \\
 &= \gamma^2L^2[1 - v^2(1 - \cos^2\theta)] \\
 \text{or, } L^2 &= \frac{L_0^2}{\gamma^2[1 - v^2(1 - \cos^2\theta)]} \\
 \therefore L_x^2 &= \frac{L_{0x}^2}{\gamma^2[1 - v^2(1 - \cos^2\theta)]}, \quad L_y^2 = \frac{L_{0y}^2}{\gamma^2[1 - v^2(1 - \cos^2\theta)]} \quad \text{and} \quad L_z^2 = \frac{L_{0z}^2}{\gamma^2[1 - v^2(1 - \cos^2\theta)]}
 \end{aligned}$$

The total charge as observed by an observer in S' is

$$\begin{aligned}
 Q' &= L_x L_y L_z \rho' \\
 &= \sqrt{\frac{L_{0x}^2}{\gamma^2[1 - v^2(1 - \cos^2\theta)]}} \cdot \sqrt{\frac{L_{0y}^2}{\gamma^2[1 - v^2(1 - \cos^2\theta)]}} \cdot \sqrt{\frac{L_{0z}^2}{\gamma^2[1 - v^2(1 - \cos^2\theta)]}} \rho'
 \end{aligned}$$

$$= \frac{L_0^3}{\gamma^2 [1 - v^2 (1 - \cos^2 \theta)]^{\frac{3}{2}}} \rho'$$

According to the principle of conservation of charge

$$\begin{aligned} Q' &= Q_0 \\ \frac{L_0^3}{\gamma^2 [1 - v^2 (1 - \cos^2 \theta)]^{\frac{3}{2}}} \rho' &= L_0^3 \rho \\ \text{or, } \rho' &= [\gamma^2 \{1 - v^2 (1 - \cos^2 \theta)\}]^{\frac{3}{2}} \rho \end{aligned} \tag{14}$$

This equation represents the transformation equation for the volume charge density in Mixed number Lorentz transformation.

4. Table 1

Table1. Transformation equation for VCD in MNLT.

Length contraction	$L_0 = \gamma(L - iL \times v)$
Volume charge density	$\rho' = [\gamma^2 \{1 - v^2 (1 - \cos^2 \theta)\}]^{\frac{3}{2}} \rho$

5. Graphical Representation of VCD in MNLT

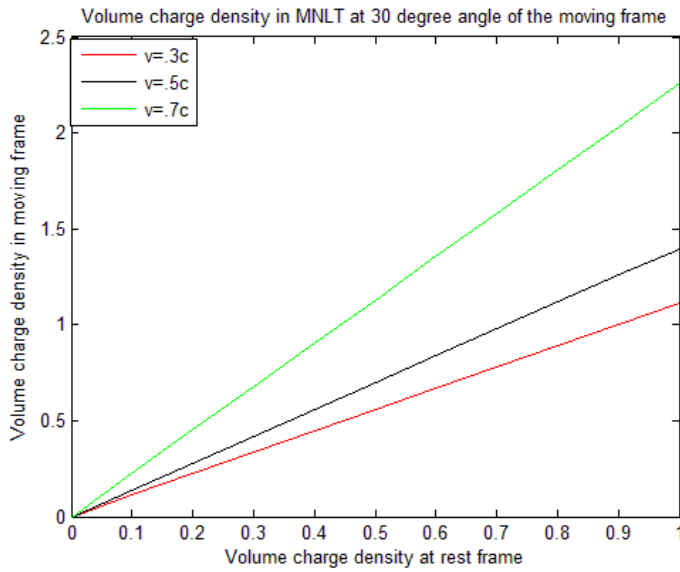


Fig. 2. Volume Charge Density in MNLT at 30 degree angle of the moving frame.

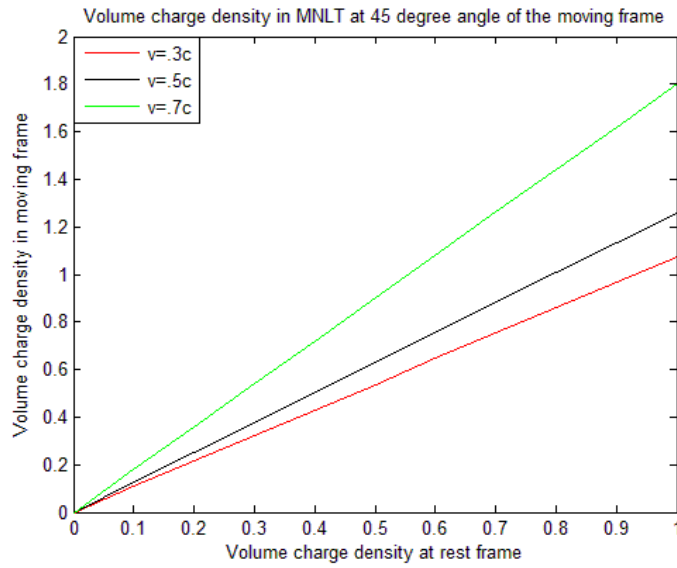


Fig. 3. Volume Charge Density in MNL T at 45 degree angle of the moving frame.

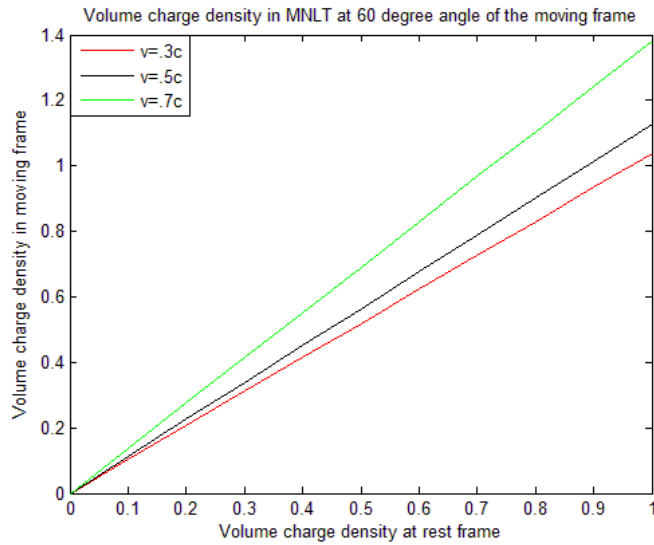


Fig. 4. Volume Charge Density in MNL T at 60 degree angle of the moving frame.

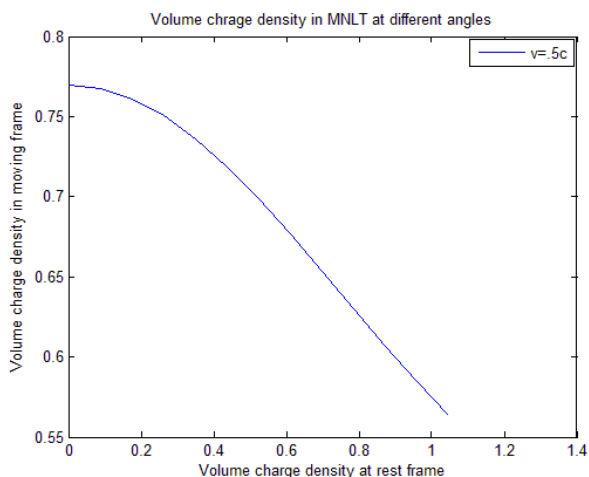


Fig. 5. Volume Charge Density in MNLT at different angles of the moving frame.

5. Conclusion

The transformation formula for volume charge density in Mixed number Lorentz transformation is illustrated in Table 1. The numerical values of volume charge densities of the moving system in terms of a system at rest for mixed number Lorentz transformation have been calculated and it is observed graphically that for same angle (θ) between the system S and S' , the values of volume charge density of moving system increases with increasing velocity v of the moving system. On the other hand for the same velocity of the moving system the values of volume charge density decreases with increasing the angle θ .

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