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A New Class on $Ng^{\#}\alpha$-Quotient Mappings in Nano Topological Space

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Abstract

The primary intend of this article is to define a new class of mappings called $Ng^{\#}\alpha$-quotient mappings in nano topological space. The intention is to analyze characterizations and inter relationship of $Ng^{\#}\alpha$-quotient mappings with nano $T_{g^{\#}\alpha}$-space, $Ng^{\#}\alpha$-continuous, $Ng^{\#}\alpha$-open, $Ng^{\#}\alpha$-irresolute, $Ng^{\#}\alpha$-homeomorphism and nano α-quotient mapping. Also several properties of strongly $Ng^{\#}\alpha$-quotient mapping are derived and the relationships among them are illustrated with the help of examples. Their interesting composition with strongly $Ng^{\#}\alpha$-irresolute are established. The concept of $Ng^{\#}\alpha*$-quotient mapping is explored and composition of mappings under strongly $Ng^{\#}\alpha$-quotient mapping and $Ng^{\#}\alpha*$-quotient mapping are discussed. Furthermore, to emphasize $Ng^{\#}\alpha*$-quotient mapping a few examples are considered and derived in detail.

Keywords: $Ng^{\#}\alpha$-Quotient mapping; Strongly $Ng^{\#}\alpha$-quotient mapping; $Ng^{\#}\alpha*$-Quotient mapping.

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1. Introduction

The class of quotient mappings is one of the most important classes of mappings in topology. Bhuvaneshwari *et al.* presented nano g-closed sets in nano topological space [1]. M. K. Gosh introduced separation axioms and graphs of functions in nano topological spaces [2]. Thivagar *et al.* established nano continuity in 2013 [3]. In another article the author introduced a nano topological space with respect to a subset X of an universe which is defined in terms of lower approximation, upper approximation and boundary region [4]. Thivagar *et al.* also defined quotient mappings in topological space [5]. The nano α-continuity was introduced by Nachiyar and K. Bhuvaneswari [6]. Nono *et al.* introduced the concept of $g^{\#}\alpha$-closed sets to investigate some of their topological properties [7]. The primary intention of this

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article is to introduce $Ng^{\#}\alpha$ -quotient mappings in nano topological spaces. Likewise, the concept of strongly $Ng^{\#}\alpha$-quotient mapping and $Ng^{\#}\alpha^*$-quotient mapping are explored to study their fundamental properties in nano topological space.

2. Preliminaries

Definition 2.1 ([8]). Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space and Let $G \subseteq U$. Then

(1). The lower approximation of G with respect to R is the set of all objects, which can be for certain classified as G with respect to R and it is denoted by $L_R(G)$. That is, $L_R(G) = \cup\{R(a): R(a) \subseteq G, a \in U\}$, where $R(a)$ denotes the equivalence class determined by a.

(2). The upper approximation of G with respect to R is the set of all objects, which can be possibly classified as G with respect to R and it is denoted by $U_R(G)$. That is, $U_R(G) = \cup\{R(a): R(a) \cap G \neq \emptyset, a \in U\}$.

(3). The boundary region of G with respect to R is the set of all objects, which can be classified neither as G nor as not G with respect to R and it is denoted by $B_R(G)$. That is, $B_R(G) = U_R(G) - L_R(G)$.

Proposition 2.2 ([4]). If (U, R) is an approximation space and $G, H \subseteq U$. Then

(1) $L_R(G) \subseteq G \subseteq U_R(G)$
(2) $L_R(\emptyset) = U_R(\emptyset) = \emptyset$ and $L_R(U) = U = U_R(U)$
(3) $U_R(G \cup H) = U_R(G) \cup U_R(H)$
(4) $U_R(G \cap H) = U_R(G) \cap U_R(H)$
(5) $L_R(G \cup H) = L_R(G) \cup L_R(H)$
(6) $L_R(G \cap H) = L_R(G) \cap L_R(H)$
(7) $L_R(G) \subseteq L_R(H)$ and $U_R(G) \subseteq U_R(H)$ whenever $G \subseteq H$
(8) $U_R(G^c) = (L_R(G))^c$ and $L_R(G^c) = (U_R(G))^c$
(9) $U_R(G) = L_R U_R(G) = U_R(G)$
(10) $L_R L_R(G) = U_R L_R(G) = L_R(G)$.

Definition 2.3. A subset H of $(U, \tau_R(G))$ is called Nano $g^{\#}\alpha$-closed set [10] (briefly $Ng^{\#}\alpha$-closed) if $N\alpha cl(H) \subseteq V$ whenever $H \subseteq V$ and V is Nano g-open in $(U, \tau_R(G))$. The complements of Nano $g^{\#}\alpha$-closed set is Nano $g^{\#}\alpha$-open set in $(U, \tau_R(G))$.

Definition 2.4. Let $(U, \tau_R(G))$ and $(V, \sigma_R(H))$ be nano topological space. Then the function f: $(U, \tau_R(G)) \rightarrow (V, \sigma_R(H))$ is called:

1. nano $g^{\#}\alpha$-continuous [10] if $f^{-1}(h)$ is a $Ng^{\#}\alpha$-closed set in $(U, \tau_R(G))$ for every nano closed set h in $(V, \sigma_R(H))$.

2. strongly $Ng^{\#}$ α-open mapping [9] if for every $Ng^{\#}$ α-open set D of (U, $\tau_R(G)$), $f(D)$ is $Ng^{\#}$ α -open set D of (V, $\sigma_R(H)$).

Definition 2.5. Let f: (U, $\tau_R(G)$) → (V, $\sigma_R(H)$) be a surjective function. Then f is said to be:
1. nano-quotient mapping [5] if f is nanocontinuous and $f^{-1}(E)$ is nano open in (U, $\tau_R(G)$) implies E is an nanoopen set in (V, $\sigma_R(H)$).
2. nano α-quotient mapping [5] if f is nano α-continuous and $f^{-1}(E)$ is nano open in (U, $\tau_R(G)$) implies E is an nano α-open set in(V, $\sigma_R(H)$).

3. Ng$^{\#}$α-Quotient Mappings

Definition 3.1. Let j: (U, $\tau_R(G)$) → (V, $\sigma_R(H)$) be a surjective function. Then j is said to be $Ng^{\#}$α-quotient mapping if j is Ng$^{\#}$α-continuous and $j^{-1}(D)$ is nano open in (U, $\tau_R(G)$) implies D is an $Ng^{\#}$α-open set in(V, $\sigma_R(H)$).

Example 3.2. Let U= $\{\alpha,\beta,\gamma,\delta\}$ with U/R = $\{\{\alpha\},\{\beta,\gamma\}, \{\delta\}\}$ and G = $\{\beta,\delta\}$. Let $\tau_R(G)$ = {Ø, $\{\delta\}$, $\{\beta,\gamma\}$, $\{\beta,\gamma,\delta\}$, U} and $\tau_R{}^c(G)$ = {Ø, $\{\alpha\}$, $\{\alpha,\delta\}$, $\{\alpha,\beta,\gamma\}$, U} be the nano topological space. Then the $Ng^{\#}$α-open sets are {Ø, $\{\beta\}$, $\{\gamma\}$, $\{\delta\}$, $\{\beta,\delta\}$, $\{\beta,\gamma\}$, $\{\gamma,\delta\}$, $\{\beta,\gamma,\delta\}$, U}.

Let V = {t, u, v, w} with V/R = {{t,u}, {v}, {w}} and H = {t, v}. Let $\sigma_R(H)$= {Ø, {v}, {t, u}, {t, u, v}, V} and $\sigma_R{}^c(H)$ = {Ø, {w}, {v, w}, {t, u, w}, V} be the nano topological space. Then the $Ng^{\#}$α-open sets are {Ø, {a}, {b}, {c}, {a, c}, {b, c}, {a, b}, {c, d}, {a, b, c}, {a, b, d}, V}. Define a functionj: (U, $\tau_R(G)$) → (V, $\sigma_R(H)$)such that j: (U, $\tau_R(G)$) → (V, $\sigma_R(H)$) as $j(\alpha) = w$, $j(\beta) = u$, $j(\gamma) = v$, $j(\delta) = t$. Then j is $Ng^{\#}$α-quotient mapping.

Definition 3.3. A nano topological space (U, $\tau_R(G)$) is said to be nano $T_{g^{\#}\alpha}$-space [2] (in short $NT_{g^{\#}\alpha}$-space) if every nano $g^{\#}\alpha$-closed set in it is nano closed set.

Definition 3.4. Let (U, $\tau_R(G)$) and (V, $\sigma_R(H)$) be nano topological space. Then the function f: (U, $\tau_R(G)$) → (V, $\sigma_R(H)$) is called
(i) $Ng^{\#}$α-open mappingif for every nano open set A of (U, $\tau_R(G)$), its image $j(A)$ is $Ng^{\#}$α-open in (V, $\sigma_R(H)$).
(ii). $Ng^{\#}\alpha$-irresolute if the inverse image of every $Ng^{\#}\alpha$-closed set in (V, $\sigma_R(H)$) is $Ng^{\#}\alpha$-closed in (U, $\tau_R(G)$).
(iii). $Ng^{\#}$α-homeomorphism if f is 1-1 and onto, $Ng^{\#}$α-continuous and $Ng^{\#}$α-open.

Theorem 3.5. Let j: (U, $\tau_R(G)$) → (V, $\sigma_R(H)$) bea nano quotient mapping, then j is a $Ng^{\#}$α-quotient mapping.

Proof. Suppose j: (U, $\tau_R(G)$) → (V, $\sigma_R(H)$) is a nano quotient mapping. Let E be nano open set in (V, $\sigma_R(H)$), since j is nano quotient mapping $j^{-1}(E)$ is $Ng^{\#}$α-open in (U,

$\tau_R(G)$). Therefore, j is $Ng^{\#}\alpha$-continuous. Let $E \subseteq (V, \sigma_R(H))$ and $j^{-1}(E)$ be nano open in $(V, \sigma_R(H))$. Since j is a nano quotient mapping, E is a nano open set in $(V, \sigma_R(H))$, that is $Ng^{\#}\alpha$-open set in $(V, \sigma_R(H))$. Hence j is $Ng^{\#}\alpha$-quotient mapping.

Remark 3.6. Let j: $(U, \tau_R(G)) \to (V, \sigma_R(H))$ be a $Ng^{\#}\alpha$-quotient mapping, then j need not be nano quotient mapping as shown by the following example.

Example 3.7. In example 3.2, j is a $Ng^{\#}\alpha$-quotient mapping but j is not nano quotient mapping since $j^{-1}(\{t, u\}) = (\{\beta, \delta\}) \notin \tau_R(G)$.

Theorem 3.8. Every nano α-quotient mapping is a $Ng^{\#}\alpha$-quotient mapping.

Proof. Since every nano α - continuous is $Ng^{\#}\alpha$-continuous [10]. Also every nano α - open set is $Ng^{\#}\alpha$-open [10] and hence the proof follows from the definitions ([3.1], [2.6]).

Remark 3.9. The converse of the preceding theorem need not be true which is shown by the following example.

Example 3.10. In example 3.2, the nano α-open sets in $(U, \tau_R(G))$ are $\{\emptyset, \{\delta\}, \{\beta,\gamma\}, \{\beta,\gamma,\delta\}, U\}$ and the nano α-closed sets are $\{\emptyset, \{\alpha\}, \{\alpha,\delta\}, \{\alpha,\beta,\gamma\}, U\}$. Also the nano α-open sets in $(V, \sigma_R(H))$ are $\{\emptyset, \{v\}, \{t, u\}, \{t, u, v\}, \{t, u, w\}, V\}$ and the nano α-closed sets are $\{\emptyset, \{v\}, \{w\}, \{v, w\}, \{t, u, w\}, V\}$. Therefore j: $(U, \tau_R(G)) \to (V, \sigma_R(H))$ is $Ng^{\#}\alpha$-quotient mapping but not nano α-quotient mapping since $j^{-1}(\{t, u, w\}) = (\{\alpha, \beta, \delta\})$ is not nano α-open set in $(U, \tau_R(G))$.

Theorem 3.11. If f: $(U, \tau_R(G)) \to (V, \sigma_R(H))$ is surjective $Ng^{\#}\alpha$-continuous and $Ng^{\#}\alpha$-open, then f is an $Ng^{\#}\alpha$-quotient mapping.

Proof. Let $f^{-1}(D)$ be nano open in $(U, \tau_R(G))$. Then $f(f^{-1}(D))$ is an $Ng^{\#}\alpha$-open set. Since f is an $Ng^{\#}\alpha$-open set. Hence D is an $Ng^{\#}\alpha$-open set, as f is surjective, $f(f^{-1}(D)) = D$. Thus f is an $Ng^{\#}\alpha$-quotient map.

Theorem 3.12. If f: $(U, \tau_R(G)) \to (V, \sigma_R(H))$ be an nano open surjective $Ng^{\#}\alpha$-irresolute and h: $(V, \sigma_R(H)) \to (W, \eta_R(I))$ be an $Ng^{\#}\alpha$-quotient mapping. Then $h \circ f$: $(U, \tau_R(G)) \to (W, \eta_R(I))$ is an $Ng^{\#}\alpha$-quotient mapping.

Proof. Let D be any nano open set in $(W, \eta_R(I))$. Then $h^{-1}(D)$ is an $Ng^{\#}\alpha$-open, since h is an $Ng^{\#}\alpha$-quotient mapping. And also since f is $Ng^{\#}\alpha$-irresolute, $f^{-1}(h^{-1}(D))$ is an $Ng^{\#}\alpha$-open set. Hence $(h \circ f)^{-1}(D)$ is an $Ng^{\#}\alpha$-open set implies $h \circ f$ is an $Ng^{\#}\alpha$-open set. Hence $h \circ f$ is an $Ng^{\#}\alpha$-continuous. Also, assume that $(h \circ f)^{-1}(D)$ be nano open in $(U, \tau_R(G))$ for $V \subseteq W$, that is $f^{-1}((h^{-1})(D))$ is nano open in $(U, \tau_R(G))$. Since f: $(U, \tau_R(G)) \to (V, \sigma_R(H))$ is nano open, $f(f^{-1}(h^{-1}(D))$ is nano open in $(V, \sigma_R(H))$. It follows that

$h^{-1}(D)$ is nano open in (V, $\sigma_R(H)$), f is surjective. Since h is an $Ng^{\#}$α-quotient mapping, (V, $\sigma_R(H)$) is an $Ng^{\#}$α-quotient mapping, (V, $\sigma_R(H)$) is an $Ng^{\#}$α-open set. Thus $h \circ f$: (U, $\tau_R(G)$) → (W, $\eta_R(I)$) is an $Ng^{\#}$α-quotient mapping.

Definition 3.13. Let f: (U, $\tau_R(G)$) → (V, $\sigma_R(H)$) be an on-to map. Then f is called strongly $Ng^{\#}$α-quotient mapping provided a set E of (V, $\sigma_R(H)$) is nano-open in (V, $\sigma_R(H)$) if and only if $f^{-1}(E)$ is $Ng^{\#}$α-open in (U, $\tau_R(G)$).

Example 3.14. Let $U = \{\alpha, \beta, \gamma, \delta\}$ be the universe with $U/R = \{\{\alpha, \beta\}, \{\gamma\}, \{\delta\}\}$ and let $G = \{\alpha, \gamma, \delta\}$. Then the nano open sets are {Ø, $\{\alpha, \beta\}$, $\{\gamma, \delta\}$, U}. Let V = {1, 2, 3} be another universe with V/R = {1, 2} and let H = {{1}, {2, 3}}. Then the nano open sets are {Ø, {1}, {2, 3}, V}. Define the function f: (U, $\tau_R(G)$) → (V, $\sigma_R(H)$) as $f(\alpha) = 1 = f(\beta) =$, $f(\gamma) = 3$, $f(\delta) = 2$. Then f is strongly $Ng^{\#}$α-quotient mapping.

Theorem 3.15. Let j: (U, $\tau_R(G)$) → (V, $\sigma_R(H)$) be strongly $Ng^{\#}$α-quotient mapping, then j is strongly $Ng^{\#}$α-open mapping.

Proof. Let j: (U, $\tau_R(G)$) → (V, $\sigma_R(H)$) be a strongly $Ng^{\#}$α-quotient mapping. Let D be a $Ng^{\#}$α-open set in (U, $\tau_R(G)$). That is $j^{-1}(j(D))$ is $Ng^{\#}$α-open in (U, $\tau_R(G)$). Since j is strongly $Ng^{\#}$α-quotient, $j(D)$ is open and hence $Ng^{\#}$α-open in (V, $\sigma_R(H)$). This shows that j is strongly $Ng^{\#}$α-open mapping.

Remark 3.16. The converse need not be true which is shown by the succeeding example.

Example 3.17. Let U = {e, f, g, h} be the universe with U/R = {{e}, {g}, {f, h}} and let G = {e, f}. Then the nano open sets are {Ø, {e}, {f, h}, {e, f, h}, U}. The $Ng^{\#}$α-open sets are {Ø, {g}, {f}, {e}, {e, g}, {f, h}, {e, h}, {e, f, h}, U}.

Let V = {e, f, g, h} be the universe with V/R = {{e}, {f, g}, {h}} and let H = {f, h}. Then the nano open sets are {Ø, {h}, {f, g}, {f, g, h}, V}. The $Ng^{\#}$α-open sets are {Ø, {f}, {g}, {h}, {f, g}, {f, h}, {g, h}, {f, g, h}, V}. Define the function j: (U, $\tau_R(G)$) → (V, $\sigma_R(H)$) as $j(e) = f$, $j(f) = h$, $j(g) = e$, $j(h) = g$. Then, the function j is strongly $Ng^{\#}$α-open mapping but not strongly $Ng^{\#}$α-quotient mapping since the set {e, g} is $Ng^{\#}$α-open in (U, $\tau_R(G)$) but not nano open in (V, $\sigma_R(H)$).

Theorem 3.18. Let j: (U, $\tau_R(G)$) → (V, $\sigma_R(H)$) be strongly $Ng^{\#}$α-quotient mapping, then j is a $Ng^{\#}$α-quotient mapping.

Proof. Let W be an nano open set in (V, $\sigma_R(H)$). Since j is nano strongly $Ng^{\#}$α-quotient, $j^{-1}(W)$ is an $Ng^{\#}$α-openset in (U, $\tau_R(G)$). Let $j^{-1}(W)$ be nano open in (U, $\tau_R(G)$), then $j^{-1}(W)$ is an $Ng^{\#}$α-open set in (U, $\tau_R(G)$). Hence j is $Ng^{\#}$α-quotient mapping.

Remark 3.19. Let j: $(U, \tau_R(G)) \to (V, \sigma_R(H))$ be $Ng^{\#}\alpha$-quotient mapping then j need not be strongly $Ng^{\#}\alpha$-quotient mapping.

Example 3.20. In example 3.2, $j^{-1}(u) = \{\beta\}$ is $Ng^{\#}\alpha$-open set in $(U, \tau_R(G))$, but $\{u\}$ is not nano open set in $(V, \sigma_R(H))$. Hence j is $Ng^{\#}\alpha$-quotient mapping but not strongly $Ng^{\#}\alpha$-quotient mapping.

Theorem 3.21. If a function p: $(U, \tau_R(G)) \to (V, \sigma_R(H))$ is $Ng^{\#}\alpha$-homeomorphism, then p is a $Ng^{\#}\alpha$-quotient mapping.

Proof. Since p is $Ng^{\#}\alpha$-homeomorphism, p is bijective and p is $Ng^{\#}\alpha$-continuous. Let $p^{-1}(D)$ be open in $(U, \tau_R(G))$. Since $p^{-1}(D)$ is $Ng^{\#}\alpha$-continuous, $p(p^{-1}(D)) = D$ is $Ng^{\#}\alpha$-open in $(V, \sigma_R(H))$. Hence p is a $Ng^{\#}\alpha$-quotient mapping.

Definition 3.22. Let $(U, \tau_R(G))$ and $(V, \sigma_R(H))$ be a nano topological spaces, then h: $(U, \tau_R(G)) \to (V, \sigma_R(H))$ is a strongly $Ng^{\#}\alpha$-irresolute function if $h^{-1}(E)$ is open in $(U, \tau_R(G))$ for every $Ng^{\#}\alpha$-open set E in $(V, \sigma_R(H))$.

Theorem 3.23. Let j: $(U, \tau_R(G)) \to (V, \sigma_R(H))$ be a $Ng^{\#}\alpha$-quotient mapping where $(U, \tau_R(G))$ and $(V, \sigma_R(H))$ are nano $T_{g^{\#}\alpha}$-spaces. Then h: $(V, \sigma_R(H)) \to (W, \eta_R(I))$ is strongly $Ng^{\#}\alpha$-irresolute if and only if the composite mapping $h \circ j$: $(U, \tau_R(G)) \to (W, \eta_R(I))$ is strongly $Ng^{\#}\alpha$-irresolute.

Proof. Let j: $(U, \tau_R(G)) \to (V, \sigma_R(H))$ be strongly $Ng^{\#}\alpha$-irresolute and E be a $Ng^{\#}\alpha$-open set in $(W, \eta_R(I))$. Since j is strongly $Ng^{\#}\alpha$-irresolute, $j^{-1}(E)$ is nano open in $(V, \sigma_R(H))$. Then $(h \circ j)^{-1}(D) = j^{-1}(h^{-1}(E))$ is $Ng^{\#}\alpha$-open in $(U, \tau_R(G))$ (since j is $Ng^{\#}\alpha$-quotient). Since $(U, \tau_R(G))$ is nano $T_{g^{\#}\alpha}$-space, $j^{-1}(h^{-1}(E))$ is nano open in $(U, \tau_R(G))$ and hence the composite map $h \circ j$ is strongly $Ng^{\#}\alpha$-irresolute.

Conversely, suppose that the composite function $h \circ j$ is strongly $Ng^{\#}\alpha$-irresolute. Let E be a $Ng^{\#}\alpha$-open set in $(W, \eta_R(I))$, $j^{-1}(h^{-1}(E))$ is nano open in $(U, \tau_R(G))$. Since j is $Ng^{\#}\alpha$-quotient mapping, it implies that, $j^{-1}(E)$ is $Ng^{\#}\alpha$-open in $(V, \sigma_R(H))$. Since $(V, \sigma_R(H))$ is nano $T_{g^{\#}\alpha}$-space, it implies that $j^{-1}(E)$ is nano open in $(V, \sigma_R(H))$. Hence, j is strongly $Ng^{\#}\alpha$-irresolute.

Theorem 3.24. Let f: $(U, \tau_R(G)) \to (V, \sigma_R(H))$ be a $Ng^{\#}\alpha$-open, surjective and $Ng^{\#}\alpha$-irresolute map and h: $(V, \sigma_R(H)) \to (W, \eta_R(I))$ be a strongly $Ng^{\#}\alpha$-quotient mapping. Then $h \circ f$: $(U, \tau_R(G)) \to (W, \eta_R(I))$ is a strongly $Ng^{\#}\alpha$-quotient mapping.

Proof. Let D be an nano open set in $(W, \eta_R(I))$. Then D is $Ng^{\#}\alpha$-open. Since h is a strongly $Ng^{\#}\alpha$-quotient mapping, $h^{-1}(D)$ is $Ng^{\#}\alpha$-open in $(V, \sigma_R(H))$. Then $f^{-1}(h^{-1}(D))$ is $Ng^{\#}\alpha$-open in $(U, \tau_R(G))$ (Since f is $Ng^{\#}\alpha$-irresolute). Hence $(h \circ f)^{-1}(D)$ is $Ng^{\#}\alpha$-open in $(V, \sigma_R(H))$. Let $(h \circ f)^{-1}(D)$ is $Ng^{\#}\alpha$-open in $(U, \tau_R(G))$. That is, $f^{-1}(h^{-1}(D))$ is $Ng^{\#}\alpha$-open in $(U, \tau_R(G))$. Since f is a $Ng^{\#}\alpha$-open mapping, $(f^{-1}(h^{-1}(D)))$ is $Ng^{\#}\alpha$-open in (V, $\sigma_R(H)$) and hence $h^{-1}(D)$ is $Ng^{\#}$α-open in (V, $\sigma_R(H)$). Since h is a strongly $Ng^{\#}$α-quotient mapping, D is nano open in (W, $\eta_R(I)$) and therefore $h∘f$: (U, $\tau_R(G)$) → (W, $\eta_R(I)$) is a strongly $Ng^{\#}$α-quotient mapping.

4. Ng$^{\#}$α*-Quotient Map

Definition 4.1. Let f: (U, $\tau_R(G)$) → (V, $\sigma_R(H)$) be a surjective function. Then f is called $Ng^{\#}$α*-quotient mapping if f is $Ng^{\#}$α-irresolute and $f^{-1}(D)$ is $Ng^{\#}$α-open in (U, $\tau_R(G)$) implies D is nano-open in (V, $\sigma_R(H)$).

Example 4.2. Let U = $\{\alpha,\beta,\gamma,\delta\}$ be the universe with U/R = $\{\{\alpha,\beta\}, \{\gamma\}, \{\delta\}\}$ and let G = $\{\alpha,\gamma,\delta\}$. Then the nano open sets are {Ø, $\{\alpha,\beta\}$, $\{\gamma,\delta\}$, U}. The $Ng^{\#}$α-closed sets are {Ø, $\{\alpha,\beta\}$, $\{\gamma,\delta\}$, U}.

Let V = {t, u, w} with V/R = {t, u} and H = {t, u}. Let {Ø, {t}, {u, w}, V} be the nano open sets. The $Ng^{\#}$α-closed are {Ø, {t}, {u, w}, V}. Define the function f: (U, $\tau_R(G)$) → (V, $\sigma_R(H)$) as $f(\alpha)$ = t = $f(\beta)$, $f(\gamma)$ = u, $f(\delta)$ = w. Then, the function f is $Ng^{\#}$α*-quotient mapping.

Theorem 4.3. Let f: (U, $\tau_R(G)$) → (V, $\sigma_R(H)$) be a $Ng^{\#}$α*-quotient mapping then f is strongly $Ng^{\#}$α-quotient mapping.

Proof. If f: (U, $\tau_R(G)$) → (V, $\sigma_R(H)$) is $Ng^{\#}$α-irresolute and D is nano open set in (V, $\sigma_R(H)$) then $f^{-1}(D)$ is $Ng^{\#}$α-open set in (U, $\tau_R(G)$). Suppose $f^{-1}(D)$ is $Ng^{\#}$α-open set in (U, $\tau_R(G)$), since f is an $Ng^{\#}$α*-quotient mapping, D is a nano open set in (V, $\sigma_R(H)$). Hence f is strongly $Ng^{\#}$α-quotient mapping.

Theorem 4.4. Every $Ng^{\#}$α*-quotient mapping is $Ng^{\#}$α-irresolute function.

Proof. Let f: (U, $\tau_R(G)$) → (V, $\sigma_R(H)$) be a $Ng^{\#}$α*-quotient mapping. Let S be a $Ng^{\#}$α-open set in (U, $\tau_R(G)$). That is $f^{-1}(f(S))$ is $Ng^{\#}$α-open in (U, $\tau_R(G)$). Since f: (U, $\tau_R(G)$) → (V, $\sigma_R(H)$) is $Ng^{\#}$α*-quotient mapping, thus $f(S)$ is nano open in (V, $\sigma_R(H)$) and hence $Ng^{\#}$α-open in (V, $\sigma_R(H)$). Therefore, f is $Ng^{\#}$α-irresolute.

Remark 4.5. The reverse implication of the above theorem need not be true as shown in the succeeding example.

Example 4.6. Let U = {e, i, g, h} with U/R = {{e}, {g}, {i, h}} and G = {e, i}. Then $\tau_R(G)$ = {Ø, {g}, {e, i}, {e, i, g}, U} and $\tau_R{}^c(G)$ = {Ø, {h}, {g, h}, {e, i, h}, U} be the nano closed set.

Let V = $\{\alpha,\beta,\gamma,\delta\}$ be the universe with V/R = $\{\{\alpha,\beta\}, \{\gamma\}, \{\delta\}\}$ and let H = $\{\alpha,\gamma\}$. Then $\sigma_R(H)$ = {Ø, $\{\alpha,\beta\}$, V} and $\sigma_R{}^c(H)$ = {Ø, $\{\gamma,\delta\}$, V}. Define the function f: (U, $\tau_R(G)$) → (V, $\sigma_R(H)$) as $f(e)$ = α, $f(i)$ = β, f(g) = γ, $f(h)$ = δ. Then, the function f

is $Ng^{\#}\alpha$-irresolute function but not $Ng^{\#}\alpha$ *-quotient mapping since the set $f^{-1}(\{\gamma\})$ = $\{g\}$ is $Ng^{\#}\alpha$-open in $(U, \tau_R(G))$ but not nano open in $(V, \sigma_R(H))$.

Remark 4.7. The following example shows that a nano quotient mapping is neither $Ng^{\#}\alpha^*$-quotient nor strongly $Ng^{\#}\alpha$-quotient mapping.

Example 4.8. Let $U = \{\alpha, \beta, \gamma, \delta\}$ be the universe with $U/R = \{\{\alpha, \beta\}, \{\gamma\}, \{\delta\}\}$ and let $G = \{\alpha, \gamma\}$. Then the nano open sets are $\{\emptyset, \{\gamma\}, \{\alpha, \beta\}, \{\alpha, \beta, \gamma\}, U\}$. The $Ng^{\#}\alpha$-closed sets are $\{\emptyset, \{\gamma\}, \{\delta\}, \{\alpha, \beta\}, \{\gamma, \delta\}, \{\alpha, \delta\}, \{\beta, \delta\}, \{\beta, \gamma, \delta\}, \{\alpha, \gamma, \delta\}, \{\alpha, \beta, \delta\}, U\}$ and the $Ng^{\#}\alpha$-open sets are $\{\emptyset, \{\gamma\}, \{\alpha\}, \{\beta\}, \{\alpha, \gamma\}, \{\beta, \gamma\}, \{\alpha, \beta\}, \{\gamma, \delta\}, \{\alpha, \beta, \gamma\}, \{\alpha, \beta, \delta\}, U\}$.

Let $V = \{a, b, c, d\}$ be another universe with $V/R = \{\{a\}, \{b, c\}, \{d\}\}$ and let $H = \{b, d\}$. Then the nano open sets are $\{\emptyset, \{d\}, \{b, c\}, \{b, c, d\}, V\}$ and the nano closed sets are $\{\emptyset, \{a\}, \{a, d\}, \{a, b, c\}, V\}$. The $Ng^{\#}\alpha$-closed are $\{\emptyset, \{a\}, \{a, b\}, \{a, d\}, \{a, c\}, \{a, b, c\}, \{a, c, d\}, \{a, b, d\}, V\}$ and $Ng^{\#}\alpha$-open sets are $\{\emptyset, \{b\}, \{c\}, \{d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{b, c, d\}, V\}$. Define the function $j: (U, \tau_R(G)) \rightarrow (V, \sigma_R(H))$ as $j(\alpha) = c$, $j(\beta) = b$, $j(\gamma) = d$, $j(\delta) = a$. Then, the function j is nano quotient mapping but not $Ng^{\#}\alpha^*$-quotient mapping and strongly $Ng^{\#}\alpha$-quotient mapping since $j^{-1}(\{b\}) = \{\beta\}$ is not nano open in $(V, \sigma_R(H))$.

Theorem 4.9. Let f: $(U, \tau_R(G)) \rightarrow (V, \sigma_R(H))$ be a surjective strongly $Ng^{\#}\alpha$-open mapping and $Ng^{\#}\alpha$-irresolute function and h: $(V, \sigma_R(H)) \rightarrow (W, \eta_R(I))$ be a $Ng^{\#}\alpha^*$-quotient mapping. Then $h \circ f$ is an $Ng^{\#}\alpha^*$-quotient mapping.

Proof. Let D be a $Ng^{\#}\alpha$-open set in $(W, \eta_R(I))$. Then $h^{-1}(D)$ is an $Ng^{\#}\alpha$-open set in $(V, \sigma_R(H))$. Since f is $Ng^{\#}\alpha$-irresolute, $f^{-1}(h^{-1}(D))$ is an $Ng^{\#}\alpha$-open set in $(U, \tau_R(G))$ implies $h \circ f$ is $Ng^{\#}\alpha$-irresolute. Suppose $(h \circ f)^{-1}(D)$ is an $Ng^{\#}\alpha$-open set in $(U, \tau_R(G))$ for $V \subseteq W$, that is, $f^{-1}(h^{-1}(D))$ is an $Ng^{\#}\alpha$-open in $(U, \tau_R(G))$. Since f is strongly $Ng^{\#}\alpha$-open, $f(f^{-1}(h^{-1}(D)))$ is an $Ng^{\#}\alpha$-open set in $(V, \sigma_R(H))$ and since f is surjective, $h^{-1}(D)$ is an $Ng^{\#}\alpha$-open set in $(V, \sigma_R(H))$. Since his a $Ng^{\#}\alpha^*$-quotient mapping, D is a nano open set in $(W, \eta_R(I))$. Thus $h \circ f$ is an $Ng^{\#}\alpha^*$-quotient mapping.

Theorem 4.10. Let p: $(U, \tau_R(G)) \rightarrow (V, \sigma_R(H))$ be a strongly $Ng^{\#}\alpha$-quotient mapping and q: $(V, \sigma_R(H)) \rightarrow (W, \eta_R(I))$ be a $Ng^{\#}\alpha^*$-quotient mapping and $(V, \sigma_R(H))$ be an nano $T_{g^{\#}\alpha}$-space. Then $q \circ p$: $(U, \tau_R(G)) \rightarrow (W, \eta_R(I))$ is *a* $Ng^{\#}\alpha^*$-quotient mapping.

Proof. Let E be a $Ng^{\#}\alpha$-open set in $(W, \eta_R(I))$. Then $q^{-1}(E)$ is $Ng^{\#}\alpha$-open in $(V, \sigma_R(H))$ (since q is $Ng^{\#}\alpha^*$-quotient map). Since $(V, \sigma_R(H))$ is a nano $T_{g^{\#}\alpha}$-space, $q^{-1}(E)$ is an nano open set in $(V, \sigma_R(H))$. Since p is strongly $Ng^{\#}\alpha$-quotient, $p^{-1}(q^{-1}(E))$ is $Ng^{\#}\alpha$-open in $(U, \tau_R(G))$. That is, $(q \circ p)^{-1}(E)$ is $Ng^{\#}\alpha$-open in $(U, \tau_R(G))$ and hence $q \circ p$: $(U, \tau_R(G)) \rightarrow (W, \eta_R(I))$ is $Ng^{\#}\alpha$-irresolute. Let $(q \circ p)^{-1}(E)$ be a $Ng^{\#}\alpha$-open set in $(U, \tau_R(G))$. That is, $p^{-1}(q^{-1}(E))$ is $Ng^{\#}\alpha$-open in $(U, \tau_R(G))$. This implies $q^{-1}(E)$ is nano open in (V, $\sigma_R(H)$). Hence $q^{-1}(E)$ is a $Ng^{\#}\alpha$-open set. Since q is $Ng^{\#}\alpha$*-quotient, E is nano open and hence $q \circ p$: (U, $\tau_R(G)$) → (W, $\eta_R(I)$) is a $Ng^{\#}\alpha$*-quotient mapping.

Theorem 4.11. The composition of two $Ng^{\#}\alpha$*-quotient mappings is also a $Ng^{\#}\alpha$*-quotient mapping.

Proof. Let f: (U, $\tau_R(G)$) → (V, $\sigma_R(H)$) and j: (V, $\sigma_R(H)$) → (W, $\eta_R(I)$ be two $Ng^{\#}\alpha$*-quotient mappings. Let E be a $Ng^{\#}\alpha$-open set in (W, $\eta_R(I)$). Then $j^{-1}(E)$ is a $Ng^{\#}\alpha$-open set in (V, $\sigma_R(H)$). Since f is a $Ng^{\#}\alpha$*-quotient mapping, $f^{-1}(j^{-1}(E))$ is a $Ng^{\#}\alpha$-open set in (U, $\tau_R(G)$). That is, $(j \circ f)^{-1}(E)$ is $Ng^{\#}\alpha$-open in (U, $\tau_R(G)$). Hence $j \circ f$: (U, $\tau_R(G)$) → (W, $\eta_R(I)$) is $Ng^{\#}\alpha$-irresolute. Let $(j \circ f)^{-1}$ (E) be a $Ng^{\#}\alpha$-open set in (U, $\tau_R(G)$). Then $f^{-1}(j^{-1}(E))$ is $Ng^{\#}\alpha$-open in (U, $\tau_R(G)$). This implies $j^{-1}(E)$ is nano open in (V, $\sigma_R(H)$) and hence $j^{-1}(E)$ is $Ng^{\#}\alpha$-open. Since j is a $Ng^{\#}\alpha$*-quotient mapping, E is nano open. Hence $j \circ f$ is a $Ng^{\#}\alpha$*-quotient mapping.

5. Conclusion

The current study yields $Ng^{\#}\alpha$-quotient mappings in nano topological space. The relationships among these introduced concepts are illustrated and their relationships with some nano topological notions such as $Ng^{\#}\alpha$-irresolute, nano α-quotient mapping are shown with the help of examples. The notion of strongly $Ng^{\#}\alpha$-irresolute function was presented which was helpful to verify the interesting results such as theorem 3.23. Some findings concerning strongly $Ng^{\#}\alpha$-quotient mapping and enriched $Ng^{\#}\alpha$*-quotient mappings are investigated and illustrated with a few of their examples in detail. The presented concepts in this study are fundamental for further researches and will open a way to improve more applications on nano topology.

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