

Synchronization of Non-Integer Complex Chaotic Systems with Uncertainty and Disturbances via Adaptive Sliding Mode Technique

L. S. Jahanzaib, P. Trikha, Nasreen*

Department of Mathematics, Jamia Millia Islamia, New Delhi

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Abstract

In this paper we concern the adaptive sliding mode control technique for synchronization of fractional order chaotic systems with uncertainties and disturbances. This technique is used to design control law through suitable sliding surface and estimate the external disturbances. Computational results using MATLAB verified the effectiveness of the proposed scheme.

Keywords: Chaos synchronization; Fractional order complex chaotic system; Adaptive sliding mode control technique.

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1. Introduction

Chaotic dynamics [1] has grown into very curious and attractive area for researchers. Chaotic dynamical systems are unstable and ambiguous. Generally, chaos being the intrinsic property of non-linear systems has numerous applications such as in viscoelasticity [2], dielectric polarization, electromagnetic waves [3], diffusion, signal processing, mathematical biology and in many more disciplines.

1.1. Literature review

Around us all the system that we account are non-linear. Different techniques are used to investigate the chaotic behaviour few of them are by plotting phase portraits, poincare section, bifurcation diagram or by finding Lyapunov exponents. The most reliable and widely used among the above technique is Lyapunov exponent spectrum. If the largest Lyapunov exponent is positive, we say that the system is chaotic and if more than one Lyapunov exponents are positive, then the system is said to be hyperchaotic.

The worldwide researchers got attraction in conquer chaos either by controlling chaos or by synchronizing chaotic systems. It was the pioneering work of Pecora and Caroll [4]

* Corresponding author: nasreen899@gmail.com

who gave the concept of synchronization to control and utilize the chaos in the proper way. Synchronization means the trajectories of the coupled systems evolve with time to a usual pattern. Various techniques have been developed by researchers in this direction during last two decades. Numerous synchronization schemes have been proposed such as lag synchronization [5], complete synchronization [6], phase and anti-phase synchronization [7], anti-synchronization [8], hybrid synchronization [9], projective synchronization [10], hybrid function projective synchronization [11], generalised synchronization [12], multi-switching synchronization [13] etc. To achieve synchronization different techniques have been designed some of them are adaptive feedback control, optimal control, linear and nonlinear feedback synchronization [14], active control [15], sliding mode control [16,17], adaptive sliding mode technique [18], time delay feedback approach [19], tracking control [20], back-stepping design method [20] and so on.

In recent years, a lot of pioneering work has been done in the field of fractional calculus [21]. It was first suggested by Leibnitz and L'Hospital in 1675 and they gave the theory of integrals and derivatives of random order which combines the concept of integer order differentiation and n-fold integration. These studies describe the significant work in the real life systems and have a lot of multidisciplinary applications. As compared to integer order network the fractional order system add a degree of freedom by employing fractional derivative. Many types of fractional order chaotic and hyperchaotic systems have been introduced by researchers like Lorenz system [22], Chen system [23], Rossler system [23], Lu-system [24], Lui-system [17], Chua system [25] etc. to explain the various physical processes. In order to increase the complexity, researchers also introduced numerous fractional order complex chaotic systems like complex Lorenz system [26], T-system [27], Lu-system [28], Chen system [26] etc.

1.2. Prime objectives and novelty of this manuscript

Prime objectives of this manuscript are

- To achieve synchronization of fractional order complex system in presence of external disturbances and uncertainties.
- To compare our technique with previous published literatures techniques.
- To show the effectiveness of this scheme in presence of external disturbances.

The novelty of this manuscript lies in proposing the adaptive sliding mode technique [29,30] to synchronize fractional order complex chaotic systems. To the best of authors knowledge synchronization of fractional order complex system in the presence of external disturbances and uncertainties does not examined in the prior literatures. Also, we have compared our result with previous published literatures which shows that our synchronization time is much lesser than that reported [5,31,32]. Numerical simulations have been done to validate and visualize our results in the form of plots and demonstrates that our results are in excellent agreement with the theoretical results.

2. Preliminaries

The fractional order system is continuation to the integer order calculus. As compared to integer order network the fractional order system add a degree of freedom by employing fractional derivative. Also, fractional order derivatives show better results when modelling real life processes as compared to integer order derivatives. The fractional order derivative can be defined in various forms [4], such as Riemann-Liouville's derivative, Grunwald Letnikov's derivative, Caputo's derivative etc.

The Riemann Liouville's derivative is defined as

$${}_{t_0}D_t^\alpha f(t) = \frac{d^n}{dt^n} \left[\frac{1}{\Gamma(n-\alpha)} \int_{t_0}^t \frac{f(\tau)}{(t-\tau)^{\alpha-n+1}} d\tau \right], \quad t > t_0$$

Where α is fractional derivative, $n - 1 < \alpha < n, n \in \mathbb{N}, \Gamma(\alpha) = \int_0^\infty x^{\alpha-1} e^{-x}$ is the Gamma function.

The Caputo's derivative is defined as

$${}_{t_0}D_t^\alpha f(t) = \frac{1}{\Gamma(n-\alpha)} \int_{t_0}^t \frac{f(\tau)}{(t-\tau)^{\alpha-n+1}} d\tau, \quad t > t_0$$

The Grunwald Letnikov's derivative is defined as

$${}_{t_0}D_t^\alpha f(t)|_t = kh = \lim_{h \rightarrow 0} \frac{1}{h^\alpha} \sum_{j=0}^{\lfloor \frac{t-c}{h} \rfloor} \omega_j(\alpha) f(kh - jh)$$

Where h shows the sample time. $\lfloor \cdot \rfloor$ is the floor function and the coefficients

$$\omega_j^\alpha = \frac{(-1)^j \Gamma(\alpha + 1)}{\Gamma(j + 1) \Gamma(\alpha - j + 1)}, j = 0, 1, 2, \dots, k.$$

Since the Caputo's fractional derivative of a constant is zero, in this paper we choose Caputo's definition.

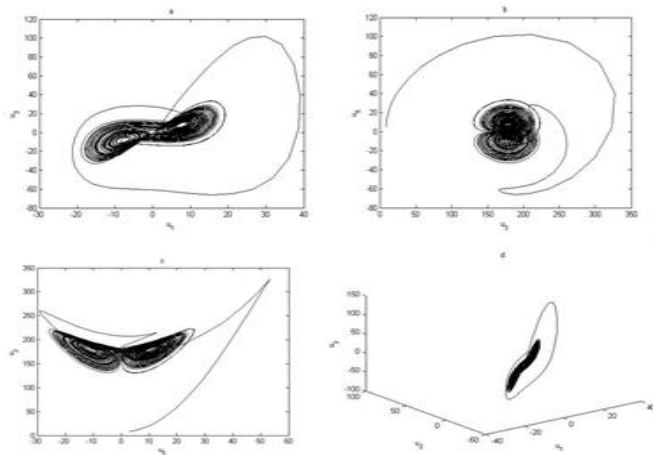


Fig. 1. Phase portraits of fractional order complex system (1) for fractional order order $\alpha = 0.95$, $u(0) = [2, 3, 5, 6, 9]^T$ (a) $u_1 - u_3$ (b) $u_3 - u_5$ (c) $u_5 - u_2$ (d) $u_1 - u_2 - u_3$

2. System Description

Considering the fractional order complex Lorenz system [26] given by

$$\begin{aligned} D^\alpha u'_1 &= a_1(u'_2 - u'_1) \\ D^\alpha u'_2 &= a_2 u'_1 - u'_2 - u'_1 u'_3 \\ D^\alpha u'_3 &= 1/2(\overline{u'_1} u'_2 - u'_1 \overline{u'_2}) - a_3 u'_3 \end{aligned} \quad (1)$$

Where $u' = [u'_1, u'_2, u'_3]^T$ is the state variable vector, $u'_1 = u_1 + iu_2$, $u'_2 = u_3 + iu_4$ are complex variables, $u'_3 = u_5$ is the real variable and a_1, a_2, a_3 are real constant parameters.

The Lorenz system with external disturbances and uncertainties is given as

$$\begin{aligned} D^\alpha u'_1 &= a_1(u'_2 - u'_1) + \Delta f'_1 \\ D^\alpha u'_2 &= a_2 u'_1 - u'_2 - u'_1 u'_3 + \Delta f'_2 \\ D^\alpha u'_3 &= 1/2(\overline{u'_1} u'_2 - u'_1 \overline{u'_2}) - a_3 u'_3 - \Delta f'_3 \end{aligned} \quad (2)$$

The external disturbances and uncertainties are taken as $\Delta f'_1 = e^{i\pi t m(u'_1)} - e^{i\frac{\pi}{2}t}$, $\Delta f'_2 = e^{i\pi t m(u'_2)} + 0.2e^{i\frac{\pi}{2}t}$ and $\Delta f'_3 = -\sin\frac{\pi}{2}u'_3 - 0.1 \cos t$.

Separating the real and imaginary parts, we obtain the system (2) as

$$\begin{aligned} D^\alpha u_1 &= a_1(u_3 - u_1) + \Delta f_1 \\ D^\alpha u_2 &= a_1(u_4 - u_2) + \Delta f_2 \\ D^\alpha u_3 &= a_2 u_1 - u_3 - u_1 u_5 + \Delta f_3 \\ D^\alpha u_4 &= a_2 u_2 - u_4 - u_2 u_5 + \Delta f_4 \\ D^\alpha u_5 &= u_1 u_3 + u_2 u_4 - a_3 u_5 + \Delta f_5 \end{aligned} \quad (3)$$

Where $\Delta f_1 = \cos \pi u_2 - \cos \pi/2t$, $\Delta f_2 = \sin \pi u_2 - \sin \pi/2t$, $\Delta f_3 = \cos \pi u_4 + 0.2 \cos \pi t$, $\Delta f_4 = \sin \pi u_4 + 0.2 \sin \pi t$ and $\Delta f_5 = \sin \pi/2 u_5 - 0.1 \cos t$.

For the values of parameters as $a_1 = 10$, $a_2 = 180$, $a_3 = 1$, initial conditions as $u(0) = [2, 3, 5, 6, 9]^T$ and $\alpha = 0.95$, the system (1) is chaotic.

Considering the fractional order complex T system [27] given by

$$\begin{aligned} D^\alpha v'_1 &= b_1(v'_2 - v'_1) \\ D^\alpha v'_2 &= (b_2 - b_1)v'_1 - b_1 v'_1 v'_3 \\ D^\alpha v'_3 &= 1/2(\overline{v'_1} v'_2 - v'_1 \overline{v'_2}) - b_3 v'_3 \end{aligned} \quad (4)$$

Where $v' = [v'_1, v'_2, v'_3]^T$ is the state variable vector, $v'_1 = v_1 + iv_2$, $v'_2 = v_3 + iv_4$ are complex variables, $v'_3 = v_5$ is the real variable and b_1, b_2, b_3 are real constant parameters.

The T system with external disturbances and uncertainties is given as

$$\begin{aligned} D^\alpha v'_1 &= b_1(v'_2 - v'_1) + \Delta g'_1 \\ D^\alpha v'_2 &= (b_2 - b_1)v'_1 - b_1 v'_1 v'_3 + \Delta g'_2 \\ D^\alpha v'_3 &= 1/2(\overline{v'_1} v'_2 - v'_1 \overline{v'_2}) - b_3 v'_3 + \Delta g'_3 \end{aligned} \quad (5)$$

The external disturbances and uncertainties are taken as $\Delta g'_1 = e^{i\pi/2 \text{Re}(v'_1)} - e^{0.1\pi t}$, $\Delta g'_2 = e^{i\pi t m(v'_2)} - e^{it}$ and $\Delta g'_3 = \cos 0.1 \pi v'_3 + 0.5 \cos t$.

Separating the real and imaginary parts, we obtain the system (5) as

$$\begin{aligned} D^\alpha v_1 &= b_1(v_3 - v_1) + \Delta g_1 \\ D^\alpha v_2 &= b_1(v_4 - v_2) + \Delta g_2 \\ D^\alpha v_3 &= (b_2 - b_1)v_1 - b_1 v_1 v_5 + \Delta g_3 \\ D^\alpha v_4 &= (b_2 - b_1)v_2 - b_1 v_2 v_5 + \Delta g_4 \\ D^\alpha v_5 &= v_1 v_3 + v_2 v_4 - b_3 v_5 + \Delta g_5 \end{aligned}$$

Where $\Delta g_1 = \cos \pi/2 v_1 - \cos 0.1\pi t$, $\Delta g_2 = \sin \pi/2 v_1 - \sin 0.1\pi t$, $\Delta g_3 = \cos \pi v_4 - \cos t$, $\Delta g_4 = \sin \pi v_4 - \sin t$ and $\Delta g_5 = \cos 0.1\pi v_5 + 0.5 \cos t$.

For the values of parameters as $b_1 = 21$, $b_2 = 30$, $b_3 = 0.6$, initial conditions as $v(0) = [8,7,5,6,10]^T$ and $\alpha = 0.95$, the system (4) is chaotic.

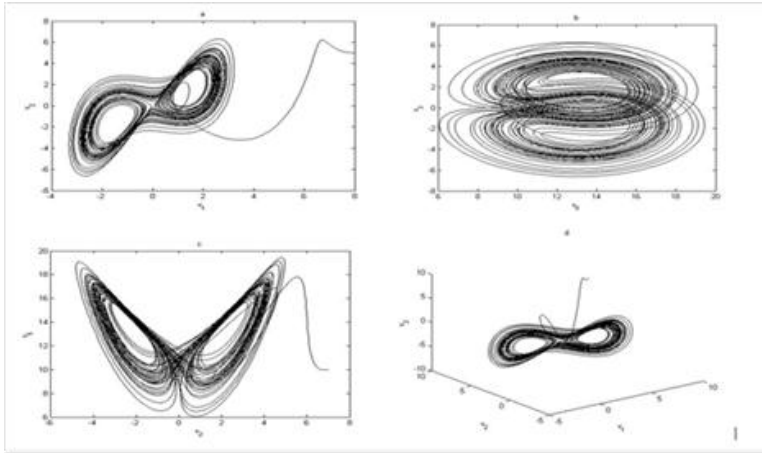


Fig. 2. Phase portraits of fractional order complex system (4) for fractional order $\alpha = 0.95$, $v(0) = [2,3,5,6,9]^T$ (a) $v_1 - v_3$ (b) $v_5 - v_3$ (c) $v_2 - v_5$ (d) $v_1 - v_2 - v_3$

3. Synchronization Scheme

The fractional order complex chaotic system (3) is taken as drive system and the system (6) is taken as response system which is given by

$$\begin{aligned}
 D^\alpha v_1 &= b_1(v_3 - v_1) + \Delta g_1 + U_1 \\
 D^\alpha v_2 &= b_1(v_4 - v_2) + \Delta g_2 + U_2 \\
 D^\alpha v_3 &= (b_2 - b_1)v_1 - b_1v_1v_5 + \Delta g_3 + U_3 \\
 D^\alpha v_4 &= (b_2 - b_1)v_2 - b_1v_2v_5 + \Delta g_4 + U_4 \\
 D^\alpha v_5 &= v_1v_3 + v_2v_4 - b_3v_5 + \Delta g_5 + U_5
 \end{aligned} \tag{7}$$

Where U_i are appropriate control inputs of the response system for $i=1,2,3,4,5$. which will be designed later.

Here we assume that $|\Delta f_i| \leq \psi_i$ and $|\Delta g_i| \leq \chi_i$, where ψ_i and χ_i are positive constants. Also $\hat{\psi}_i$ and $\hat{\chi}_i$ represents the estimated values of ψ_i and χ_i respectively.

Now the error state is defined as

$$e_i = v_i - u_i, i = 1,2,3,4,5.$$

Definition: The drive system (3) and response system (6) are said to be in synchronization, if there exists suitable controller $U = (U_1, U_2, \dots, U_m)$, such that $\lim_{t \rightarrow \infty} \|e(t)\| = \lim_{t \rightarrow \infty} \|v_i(t) - u_i(t)\|, i = 1,2,3,4,5$

The synchronization error is asymptotically stable between the state variables of drive system (3) and state variables of response system (6). The error dynamics is obtained as

$$\begin{aligned}
 D^\alpha e_1 &= b_1(e_3 - e_1) + (b_1 - a_1)u_3 + (a_1 - b_1)u_1 + \Delta g_1 - \Delta f_1 + U_1 \\
 D^\alpha e_2 &= b_1(e_4 - e_2) + (b_1 - a_1)u_4 + (a_1 - b_1)u_2 + \Delta g_2 - \Delta f_2 + U_2 \\
 D^\alpha e_3 &= (b_2 - b_1)e_1 - b_1v_1v_5 - (b_2 - b_1)u_1 - a_2u_1 + u_3 + u_1u_5 + \Delta g_3 - \Delta f_3 + U_3
 \end{aligned} \tag{8}$$

$$D^\alpha e_4 = (b_2 - b_1)e_2 - b_1 v_2 v_5 - (b_2 - b_1)u_2 - a_2 u_2 + u_4 + u_2 u_5 + \Delta g_4 - \Delta f_4 + U_4$$

$$D^\alpha e_5 = v_1 v_3 + v_2 v_4 - b_3 e_5 + b_3 u_5 - (u_1 u_3 + u_2 u_4) - a_3 u_5 + \Delta g_5 - \Delta f_5 + U_5$$

In order to minimize the error, we choose the suitable sliding surface which is as follows:

$$s_i(t) = D^{\alpha-1} e_i(t) + \kappa_i \int_0^t e_i(\xi) d\xi \tag{9}$$

To accomplish the error dynamic system (8) at chosen sliding surface (9), it is necessary that it should satisfy the following condition

$$s_i(t) = 0, \dot{s}_i(t) = 0, i = 1, 2, 3, 4, 5. \tag{10}$$

The derivative of (9), yields the following equation

$$\dot{s}_i(t) = D^\alpha e_i(t) + \kappa_i e_i(t), i = 1, 2, 3, 4, 5. \tag{11}$$

Then by considering the necessary condition $\dot{s}_i(t) = 0$, we obtain

$$D^\alpha e_i(t) = -\kappa_i e_i(t), i = 1, 2, 3, 4, 5. \tag{12}$$

Hence, the system (12) is asymptotically stable by using Matignon theorem [33]. Therefore, the control laws by using (10), (8) and SMC theory are obtained as follows

$$U_1 = -b_1(e_3 - e_1) - (b_1 - a_1)u_3 - (a_1 - b_1)u_1 - \kappa_1 e_1 - (\hat{\psi}_1 + \hat{\chi}_2 + \phi_1)signs_1$$

$$U_2 = -b_1(e_4 - e_2) - (b_1 - a_1)u_4 - (a_1 - b_1)u_2 - \kappa_2 e_2 - (\hat{\psi}_2 + \hat{\chi}_2 + \phi_2)signs_2$$

$$U_3 = -(b_2 - b_1)e_1 + b_1 v_1 v_5 + (b_2 - b_1)u_1 + a_2 u_1 - u_3 - u_1 u_5 - \kappa_3 e_3 - (\hat{\psi}_3 + \hat{\chi}_3 + \phi_3)signs_3 \tag{13}$$

$$U_4 = -(b_2 - b_1)e_2 + b_1 v_2 v_5 + (b_2 - b_1)u_2 + a_2 u_1 - u_4 - u_2 u_5 - \kappa_4 e_4 - (\hat{\psi}_4 + \hat{\chi}_4 + \phi_4)signs_4$$

$$U_5 = -v_1 v_3 - v_2 v_4 + b_3 e_5 - b_3 u_5 + (u_1 u_3 + u_2 u_4) + a_3 u_5 - \kappa_5 e_5 - (\hat{\psi}_5 + \hat{\chi}_5 + \phi_5)signs_5$$

Where sign (.) denotes the signum function and γ_i are positive constant parameters.

The adaptive parameter update laws are

$$\dot{\hat{\psi}}_i = \rho_i |s_i|, i = 1, 2, 3, 4, 5.$$

$$\dot{\hat{\chi}}_i = \delta_i |s_i|, i = 1, 2, 3, 4, 5. \tag{14}$$

Where m_i and n_i are positive constants and γ_i are gain constants of the controllers for $i = 1, 2, 3, 4, 5$.

Theorem 3.1. The fractional order complex chaotic system (3) and the slave system (6) with uncertain dynamics are globally and asymptotically stable and synchronized with adaptive sliding mode control laws (13) and parameter update laws (14).

Proof. To discuss the stability of the fractional order chaotic systems, we have used Lyapunov's direct method [34] Ch-5. Here our main focus is to take a positive definite function V and would show the derivative of V negative definite which would imply that our error converges asymptotically to zero.

$$V = V_1 + V_2 + V_3 + V_4 + V_5 \tag{15}$$

Where

$$V_1 = \frac{1}{2} s_1^2 + \frac{1}{\rho_1} (\hat{\psi}_1 - \psi_1)^2 + \frac{1}{\delta_1} (\hat{\chi}_1 - \chi_1)^2$$

$$V_2 = \frac{1}{2} s_2^2 + \frac{1}{\rho_2} (\hat{\psi}_2 - \psi_2)^2 + \frac{1}{\delta_2} (\hat{\chi}_2 - \chi_2)^2$$

$$V_3 = \frac{1}{2} s_3^2 + \frac{1}{\rho_3} (\hat{\psi}_3 - \psi_3)^2 + \frac{1}{\delta_3} (\hat{\chi}_3 - \chi_3)^2 \tag{16}$$

$$V_4 = \frac{1}{2}s_4^2 + \frac{1}{\rho_4}(\hat{\psi}_4 - \psi_4)^2 + \frac{1}{\delta_4}(\hat{\chi}_4 - \chi_4)^2$$

$$V_5 = \frac{1}{2}s_5^2 + \frac{1}{\rho_5}(\hat{\psi}_5 - \psi_5)^2 + \frac{1}{\delta_5}(\hat{\chi}_5 - \chi_5)^2$$

The dynamics of Lyapunov function is

$$\begin{aligned} \dot{V}_1 &= s_1\dot{s}_1 + \frac{1}{\rho_1}(\hat{\psi}_1 - \psi_1)^2\dot{\hat{\psi}}_1 + \frac{1}{\delta_1}(\hat{\chi}_1 - \chi_1)^2\dot{\hat{\chi}}_1 \\ \dot{V}_2 &= s_2\dot{s}_2 + \frac{1}{\rho_2}(\hat{\psi}_2 - \psi_2)^2\dot{\hat{\psi}}_2 + \frac{1}{\delta_2}(\hat{\chi}_2 - \chi_2)^2\dot{\hat{\chi}}_2 \\ \dot{V}_3 &= s_3\dot{s}_3 + \frac{1}{\rho_3}(\hat{\psi}_3 - \psi_3)^2\dot{\hat{\psi}}_3 + \frac{1}{\delta_3}(\hat{\chi}_3 - \chi_3)^2\dot{\hat{\chi}}_3 \\ \dot{V}_4 &= s_4\dot{s}_4 + \frac{1}{\rho_4}(\hat{\psi}_4 - \psi_4)^2\dot{\hat{\psi}}_4 + \frac{1}{\delta_4}(\hat{\chi}_4 - \chi_4)^2\dot{\hat{\chi}}_4 \\ \dot{V}_5 &= s_5\dot{s}_5 + \frac{1}{\rho_5}(\hat{\psi}_5 - \psi_5)^2\dot{\hat{\psi}}_5 + \frac{1}{\delta_5}(\hat{\chi}_5 - \chi_5)^2\dot{\hat{\chi}}_5 \end{aligned} \tag{17}$$

Substituting the values of s'_i s, we obtain

$$\begin{aligned} \dot{V}_1 &= s_1(D^\alpha e_1 + \phi_1 e_1) + \frac{1}{\rho_1}(\hat{\psi}_1 - \psi_1)^2\dot{\hat{\psi}}_1 + \frac{1}{\delta_1}(\hat{\chi}_1 - \chi_1)^2\dot{\hat{\chi}}_1 \\ \dot{V}_2 &= s_2(D^\alpha e_2 + \phi_2 e_2) + \frac{1}{\rho_2}(\hat{\psi}_2 - \psi_2)^2\dot{\hat{\psi}}_2 + \frac{1}{\delta_2}(\hat{\chi}_2 - \chi_2)^2\dot{\hat{\chi}}_2 \\ \dot{V}_3 &= s_3(D^\alpha e_3 + \phi_3 e_3) + \frac{1}{\rho_3}(\hat{\psi}_3 - \psi_3)^2\dot{\hat{\psi}}_3 + \frac{1}{\delta_3}(\hat{\chi}_3 - \chi_3)^2\dot{\hat{\chi}}_3 \\ \dot{V}_4 &= s_4(D^\alpha e_4 + \phi_4 e_4) + \frac{1}{\rho_4}(\hat{\psi}_4 - \psi_4)^2\dot{\hat{\psi}}_4 + \frac{1}{\delta_4}(\hat{\chi}_4 - \chi_4)^2\dot{\hat{\chi}}_4 \\ \dot{V}_5 &= s_5(D^\alpha e_5 + \phi_5 e_5) + \frac{1}{\rho_5}(\hat{\psi}_5 - \psi_5)^2\dot{\hat{\psi}}_5 + \frac{1}{\delta_5}(\hat{\chi}_5 - \chi_5)^2\dot{\hat{\chi}}_5 \end{aligned} \tag{18}$$

By substituting the values of $D^\alpha e_i$, $\dot{\hat{\psi}}_i$ and $\dot{\hat{\chi}}_i$ in (17), we obtain

$$\begin{aligned} \dot{V}_i &= s_i[\Delta g_i + \psi_i - (\hat{\psi}_i + \hat{\chi}_i + \gamma_i)signs_i] + (\hat{\psi}_i - \psi_i)|s_i| + (\hat{\chi}_i - \chi_i)|s_i| \\ &\leq (|\Delta g_i| + |\psi_i|)|s_i| - (\hat{\psi}_i + \hat{\chi}_i + \gamma_i)|signs_i| + (\hat{\psi}_i - \psi_i)|s_i| + (\hat{\chi}_i - \chi_i)|s_i| \\ &< (\psi_i + \chi_i)|s_i| - (\hat{\psi}_i + \hat{\chi}_i + \gamma_i)|signs_i| + (\hat{\psi}_i - \psi_i)|s_i| + (\hat{\chi}_i - \chi_i)|s_i| \\ &= -P_i|s_i| \end{aligned} \tag{19}$$

Finally, we get

$$\begin{aligned} \dot{V} &= \dot{V}_1 + \dot{V}_2 + \dot{V}_3 + \dot{V}_4 + \dot{V}_5 \\ &< -(G_1|s_1| + G_2|s_2| + G_3|s_3| + G_4|s_4| + G_5|s_5|) \end{aligned} \tag{20}$$

Thus there exist a non-negative real number P such that

$$(G_1|s_1| + G_2|s_2| + G_3|s_3| + G_4|s_4| + G_5|s_5|) > G, \text{ then (20) becomes}$$

$$\begin{aligned} \dot{V} &< -G\sqrt{s_1^2 + s_2^2 + s_3^2 + s_4^2 + s_5^2} \\ &< 0 \end{aligned}$$

Hence, by Lyapunov stability theory $\|s_i\| \rightarrow 0$ as $t \rightarrow \infty$. Thus the error dynamical system (10) asymptotically converges to $s_i = 0$. Therefore, the trajectories of state variables of system (3) and system (6) are asymptotically and globally adjusted to desired set of points with control laws (13) and adaptive laws (14).

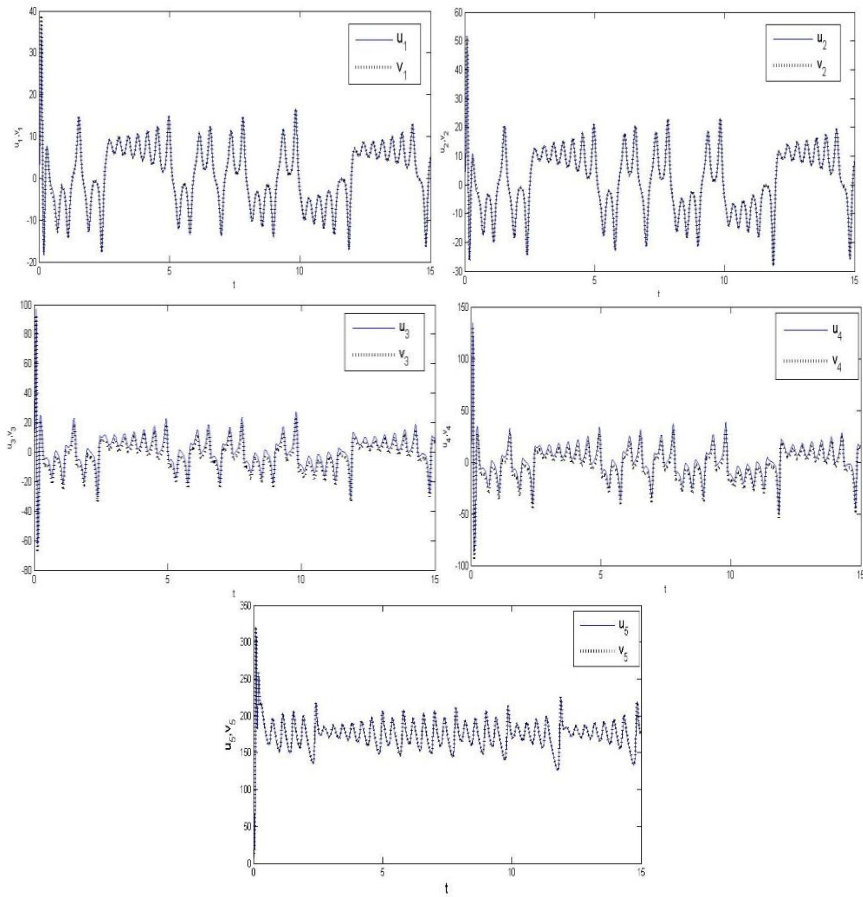


Fig. 3. Different synchronized state trajectories.

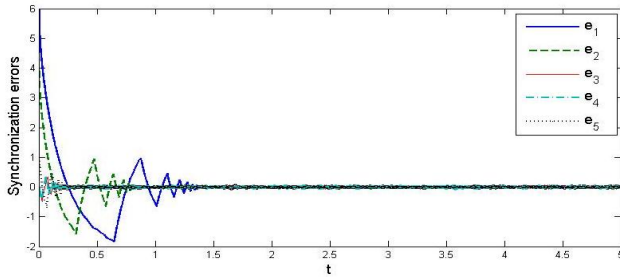


Fig. 4. Synchronization error tends to zero at time $t=1.5$ sec (approx.).

4. Numerical Simulations

Simulations have been performed (using MATLAB) to validate the effectiveness of the proposed scheme for the synchronization between drive system (3) and response system (6). In simulations we have taken fractional order $\alpha=0.95$ with step size=0.001. The

parameters of drive system system (3) are taken as $a_1 = 10, a_2 = 180, a_3 = 1$ and of response system as $b_1 = 21, b_2 = 30, b_3 = 0.6$. Initial conditions for response and drive system are $u(0) = [2,3,5,6,9]^T$ and $v(0) = [8,7,5,6,10]^T$ respectively. Figs. 1 and 2 show the phase portraits of the respective drive and response systems.

The bounded disturbances and uncertainties for the drive are taken as $\Delta f_1 = \cos \pi u_2 - \cos \pi/2t, \Delta f_2 = \sin \pi u_2 - \sin \pi/2t, \Delta f_3 = \cos \pi u_4 + 0.2 \cos \pi t, \Delta f_4 = \sin \pi u_4 + 0.2 \sin \pi t$ and $\Delta f_5 = \sin \pi/2u_5 - 0.1 \cos t$ and for response system are taken as $\Delta g_1 = \cos \pi/2v_1 - \cos 0.1\pi t, \Delta g_2 = \sin \pi/2 v_1 - \sin 0.1 \pi t, \Delta g_3 = \cos \pi v_4 - \cos t, \Delta g_4 = \sin \pi v_4 - \sin t$ and $\Delta g_5 = \cos 0.1\pi v_5 + 0.5 \cos t$. The initial condition for estimating the parameters as $\hat{\psi}(0) = (0.1,0.1,0.1)$ and $\hat{\chi}(0) = (0.1,0.1,0.1)$ and designed control parameters as $m_i = 0.2, n_{i=0.3}, \kappa_1 =, \kappa_2 =, \kappa_3 =, \kappa_4 = \kappa_5 =$ and $\phi_1 =, \phi_2 =, \phi_3 =, \phi_4 =, \phi_5 =$. Fig. 3 represents the trajectories of the drive system and controlled response system. Fig. 4 shows the synchronization error becomes zero as time increasing. Fig. 5 shows that chosen sliding surface converges to zero and hence stable. Fig. 6 shows the estimated bounds of external disturbances of both the drive and response systems.

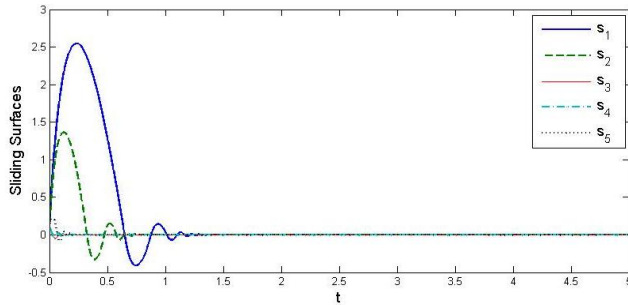


Fig. 5. Sliding surface tends to zero at time t=1.5sec(approx.)

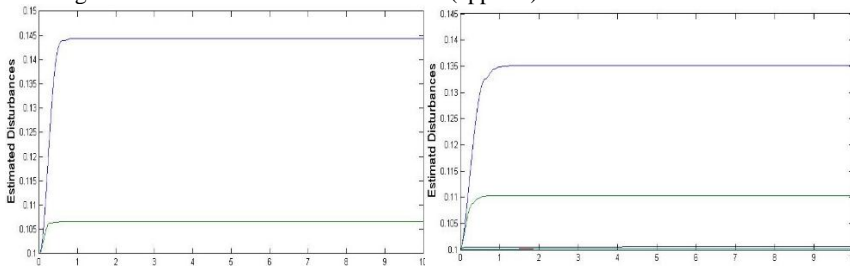


Fig. 6. Estimated Bounds of (a) drive system external disturbances and uncertainties, (b) response system external disturbances and uncertainties.

5.1. Comparison of given synchronization scheme with previous published literature

- In a reference [28], author studies active control technique for synchronization of drive fractional order complex Lorenz system and response T-system. For $\alpha = 0.95$ they achieve synchronization at time t=9 sec (approx.) with uncertain terms and at time t= 6

sec (approx.) without uncertain terms whereas in present scheme we achieve synchronization at time $t=$ sec (approx.) which is much lesser than the synchronization time as reported [28].

- In the reference [32], author studies active control technique for synchronization of fractional order complex Lorenz system and T-system. For $\alpha=0.7$, $\alpha=0.85$ and $\alpha=1$ the synchronization is achieved at time $t=5$ sec (approx.), $t=5.5$ sec (approx.) and $t=7$ sec (approx.) respectively whereas in present scheme we achieve synchronization at time $t=0.2$ sec (approx.), $t=0.6$ sec (approx.) and $t=3.8$ sec (approx.) respectively which is much lesser than the reported synchronization time [32].

Therefore, our results are far better than the result obtained by previous authors which have shown in Fig. 7.

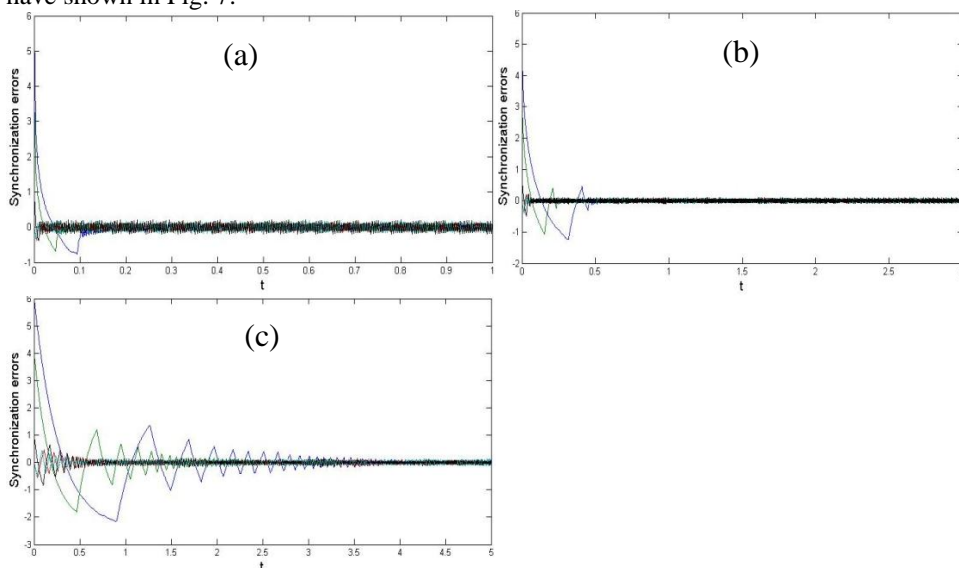


Fig. 7. Synchronization errors (a) for fractional order $\alpha = 0.7$ (b) for fractional order $\alpha = 0.85$ (c) for fractional order $\alpha = 1$.

6. Conclusion

In this paper, we have designed an adaptive sliding mode control technique to synchronize different fractional order complex chaotic systems. By choosing the suitable sliding surface and selecting parameters by update laws to carry out the desired synchronization and to deny the issues of external disturbances and chattering problem. Since the synchronization of fractional order complex chaotic system in the presence of uncertainties and disturbances has not been examined in the prior literature, we have interrogated, and synchronized the considered fractional order complex Lorenz system and complex chaotic T-systems in the presence of uncertainties and disturbances. Also we have compared our results with previous published literature results which have established that our scheme gives better synchronization time than the used technique in

prior literature. Although we have taken both complex system with uncertainties and disturbances but still our synchronization results are better. Also, this scheme will perform significant role to enhance security in communication. Computational methods evaluate the efficiency of the considered scheme.

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