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The Wave Energy Up Conversion of Plasma Wave in Inhomogeneous Ionosphereic Plasma

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Abstract

Different types of instabilities are observed in the thermodynamically nonequilibrium Earth's ionosphere. Effective energy exchange process among waves may takes place through nonlinear interaction modes because of availability of free energy. We consider gradients in density and magnetic field is present in the system which support drift wave turbulence. In this study we concern on the wave energy up conversion of electrostatic nonresonant lower hybrid wave through plasma maser instability in the mid-altitude ionospheric region. We have formulated the growth rate of lower hybrid wave by Vlasov-Poisson mathematical frame and estimated its value by observational data.

Keywords: Plasma maser effect; Lower hybrid wave; Drift wave; Density gradient; Magnetic field gradient.

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1. Introduction

The Earth's ionosphere is a wall-less plasma laboratory has weak particle collision rate and low temperature. Through experimental observations it has been found the existence of both the linear and nonlinear properties in this atmospheric region [1]. In different altitudes of this region, naturally occurring instabilities are observed and their nonlinear effects are recorded. Perturbation develops in this open ionospheric plasma system for the entry of energy and momentum fluxes through particles and waves like solar wind. Gradients of several physical parameters like density, temperature and magnetic field are existing in this thermodynamically nonequilibrium system. In this ionospheric plasma region geomagnetic field is fluctuating with respect to altitudes for the interaction among solar wind, magnetospheric dynamo, interplanetary magnetic fields and the presence of other irregularities [2]. Due to the presence of a rich class of free energy sources in the earth's environment several macro- and micro-instabilities are observed in this open

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inhomogeneous plasma system and investigated at lower and mid altitudes and in the auroral zone [3-6].

In the mid-altitude F-region weakly collisional ionospheric plasma is produced mainly through photo-ionisation process and has peak plasma density [7,8]. From satellite observational data, gradient drift instability and temperature drift instability are observed during turbulent geomagnetic field conditions within 30° to 60° geomagnetic latitude ranges in the mid-altitude ionospheric plasma [9]. Kelly [5] observed in inhomogeneous ionospheric region at the mid altitude, different entities like gravity waves, shear effects, drift waves and lower hybrid waves are contributing in turbulence processes. In several studies investigations on instabilities in an inhomogeneous magnetised plasma system associated with density and temperature gradients from nonlinear wave-particle interaction approach in both open and laboratory plasma environment as well as in homogeneous and inhomogeneous magnetised plasma system are carried on [10-17]. Deka [18] investigated on the amplification of Langmuir wave in presence of ion acoustic turbulence in inhomogeneous plasma in presence of magnetic field gradients through plasma maser effect.

In this study, for the first time, we are investigating on probable amplification of lower hybrid wave in the presence of drift wave in ionospheric region at mid-altitude taking magnetic fields gradients and density gradients through plasma-maser instability and wish to estimate the growth rate by using observational data. Here on the basis of weak turbulence theory, plasma maser effect which was also known as induced Bremsstrahlung instability is a mode coupling effect and associated with the linear Landau damping [19]. In this effect electrons are accelerated by resonant mode drift wave turbulent field. The accelerated electrons transfer its energy to nonresonant lower hybrid wave through a modulated field non-linearly. Also due to wave-particle-wave interaction a high frequency dissipative nonlinear force is developed which acts as driving force to the growth of nonresonant mode. For this theoretical investigation we are using Vlasov-Poisson mathematical framework to evaluate fluctuating parts of distribution functions, modulated electric field, nonlinear force and growth rate expression through Fourier transform and integration along unperturbed orbit methods.

2. Formulation of the Problem

Consider a semi-infinite bounded inhomogeneous plasma which is confined by a magnetic field in the presence of low frequency drift wave. As superimposing perturbation field we consider a high frequency lower hybrid wave to the system. Let the plasma density decreases in the x-direction is balanced by a magnetic field that increases with x.

For this system, the electron distribution function is considered as [20]

$$f_{jo} = f_{jo}(v_\perp^2, v_x)\left[1-\varepsilon'\left(x+\frac{v_y}{\Omega_j}\right)\right] \tag{1}$$

Here

$$\varepsilon' = -\frac{1}{f_{jo}} \frac{\partial f_{jo}}{\partial x}\bigg|_{x=x_0}$$ is density gradient.

Here "j" is used to specify plasma particle specification.

$$\Omega_j = \frac{e_j B_o}{m_j c}$$ is cyclotron frequency.

$$\varepsilon'' = -\frac{1}{B_o} \frac{dB_o}{dx}$$ is magnetic field gradient.

In this problem neglecting temperature variation and their anisotropy, the density and magnetic field with gradients are taken as–

$$n_j = n_{jo}(1-\varepsilon' x)$$

$$\vec{B}(x) = \hat{z}B_o(1+\varepsilon'' x)$$

3. Mathematical Analysis

The interaction of Lower hybrid wave with drift wave is generated by the Vlasov-Poison system of equations which are –

$$\left[\frac{\partial}{\partial t} + \vec{v}.\frac{\partial}{\partial \vec{r}} - \frac{e_j}{m_j}\left(\vec{E} + \frac{\vec{v}\times\vec{B}}{c}\right).\frac{\partial}{\partial \vec{v}}\right]\vec{F}(\vec{r},\vec{v},t) = 0 \tag{2}$$

$$\vec{\nabla}.\vec{E} = -4\pi n_j e_j \int f(\vec{r},\vec{v},t).d\vec{v} \tag{3}$$

The unperturbed distribution function for electrons, the unperturbed electric field and the unperturbed magnetic fields are taken as–

$$F_{oj} = f_{oj} + \varepsilon f_{oj} + \varepsilon^2 f_{2j} \tag{4}$$

$$\vec{E}_{ol} = \varepsilon \vec{E}_l$$

$$\vec{B}_{ol} = \vec{B}_0(x) = \hat{z}B_0(1+\varepsilon'' x)$$

Where f_{oj} is space and time averaged part of the distribution function, f_{1j} and f_{2j} are fluctuating parts due to low frequency drift wave turbulence, ε is ordering of the low-frequency drift wave turbulence field, electric field and wave vector of drift turbulence are

$\vec{E}_l = (E_{l\perp}, 0, E_{l\|})$ and $\vec{k} = (k_\perp, 0, k_\|)$.

Now, equation (2) can be written as

$$\left[\frac{\partial}{\partial t} + \vec{v}.\frac{\partial}{\partial \vec{r}} - \varepsilon\frac{e_j}{m_j}\vec{E}_l.\frac{\partial}{\partial \vec{v}} - \frac{e_j}{m_j}\left(\frac{\vec{v}\times\vec{B}}{c}\right).\frac{\partial}{\partial \vec{v}}\right](f_{oj} + \varepsilon f_{1j} + \varepsilon^2 f_{2j}) = 0 \tag{5}$$

To the order of ε , we have –

$$\left[\frac{\partial}{\partial t}+\vec{v}.\frac{\partial}{\partial \vec{r}}-\frac{e_j}{m_j}\left(\frac{\vec{v}\times\vec{B}}{c}\right).\frac{\partial}{\partial \vec{v}}\right]f_{1j}=\frac{e_j}{m_j}\vec{E}_l.\frac{\partial}{\partial \vec{v}}f_{oj} \tag{6}$$

Since the system is inhomogeneous in the x-direction, here we consider wave propagation only in the y-z plan. Equations of motion are [20]

$$\frac{d\vec{r}'}{dt'}=\vec{v}'$$

$$\frac{d\vec{v}'}{dt}=-\frac{e_j}{m_j c}\vec{v}'\times\vec{B}_o(1+\varepsilon'' x')$$

$$\vec{r}'(t'=t)=\vec{r} \text{ and } \vec{v}'(t'=t)=\vec{v} \tag{7}$$

From equations (7), the particle orbits, under the boundary conditions $x'(o)=x,\vec{v}'(\tau=o)=\vec{v}$ are [18,20]

$$x'-x=\frac{v_y}{(1+\varepsilon'' x-\frac{\varepsilon'' v_y}{\Omega_j})}\sin\left\{\theta+\left(1+\varepsilon'' x-\frac{\varepsilon'' v_y}{\Omega_j}\right)\tau\right\}-\frac{\varepsilon'' v_y}{2\Omega_j}\left[\frac{\sin 2\theta}{\Omega_j}\sin\Omega_j\tau+\frac{\cos(2\theta+2\Omega_j\tau)}{2\Omega_j}+\frac{2\cos\Omega_j\tau}{\Omega_j}\cos^2\theta\right]$$

$$-\frac{v_\perp\sin\theta}{(1+\varepsilon'' x-\frac{\varepsilon'' v_y}{\Omega_j})}+\frac{\varepsilon'' v_\perp^2}{2\Omega_j^2}\left(\frac{\cos 2\theta}{2}+2\cos^2\theta\right)$$

$$y'-y=-\frac{v_\perp}{(1+\varepsilon'' x-\frac{\varepsilon'' v_y}{\Omega_e})}\cos\{\theta+(1+\varepsilon'' x-\frac{\varepsilon'' v_y}{\Omega_e})\tau\}-\frac{\varepsilon'' v_\perp}{2\Omega_e}[\tau-\frac{\sin 2\theta}{\Omega_e}\cos\Omega_e\tau+\frac{\sin(2\theta+2\Omega_e\tau)}{2\Omega_e}-$$

$$\frac{2\sin\Omega_e\tau}{\Omega_e}\cos^2\theta]+\frac{v_\perp\cos\theta}{(1+\varepsilon'' x-\frac{\varepsilon'' v_y}{\Omega_e})}-\frac{\varepsilon'' v_\perp^2}{4\Omega_e^2}\sin 2\theta$$

$$z'-z=v_\parallel\tau \tag{8}$$

Using Fourier transform and the method of characteristics [20], from equation (6) we have

$$f_{1e}\left(\vec{k},\omega\right)=i\sum\frac{e}{m}\left[E_{l\parallel}\frac{\partial f_{oe}}{\partial v_\parallel}\mathbb{R}_{s,t}-\frac{m}{k_\perp T_e}E_{l\perp}\left\{1+\left(k_\parallel v_\parallel-\omega-\frac{\varepsilon' k_\perp T_e}{m\Omega_e}\right)\mathbb{R}_{s,t}\right\}f_{oe}\right] \tag{9}$$

where

$$\mathbb{R}_{s,t}=\sum_{s,t}\frac{J_s\left(\dfrac{v_\perp k_\perp}{1+\varepsilon''x-\dfrac{\varepsilon''v_y}{\Omega_j}}\right)J_t\left(\dfrac{v_\perp k_\perp}{1+\varepsilon''x-\dfrac{\varepsilon''v_y}{\Omega_j}}\right)e^{[-i(s+t)\theta]}}{s\left(\dfrac{v_\perp k_\perp}{1+\varepsilon''x-\dfrac{\varepsilon''v_y}{\Omega_j}}\right)+\left(k_\parallel v_\parallel-\omega\right)+i0^+}$$

Let the quasi steady state is perturbed by the test nonresonant lower hybrid wave field $\mu\delta\vec{E}_h$, propagation vector $K=(0,0,K_\parallel)$, electric field $\delta E=(0,0,\delta E_h)$ and a frequency Ω. Here electrostatic lower hybrid waves are directed perpendicular to magnetic fields [21,22]. Due to this perturbation total perturbed electric field, magnetic field and electric distribution function are

$$\delta\vec{E}=\mu\delta\vec{E}_h+\mu\varepsilon\delta\vec{E}_{lh}+\mu\varepsilon^2\Delta\vec{E}$$

$$\delta\vec{B}=0$$

$$\delta f_j=\mu\delta f_h+\mu\varepsilon\delta f_{lh}+\mu\varepsilon^2\Delta f \tag{10}$$

Where δE_{lh} and $\Delta\vec{E}$ are modulating fields, δf_h is the fluctuating part due to high frequency lower hybrid wave, δf_{lh} and Δf are particle distribution function corresponds to modulating fields

Let, the operator

$$\hat{L}=\frac{\partial}{\partial t}+\vec{v}.\frac{\partial}{\partial\vec{r}}-\frac{e_j}{m_j}\left(\frac{\vec{v}\times\vec{B}_o}{c}\right).\frac{\partial}{\partial\vec{v}}$$

Using equation (10) in Vlasov equation (2) for the perturbed state, we get –

$$\left[\hat{L}-\frac{e_j}{m_j}\left(\mu\vec{E}_l+\mu\delta\vec{E}_h+\mu\varepsilon\delta\vec{E}_{lh}+\mu\varepsilon^2\Delta\vec{E}\right).\frac{\partial}{\partial\vec{v}}\right]\times$$
$$(f_{oj}+\varepsilon f_{1j}+\varepsilon^2 f_{2j}+\mu\delta f_h+\mu\varepsilon\delta f_{lh}+\mu\varepsilon^2\Delta f)=0 \tag{11}$$

To the order of μ, we have-

$$\hat{L}\delta f_h=\frac{e_j}{m_j}\delta\vec{E}_h.\frac{\partial}{\partial\vec{v}}f_{oj} \tag{12}$$

To the order of $\mu\varepsilon$, we have –

$$\hat{L}\delta f_{lh}=\frac{e_j}{m_j}\vec{E}_l.\frac{\partial}{\partial\vec{v}}\delta f_h+\frac{e_j}{m_j}\delta\vec{E}_h.\frac{\partial}{\partial\vec{v}}\delta f_{1j}+\frac{e_j}{m_j}\delta\vec{E}_{lh}.\frac{\partial}{\partial\vec{v}}\delta f_{oj} \tag{13}$$

To the order of $\mu\varepsilon^2$ and applying random phase approximation to omit second order quantities, we have –

$$\hat{L}\Delta f = \frac{e_j}{m_j}\left[\vec{E}_l\cdot\frac{\partial}{\partial\vec{v}}\delta f_{lh} + \delta\vec{E}_{lh}\cdot\frac{\partial}{\partial\vec{v}}\delta f_{1j}\right] \tag{14}$$

Applying Fourier transform and integrating along the unperturbed orbit, we can evaluate the fluctuating part δf_h of the distribution function due to high frequency lower hybrid wave $K = (0, 0, K_\parallel)$ over the particle trajectories. Here

$$\delta f_h\left(\vec{K},\Omega\right) = \frac{e_j}{m_j}\int_{-\alpha}^{0}\left(\delta E_h\frac{\partial f_{oj}}{\partial v_\parallel}\right)e^{\left[i\{K_\parallel(z'-z)-\Omega\tau\}\right]d\tau}$$

Using

$$\frac{\partial f_{oj}}{\partial\vec{v}} = \left[-\frac{m_j\vec{v}}{T_j} - \hat{X}\frac{\varepsilon'}{\Omega_j}\right]$$

and for weak gradient

$$\frac{\varepsilon'' K_\perp T_j}{2\Omega_j m_j} \ll 1$$

We get after lengthy calculations-

$$\delta f_h\left(\vec{K},\Omega\right) = -i\sum\frac{e_j}{m_j}\delta E_h\frac{\frac{\partial f_{oj}}{\partial v_\parallel}}{(K_\parallel v_\parallel - \Omega)} \tag{15}$$

Using Fourier transform and the method of characteristics to equation (14) we get-

$$\delta f_{lh}(\vec{K}-\vec{k},\Omega-\omega) = \sum\frac{e_j}{m_j}\int_{-\alpha}^{0}\left(\delta\vec{E}_{lh}\cdot\frac{\partial}{\partial\vec{v}}f_{oj} + \delta\vec{E}_h\cdot\frac{\partial}{\partial\vec{v}}f_{1j} + \vec{E}_l\cdot\frac{\partial}{\partial\vec{v}}\delta f\right).e^{[i\{(\vec{K}-\vec{k}).(\vec{r}'-\vec{r})-(\Omega-\omega)\tau\}]d\tau}$$

$$= I_{lh}^1 + I_{lh}^2 + I_{lh}^3 \tag{16}$$

Here after lengthy calculations we get

$$I_{lh}^1 = \sum\frac{e_j}{m_j}\int_{-\alpha}^{0}\delta\vec{E}_{lh}\cdot\frac{\partial}{\partial\vec{v}}f_{oj}.e^{[i\{(\vec{K}-\vec{k}).(\vec{r}'-\vec{r})-(\Omega-\omega)\tau\}]d\tau}$$

$$= i\sum\frac{e_j}{m_j}\frac{\delta E_{lh}}{k_\perp}\frac{m_j}{T_j}\left[1+\left\{(\Omega-\omega)-\left(K_\parallel - k_\parallel\right)v_\parallel - \frac{\varepsilon' T_j k_\perp}{\Omega_j m_j}\right\}\mathbb{Z}_{a,b}\right]f_{oj} \tag{17}$$

$$I_{lh}^2 = \sum\frac{e_j}{m_j}\int_{-\alpha}^{0}\delta\vec{E}_h\cdot\frac{\partial}{\partial\vec{v}}f_{1j}.e^{[i\{(\vec{K}-\vec{k}).(\vec{r}'-\vec{r})-(\Omega-\omega)\tau\}]d\tau}$$

$$= i\sum\left(\frac{e_j}{m_j}\right)^2\delta E_h\frac{\partial}{\partial v_\parallel}\left[E_{l\parallel}\frac{\partial f_{oj}}{\partial v_\parallel}\mathbb{R}_{s,t} - \frac{m_j}{k_\perp T_j}E_{l\perp}\left\{1+\left(k_\parallel v_\parallel - \omega - \frac{\varepsilon' k_\perp T_j}{m_j\Omega_j}\right)\mathbb{R}_{s,t}\right\}f_{oj}\right]\mathbb{Z}_{a,b} \tag{18}$$

$$I_{lh}^{3}=\sum\frac{e_j}{m_j}\int_{-\alpha}^{0}\vec{E}_l.\frac{\partial}{\partial\vec{v}}\delta f_h\,e^{\left[i\left\{\left(\vec{K}-\vec{k}\right).\left(\vec{r}'-\vec{r}\right)-\left(\Omega-\omega\right)\tau\right\}\right]}d\tau$$

$$=-\sum\left(\frac{e_j}{m_j}\right)^2E_{l\perp}\delta E_h\frac{\frac{\partial f_{oj}}{\partial v_\parallel}}{\left(K_\parallel v_\parallel-\Omega\right)}\frac{m_j}{k_\perp T_j}\left[1+\left\{(\Omega-\omega)-(K_\parallel-k_\parallel)v_\parallel-\frac{\varepsilon' T_j k_\perp}{m_j\Omega_j}\right\}\mathbb{Z}_{a,b}\right]-$$

$$\sum\left(\frac{e_j}{m_j}\right)^2E_{l\parallel}\delta E_h\frac{\partial}{\partial v_\parallel}\left(\frac{\frac{\partial f_{oj}}{\partial v_\parallel}}{\left(K_\parallel v_\parallel-\Omega\right)}\right)\mathbb{Z}_{a,b}\tag{19}$$

Where

$$\mathbb{Z}_{a,b}=\sum_{a,b}\frac{J_a\left(\frac{v_\perp k_\perp}{1+\varepsilon'' x-\frac{\varepsilon'' v_y}{\Omega_j}}\right)J_b\left(\frac{v_\perp k_\perp}{1+\varepsilon'' x-\frac{\varepsilon'' v_y}{\Omega_j}}\right)e^{\left[-i(a+b)\theta\right]}}{a\left(1+\varepsilon'' x-\frac{\varepsilon'' v_y}{\Omega_j}\right)+\left(K_\parallel-k_\parallel\right)v_\parallel-\left(\Omega-\omega\right)}$$

By using Poisson's equation

$$\vec{\nabla}.\delta\vec{E}_{lh}=-4\pi\sum e_j n_j\int\delta f_{lh}.d\vec{v}\tag{20}$$

the expression of modulated electric field $\delta\vec{E}_{lh}\left(\vec{K}-\vec{k}\right)$ is

$$\delta E_{lh}=-\sum\frac{4\pi e_j n_j}{iR\left|\vec{K}-\vec{k}\right|}\left(\frac{e_j}{m_j}\right)^2\delta E_h\left\{\Delta^1+\Delta^2+\Delta^3\right\}\tag{21}$$

Where

$$\Delta^1=\int\left[-E_{l\perp}\frac{\frac{\partial f_{oj}}{\partial v_\parallel}}{\left(K_\parallel v_\parallel-\Omega\right)}\frac{m_j}{k_\perp T_j}\left[1+\left\{\left(\Omega-\omega\right)-\left(K_\parallel-k_\parallel\right)v_\parallel-\frac{\varepsilon' T_j k_\perp}{m_j\Omega_j}\right\}\mathbb{Z}_{a,b}\right]\right]dv$$

$$\Delta^2=\int\left[-E_{l\parallel}\frac{\partial}{\partial v_\parallel}\left(\frac{\frac{\partial f_{oj}}{\partial v_\parallel}}{K_\parallel v_\parallel-\Omega}\right)\mathbb{Z}_{a,b}\right]dv$$

$$\Delta^3=\int\left[\frac{\partial}{\partial v_\parallel}\left[E_{l\parallel}\frac{\partial f_{oj}}{\partial v_\parallel}\mathbb{R}_{s,t}-\frac{m_j}{k_\perp T_j}E_{l\perp}\left\{1+\left(k_\parallel v_\parallel-\omega-\frac{\varepsilon' k_\perp T_j}{m_j\Omega_j}\right)\mathbb{R}_{s,t}\right\}f_{oj}\right]\mathbb{Z}_{a,b}\right]dv$$

and

$$R = 1 + \left(\frac{m_j}{T_j}\right)\frac{4\pi n_j e_j^2}{m_j\left|\vec{K}-\vec{k}\right|}\int\left[1+\left\{(\Omega-\omega)-\left(K_{\parallel}-k_{\parallel}\right)v_{\parallel}-\frac{\varepsilon' k_{\perp}T_j}{m_j\Omega_j}\right\}\mathbb{Z}_{a,b}\right]f_{oj}dv$$

For the nonlinear interaction between low frequency resonant drift wave $\vec{k} = (k_{\perp}, 0, k_{\parallel})$ and high frequency nonresonant lower hybrid wave $\vec{K} = (0,0,K_{\parallel})$ present in the system by means of weak turbulence theory a nonlinear force develop due to acceleration of electrons in the modulated electric field $\delta\vec{E}_{lh}$ and contribute in wave energy up conversion process [23].

From equation (14)-

$$\hat{L}\Delta f = \frac{e_j}{m_j}\left[\vec{E}_l \cdot \frac{\partial}{\partial \vec{v}}\delta f_{lh} + \delta\vec{E}_{lh}\cdot\frac{\partial}{\partial\vec{v}} f_{1j}\right] = \vec{F} \text{ (Say)} \qquad (22)$$

Using Fourier transform and integrating along unperturbed orbits, we get –

$$\Delta f = \int_{-\alpha}^{0} \vec{F}(\vec{K},\Omega)\,\mathrm{e}^{\{i\{\vec{K}.(\vec{r}'-\vec{r})-\Omega\tau\}]d\tau}$$

$$= -i\frac{F}{K_{\parallel}(z'-z)-\Omega} \qquad (23)$$

The nonlinear force F_{Nh} acting on unit volume of particles can be written as

$$\vec{F}_{Nh} = m_j n_{jo}\Omega\int\frac{F}{K_{\parallel}v_{\parallel}-\Omega}\vec{v}d\vec{v}$$

$$= m_j n_{j0}\Omega\int\left\langle\frac{e_j}{m_j}\left[\vec{E}_l\cdot\frac{\partial}{\partial\vec{v}}\delta f_{lh} + \delta\vec{E}_{lh}\cdot\frac{\partial}{\partial\vec{v}}\delta f_{1j}\right]\right\rangle \times \frac{1}{K_{\parallel}v_{\parallel}-\Omega}\vec{v}d\vec{v} \qquad (24)$$

where $\langle \ldots\ldots\ldots\ldots \rangle$ represents ensemble average.

The Z-component of this nonlinear force is-

$$\vec{F}_{Nhz}(\vec{K},\Omega) = \vec{F}_{Nhz1}(\vec{K},\Omega) + \vec{F}_{Nhz2}(\vec{K},\Omega) \qquad (25)$$

where

$$F_{Nhz1} = e_j n_{j0}\Omega\int\left\langle \vec{E}_l\cdot\frac{\partial}{\partial\vec{v}}\delta f_{lh}\right\rangle\frac{1}{K_{\parallel}v_{\parallel}-\Omega}v_{\parallel}dv \qquad (26)$$

$$F_{Nhz2} = e_j n_{j0}\Omega\int\left\langle \delta\vec{E}_{lh}\cdot\frac{\partial}{\partial\vec{v}}\delta f_{1j}\right\rangle\frac{1}{K_{\parallel}v_{\parallel}-\Omega}v_{\parallel}dv \qquad (27)$$

Here, the direct coupling term contributes to the nonlinear component force F_{Nhz1} and the other high frequency nonlinear component force F_{Nhz2} comes from contributions of polarization coupling term. From earlier investigations [18,24] it is found that for the growth of plasma wave in the presence of lower hybrid wave in inhomogeneous magnetised plasma the polarization coupling term dominates over the direct coupling term. In our present discussion we retain only the nonlinear force component $F_{Nhz2}(\vec{K},\Omega)$ only.

To evaluate $F_{Nhz2}\left(\vec{K},\Omega\right)$ we consider the plasma maser effect arises from resonant electrons under the condition $\omega=\vec{k}.\vec{v}$ between lower hybrid wave and drift wave turbulence. It is also assumed that (i) $\Omega < K_{\parallel}v_{\parallel}$ (ii) the gyro angle θ should be very small and (iii) keeping a=b=s=t=0 to retaining the most dominant Bessel's terms. Here we have used the relation

$$\mathrm{Im}\frac{1}{kv-\omega+io^{+}}=-\pi i\delta(\omega-kv)$$

and

$$\int_{-\infty}^{\infty}G(v)\delta(\omega-kv)=\left(\frac{1}{|k|}G\left(\frac{\omega}{k}\right)\right)$$

Here $$J_0\left(\frac{v_{\perp}K_{\perp}}{1+\varepsilon''x-\frac{\varepsilon''v_y}{\Omega_j}}\right)=1-\frac{\left(\frac{v_{\perp}K_{\perp}}{1+\varepsilon''x-\frac{\varepsilon''v_y}{\Omega_j}}\right)^2}{4}$$

From observational data it is found that magnetic gradients in the ionospheric region is very much low, we can neglect $\varepsilon''=0$.

After lengthy calculations and keeping dominant terms only, from equation (21) we get approximate expression

$$\delta E_{lh}=\left(\frac{e_j}{m_j}\right)E_{l\perp}\delta E_h\frac{K_{\parallel}\left(K_{\parallel}-k_{\parallel}\right)^2}{k_{\parallel}^2}\frac{\omega_{pj}^2}{\Omega_j^3\left(\Omega_j-\Omega\right)} \tag{28}$$

where at the lowest order approximation we obtain

$$\frac{1}{\Re\left|\vec{K}-\vec{k}\right|^2}\simeq\frac{1}{k_{\parallel}^2} \tag{29}$$

Using equations (9) and (28) and to consider only polarization coupling term we get from equation (25)

$$\vec{F}_{Nhz}(\vec{K},\Omega)=\vec{F}_{Nhz2}(\vec{K},\Omega)$$

$$=ie_jn_{jo}\delta E_h\left(\frac{e_j}{m_j}\right)^2\frac{K_{\parallel}(K_{\parallel}-k_{\parallel})^2}{k_{\parallel}^2}\frac{\omega_{pj}^2}{\Omega_j^3(\Omega_j-\Omega)}E_{l\parallel}\left(E_{l\perp}\frac{\varepsilon'}{\Omega_j}+2E_{l\parallel}\frac{v_d}{v_j^2}\right)e^{\left\{-\left(\frac{v_d}{v_j}\right)^2\right\}} \tag{30}$$

In the presence of nonlinear force term, by using the method of Chen [25], the momentum equation of electron as

$$m_j n_j \left(\frac{\partial \vec{v}_j}{\partial t} + \vec{v}_j . \vec{\nabla} v_j \right) = -e_j n_j \vec{\nabla} \phi + \vec{F}_{Nh} \tag{31}$$

where v_j , n_j and ϕ are the density, velocity and potential of charge particles.

Equation (31) is linearized on the steady state condition $n_j = n_{jo}$ and $v_j = v_{jo} = 0$, and applying Fourier transformations, the first order z-component of momentum equation becomes

$$-im_j n_{jo} \Omega v_{j1z} = -ie_j n_{jo} \chi \phi_1 + F_{Nhz} \tag{32}$$

From Boltzmann relation, first order charge particle density fluctuation is

$$\delta n_j = n_{jo} \frac{e_j \phi_1}{k_B T_j} \tag{33}$$

The continuity equation for electron is

$$\frac{\partial n_j}{\partial t} = -\vec{\nabla}.(n_j v_j) \tag{34}$$

After linearizing equation (39) become

$$v_{jz} = -\frac{\Omega \delta n_j}{n_{jo} \chi} \tag{35}$$

Using equation (37),(38) and (40) ,we get

$$\Omega^2 = \chi^2 \left(-\frac{k_B T_j}{m_j} \right) + i \frac{\chi}{m_j} \frac{F_{Nhz}}{\delta n_j} \tag{36}$$

is the dispersion relation of electrostatic lower hybrid wave in the presence of drift wave turbulence.

As neglecting nonlinear frequency shift, from equation (36) we get real frequency of lower hybrid wave

$$\Omega_r = \chi \left(-\frac{k_B T_j}{m_j} \right)^{\frac{1}{2}} \tag{37}$$

and growth rate of lower hybrid wave

$$\gamma = \frac{\chi}{2 m_j \Omega_r} \frac{F_{Nhz}}{\delta n_j} \tag{38}$$

Now using equations (30), (33) and (37) in equation (38) we obtain

$$\frac{\gamma}{\Omega} = \frac{1}{2} \left(\frac{e_j}{m_j} \right)^2 \left(\frac{m_j}{k_B T_j} \right)^{\frac{1}{2}} \Omega \frac{\omega_{pj}^2}{\Omega_j^3} \frac{K_\parallel \left(K_\parallel - k_\parallel \right)^2}{k_\parallel^2} E_{l\parallel} \left(E_{l\perp} \frac{\varepsilon'}{\Omega_j} + 2 E_{l\parallel} \frac{v_d}{v_j^2} \right) e^{\left\{ -\left(\frac{v_d}{v_j} \right)^2 \right\}} \tag{39}$$

Equation (39) estimates the growth rate of lower hybrid wave with drift wave turbulence from polarization coupling term in inhomogeneous plasma.

From observation it is found that $|E_{l\perp}| \gg |E_{l\parallel}|$ for drift wave turbulence, for this neglecting $E_{l\parallel}$ factor, equation (39) becomes

$$\frac{\gamma}{\Omega}=\frac{1}{2}\left(\frac{e_j}{m_j}\right)^2\left(\frac{m_j}{k_B T_j}\right)^{\frac{1}{2}}\Omega\frac{\omega_{pj}^2}{\Omega_j^3}\frac{K_\parallel\left(K_\parallel-k_\parallel\right)^2}{k_\parallel^2}E_{l\parallel}E_{l\perp}\frac{\varepsilon'}{\Omega_j}e^{\left\{-\left(\frac{v_d}{v_j}\right)^2\right\}} \quad (40)$$

For the weak density plasma region we can put $\varepsilon'=0$ and equation (39) becomes

$$\frac{\gamma}{\Omega}=\left(\frac{e_j}{m_j}\right)^2\left(\frac{m_j}{k_B T_j}\right)^{\frac{1}{2}}\Omega\frac{\omega_{pj}^2}{\Omega_j^3}\frac{K_\parallel\left(K_\parallel-k_\parallel\right)^2}{k_\parallel^2}E_{l\parallel}{}^2\frac{v_d}{v_j^2}e^{\left\{-\left(\frac{v_d}{v_j}\right)^2\right\}} \quad (41)$$

4. Results and Discussion

In this study by using Vlasov-Poisson system of equations, the dispersion relation of electrostatic lower hybrid wave in the presence of drift wave turbulence is derived in inhomogeneous magnetised plasma. We have considered polarization coupling term of nonlinear dispersion for estimation of growth rate of lower hybrid wave. The issue of plasma instabilities in ionospheric region at mid-altitude in the presence of drift wave turbulence is relevant at present. To obtain an estimation of the growth rate of lower hybrid wave, we have considered the following observational data:

1) The Plasma parameters and Lower hybrid wave parameters in space [26,27]:

$T_e=400K, \Omega=\Omega_e=10^6 s^{-1}, \frac{\omega_{pe}}{\Omega_e}\approx 0.3, K_\parallel=10^{-3}m^{-1}$

2) The Drift wave Parameters in space [28]:

$k_\parallel 10^{-3}m^{-1}, k_\perp=10^{-5}m^{-1}, v_d=10^6 ms^{-1}$

3) The observational electrostatic plasma wave dc electric field intensity in the mid altitude ionospheric region [29]:

$k_\parallel=10^{-2}Vm^{-1}, E_{l\perp}=10^{-1}Vm^{-1}$

Considering density gradient $\varepsilon'=0.1$ the growth rate for lower hybrid wave in presence of drift wave turbulence in inhomogeneous ionospheric plasma from equation (40) as

$\frac{\gamma}{\Omega}\approx 10^{-5}$.

as $K_\parallel \gg K_\parallel-k_\parallel$ is assumed to take $K_\parallel-k_\parallel\approx 10^{-5}$.

5. Conclusion

In this theoretical investigating work it is found that drifting particles takes part through nonlinear energy exchange mode in the wave energy amplification process of high frequency plasma wave in the presence of both density and magnetic field strength gradients. After considering weak magnetic field gradients (≈ 0) in the inhomogeneous ionospheric zone the wave energy upconversion process of nonresonant wave may also be effective in the presence of density gradients.

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