

Unfolding Low-Energy Gamma-Ray Spectrum Obtained with NaI(Tl) in Air Using Matrix Inversion Method

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Abstract

Matrix inversion method is presented to unfold the gamma-ray spectrum obtained with an NaI(Tl) detector using several standard gamma-ray sources. The method is based on response matrix generated by Monte Carlo simulation of mono-energy gamma-ray photon ranging from 10 keV to 1 MeV in step of 10 keV. The comparison of the measured and simulated response function was also performed in order to validate the simulation response function. Good agreement was achieved around the photo-peak region of the spectrum, but slight deviation was observed at low energy region especially at Compton continuum region. The Compton continuum count was significantly transferred into the corresponding photo-peak and consequently the peak to background ratio was improved substantially by the application of the unfolding method. Therefore, small peak can be identified and analyzed that would otherwise be lost in the background.

Keywords: Gamma-ray spectrum; Unfold; NaI(Tl).

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1. Introduction

Many unfolding techniques for γ -ray spectra have been proposed in the literature. The inverse matrix method [1-3] represents the most straightforward method. The stripping method [4-8] is often applied for Ge detector and is based on a successive subtraction of Compton background from higher to lower channels. The folding iteration method [9-14] is based on successive folding of better and better trial functions.

The response matrix contains an array of values representing the over-all detector response including the possible effects of the shield and the surrounding materials on the

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pulse-height spectra of the calibrated gamma-ray sources. Researchers in gamma-ray spectrometry have developed matrix method for analysis of pulse-height spectra. The basic mathematical relationship concerning the measured pulse-height spectrum, M , the response function of the detector, R , and the true gamma-ray spectrum, T , is generally represented by the matrix equation,

$$M = RT \quad (1)$$

The direct solution to the matrix equation is:

$$T = R^{-1}M \quad (2)$$

where R^{-1} is the inverse of R .

2. Materials and Methods

2.1. Simulation of the detector response function

As mentioned before, the $R(E, E_0)$ detector response function is defined as the probability density for a γ -ray of energy E_0 emitted by the source to give rise to a signal of energy E in the detector. To obtain reliable results, this function must be determined for conditions as during the actual experiment. For the unfolding method described above, the function $R(E, E_0)$ should be known for many E_0 points. Therefore, we used Monte Carlo calculations for $E_\gamma = 0.010 - 1.0$ MeV and checked the validity of the calculations at γ -ray energies which were experimentally available. For the construction of the response matrix, one might need 500 or even 1000 spectra depending on the dimension of the problem, which is very elaborate work. To reduce the work load in making the response matrix, we simulated 100 γ -ray spectra ranging from 10 keV to 1 MeV with an interval of 10 keV.

To simulate the geometry of the set up, we consider the detector as a cylindrical NaI(Tl) cell of radius 3.81 cm and height 7.62 cm. The source is placed 10 cm away from the front face of the NaI(Tl) in air along axis. Spectra were generated by the MCNPX code for 10^7 events for the γ -ray energies ranging from 10 keV to 1 MeV with an interval of 10 keV. Then to consider spectrum broadening effect due to nonlinearity of the scintillator response, Photomultiplier tube (PMT) and electronic noise, we corrected the spectra using a Gaussian shape broadening function.

2.2. Formation of the inverse matrix

The pulse height values from each of the 100 mono-energetic gamma-ray spectra obtained from the simulated energy response function was arranged as a row and formed the

NaI(Tl) detector response matrix R (100×100) in a single file and then inverted the response matrix R using MATLAB 7 program.

2.3. Laboratory measurement

An NaI(Tl) detector of crystal size 3"×3" (CANBERRA, model: 802) used to calculate the detector response function. As shown in Fig. 1, the 3" × 3" NaI(Tl) scintillation detector is a hermetically sealed assembly which includes an NaI(Tl) crystal (D), a photomultiplier tube (PMT), a PMT base with a pre-amplifier, an internal magnetic / light shield, an aluminum housing and a 14-pin connector.

Specifications of the model 802 scintillation detector are as follows:

- Window - Aluminum, 0.5 mm thick; density 147 mg/cm².
- Reflector - Oxide; 1.6 mm thick; density 88 mg/cm².
- Magnetic /Light shield - Conetic lined steel.

The mono-energy gamma-ray standard sources, ¹³⁷Cs and ⁵⁷Co were placed in air along axis of the NaI(Tl) detector and the distance between the source & detector's front face was 10 cm. A re-binning algorithm was applied to the measured spectrum in order to fit with the simulated spectrum in the same scale.

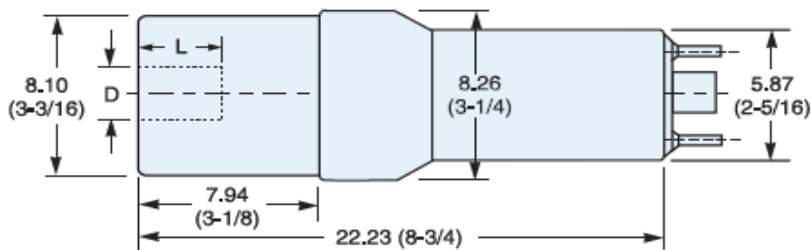


Fig. 1. NaI(Tl) detector of crystal size 3"×3" (CANBERRA, model: 802) used to calculate the detector response function. Dimensions in the outline drawings are in cm (in.).

3. Result and Discussion

3.1. Validation of simulation and experiment

The comparison of the measured and simulated spectrum is depicted in Fig. 2. Good agreement is observed around the photo-peak region. However, the discrepancy in the Compton region is mainly due to the contribution of backscattering from the surrounding materials and X-ray emission. The multiple Compton scattering and overlapping pulses created from different γ -rays are partially filled in the gap between the Compton edge and the photo-peak.

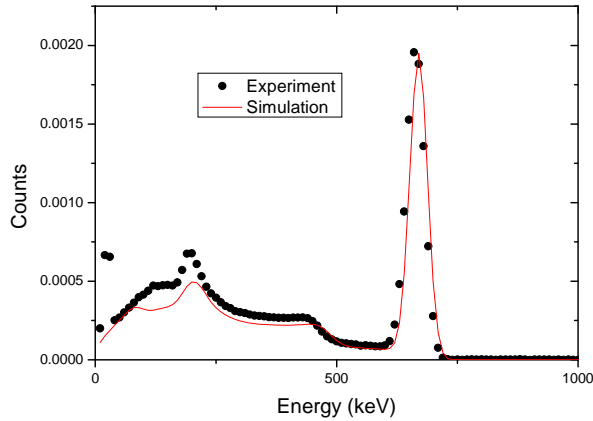


Fig. 2. Comparison of experiment and simulation for the case of standard source ^{137}Cs with gamma-ray energy of 662 keV.

3.2. *Unfolded spectrum by inversion method*

The advantage of the unfolding is that the additional peaks (backscattered peak, x-ray peaks at low energies) and the Compton continuum have been eliminated to a larger extent and consequently, signal (peak) to noise (background) ratio is substantially improved and an automatic detector efficiency correction would also be performed. Therefore, the resulting spectra can be analyzed very easily. However, one can easily see in the unfolded gamma-ray spectra as shown in Figs. 3 and 4, there is some high frequency oscillations on both sides of the resultant spectrum. This is the main drawback of the gamma-ray spectra unfolded using inversion method. This may arise due to the inaccuracy of the simulated and measured spectra, especially at low energies region.

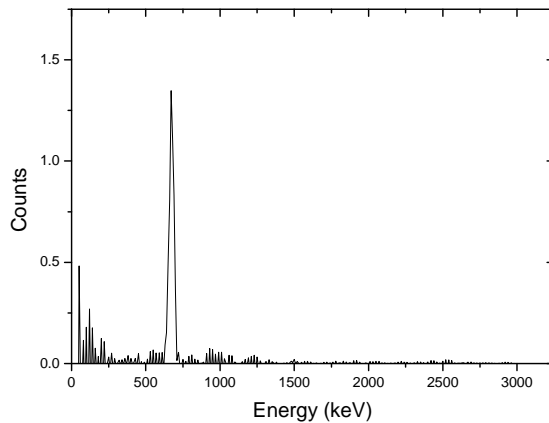


Fig. 3. Unfolded spectrum obtained from the measured spectrum with NaI(Tl) at a distance of 10 cm from the detector's front face using ^{137}Cs of energy 662 keV.

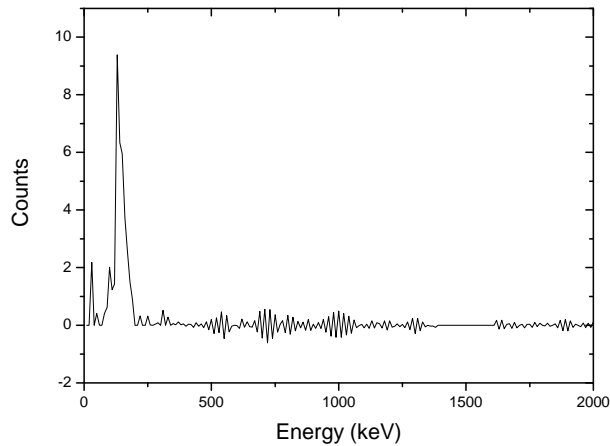


Fig. 4. Unfolded spectrum obtained from the measured spectrum with NaI(Tl) at a distance of 10 cm from the detector's front face using ^{57}Co of energy 122 keV.

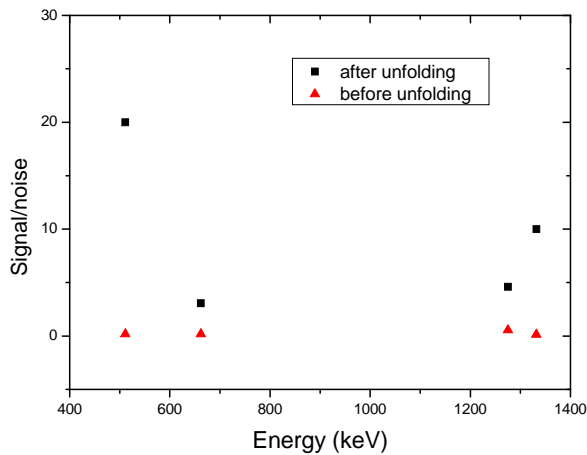


Fig. 5. Signal to noise ratio before and after unfolding of the gamma-ray spectrum.

4. Conclusion

The main goal of this work was to develop the inversion method which would remove as efficiently as possible the background counts into the corresponding photo-peaks in γ -ray spectra. We conclude that the use of appropriately Gaussian broadened Monte Carlo spectra is sufficient to produce acceptable results. The advantage of the unfolding is that the signal to noise ratio is increased about 10~100 times, consequently small signal can be detected and analyzed that would otherwise be lost in the background.

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