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Bianchi Type-VI_0 Cosmological Model with Special Form of Scale Factor in Sen-Dunn Theory of Gravitation

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Abstract

A Bianchi Type-VI_0 cosmological model with a special form of scale factor is studied. Einstein field equations in Sen-Dunn theory are obtained and solved for exact solutions. This solution gives a scenario of the dark energy model which tends to a ΛCDM model. The physical and geometrical properties are also obtained and analyzed with the present day observations.

Keywords: Sen-Dunn theory; Dark energy; ΛCDM model; Bianchi type-VI_0.

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1. Introduction

Einstein General Theory of Relativity is one of the most essential and dynamic phenomena, benefiting in describing and creating models to understand the universe better. Concerning Albert Einstein's theory of gravitation, many other authors developed an alternative theory to Einstein's theory of gravitation developed in recent decades. Like an alternative theory of gravitation by Brans-Dickie [1], the tensor field is identified by the metric tensor of Riemannian geometry and the scalar field, which is alien to the geometry. Sen and Dunn [2] proposed a theory of gravitation based on Lyra's geometry, where the field equation contains metric tensor $\boldsymbol{g_{ij}}$ and scalar function $\boldsymbol{\varphi}$ which is considered as special character of the theory are both geometrized. Dunn [4] proposed the scalar-tensor theory of gravitation using a non-Riemannian geometry in which both the metric tensor and the scalar function have an unambiguous geometric interpretation with a function $\boldsymbol{\varphi = \varphi(x^i)}$ where $\boldsymbol{x^i}$ is the coordinate in a four dimensional Riemannian manifold. Birkhoff's theorem and spherically symmetric static conformally flat solutions are studied using the Sen-Dunn scalar-tensor theory by Reddy [3,5]. Roy and Chatterjee [6] investigated symmetrically charged dust distribution in the Sen-Dunn theory of gravitation. Also, many authors like Singh and Rai [7], Mukherjee [8], Reddy and Naidu [9], Venkateswarlu *et al*. [10,11] had investigated various aspects of cosmological models

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in a different context with the Sen-Dunn scalar-tensor theory of gravitation. Ghate and Sontakke [12] studied Bianchi type-IX dark energy model in the Sen-Dunn theory of gravitation. Patra [13] also studied the string cosmological model in a new scalar-tensor theory of gravitation. Bianchi type models give us the simplicity of the field equations and better understand the universe's anisotropy, which is relatively better than FRW isotropic models. Bianchi type models are homogeneous and anisotropic, giving a better image of the early stages of the present-day observation and the behavior in the different stages. Roy and Singh [14] investigated the Bianchi type VI_0 cosmological model with a magnetic type with the free gravitational field. Patel and Koopar [15] studied viscous fluid with Bianchi type VI_0 cosmological model. Bali *et al*. [16,17], Bali and Pradhan [18] studied the Bianchi type VI_0 cosmological model relative to string theory. Many authors have mainly studied the aspects of dark energy cosmological models relative to the universe [19,20]. Mishra and Sahoo [21] investigated Bianchi type $\boldsymbol{VI_1}$ with the presence of wet, dark energy fluid cosmological model in scale-invariant theory. Anisotropic magnetized holographic Ricci dark energy cosmological models were discussed by Santhi *et al.* [22]. Also, gravity field equations are derived by Santhi *et al*. [23] with the help of a spatially homogeneous and anisotropic Bianchi type-$\boldsymbol{VI_h}$ space-time in a bulk-viscous fluid, containing one-dimensional cosmic strings. Hegazy [24], Satish and Venkateswarlu [25] studied various aspects with Bianchi type VI_0 cosmological models. Investigation of the Kantowaski-Sachs cosmological model with bulk viscous and cosmic string in *f(T)* gravity framework is done by Bhoyar *et al.* [26]. Recently, Dewri [27] studied the spatially homogeneous Robertson-Walker cosmological models with magnetized isotropic dark energy like fluid in the scalar-tensor theory of gravitation. Brahma and Dewri [28] also worked on a paper Bianchi Type – V Modified $\boldsymbol{f}(\boldsymbol{R},\ \boldsymbol{T})$ model in Lyra manifold, that approach the ΛCDM (Lambda cold dark matter) model. Works of different researchers on Sen-Dunn and other scalar-tensor theory mentioned above are the motivation behind this work on accelerating and ΛCDM model. This paper has investigated the Bianchi type VI_0 cosmological model having a variable EoS parameter and deceleration parameter in Sen-Dunn theory.

2. Metric and Field Equation

Bianchi type-VI_0 space-time metric is given by

$$ds^2 = -dt^2 + A^2dx^2 + B^2e^{2x}dy^2 + C^2e^{-2x}dz^2 \tag{1}$$

where A, B, C are function of time t.

The generalization of EoS parameter of perfect fluid separately on each spatial axis by preserving the diagonal form and examining the diagonal form of the energy momentum tensor is.

$$T_{ij} = diag[T_{00}, T_{11}, T_{22}, T_{33}] \tag{2}$$

Thus, on parameterize it formulates

$$T_{ij} = diag\left[\rho, -p_x, -p_y, -p_z\right] = diag\left[1, -w_x, -w_y, w_z\right]\rho$$
$$= diag[1, -w, -(w+\delta), -(w+\gamma)]\rho \tag{3}$$

where ρ is the energy density, p_x, p_y, p_z is the directional pressure, w_x, w_y, w_z are the directional EoS parameters along the x, y and$z$ axis respectively, w is the considered as deviation free EoS parameter, δ and γ are the skewness parameters. The parameters w, δ and γ are essentially need not be constant and may be a function of time.

The field equation given by Sen-Dunn [1] for the combined scalar and tensor fields (in natural units $c = 1, G = 1$) is

$$R_{ij} - \frac{1}{2} g_{ij} R = \omega \varphi^{-2} \left(\varphi_{,i} \varphi_{,j} - \frac{1}{2} g_{ij} \varphi_{,k} \varphi^{,k} \right) - \varphi^{-2} T_{ij} \tag{4}$$

where, $\omega = \frac{3}{2}$, R_{ij}, R, g_{ij} and T_{ij} are the Ricci tensor, Ricci scalar, metric tensor and energy momentum tensor respectively.

The field equation (4) with (3) for the line element (1) gives rise to

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} + \frac{1}{A^2} = \frac{-w\rho}{\varphi^2} + \frac{\omega}{2}\left(\frac{\dot{\varphi}}{\varphi}\right)^2 \tag{5}$$

$$\frac{\ddot{C}}{C} + \frac{\ddot{A}}{A} + \frac{\dot{A}}{A}\frac{\dot{C}}{C} - \frac{1}{A^2} = \frac{-(w+\delta)\rho}{\varphi^2} + \frac{\omega}{2}\left(\frac{\dot{\varphi}}{\varphi}\right)^2 \tag{6}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{B}}{B} - \frac{1}{A^2} = \frac{-(w+\gamma)\rho}{\varphi^2} + \frac{\omega}{2}\left(\frac{\dot{\varphi}}{\varphi}\right)^2 \tag{7}$$

$$\frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} + \frac{\dot{A}}{A}\frac{\dot{C}}{C} - \frac{1}{A^2} = \frac{-\rho}{\varphi^2} + \frac{\omega}{2}\left(\frac{\dot{\varphi}}{\varphi}\right)^2 \tag{8}$$

$$\frac{\dot{C}}{C} - \frac{\dot{B}}{B} = 0 \tag{9}$$

The average scale factor for a Bianchi type VI_0 space time is given as

$$a = (ABC)^{\frac{1}{3}} \tag{10}$$

Also, Hubble's parameter (H), expansion scalar (θ), shear scalar (σ), anisotropic parameter (A_m), and deceleration parameter (q) are defined as

$$H = \frac{\dot{a}}{a} \tag{11}$$

$$\theta = 3H \tag{12}$$

$$\sigma^2 = \frac{1}{2}\left(\sum_{i=1}^{3} H_i^2 - \frac{1}{3}\theta^2\right) \tag{13}$$

$$A_m = \frac{1}{2}\sum_1^3 \left(\frac{\Delta H_i}{H}\right)^2 = \frac{2\sigma^2}{3H^2} \tag{14}$$

$$q = \frac{-a\ddot{a}}{\dot{a}^2} = \frac{d}{dt}\left(\frac{1}{H}\right) - 1 \tag{15}$$

where, $\Delta H_i = H_i - H (i = x, y, z)$ represents the directional Hubble's parameters.

3. Solution of the Field Equation

From (9) we get by integrating

$$C = mB \tag{16}$$

where, m is a constant.

From (16), (7) and (6), the equality in the skewness parameters along the y and z axis is found.

So, the equations (5)-(9) reduces to

$$2\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} + \frac{1}{A^2} = -\frac{w\rho}{\varphi^2} + \frac{\omega}{2}\left(\frac{\dot{\varphi}}{\varphi}\right)^2 \tag{17}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{B}}{B} - \frac{1}{A^2} = \frac{-(w+\delta)\rho}{\varphi^2} + \frac{\omega}{2}\left(\frac{\dot{\varphi}}{\varphi}\right)^2 \tag{18}$$

$$2\frac{\dot{A}}{A}\frac{\dot{B}}{B}+\frac{\dot{B}}{B^2}-\frac{1}{A^2}=\frac{\rho}{\varphi^2}-\frac{\omega}{2}\left(\frac{\dot{\varphi}}{\varphi}\right)^2 \tag{19}$$

From the above three independent equations (17)-(19) five unknowns parameters A, B, ρ, w, γ can be seen. For a better and deterministic solution two more condition is essential.

First, assuming that the scalar expansion (θ) is proportional to the shear scalar (σ) and using the condition (16) gives

$$\frac{1}{\sqrt{3}}\left(\frac{\dot{A}}{A}-\frac{\dot{B}}{B}\right)=\alpha_0\left(\frac{\dot{A}}{A}+2\frac{\dot{B}}{B}\right) \tag{20}$$

where$\alpha_0$is a constant and the above equation yields to

$$\frac{\dot{A}}{A}=l\frac{\dot{B}}{B} \tag{21}$$

where $l=\frac{2\alpha_0\sqrt{3}+1}{1-\alpha_0\sqrt{3}}$. From the equation (21) gives

$$A=c_3B^l \tag{22}$$

where, c_3 is a constant and without the loss of generality taking $c_3=1$, yields

$$A=B^l \tag{23}$$

Secondly, a gauge function is considered as

$$\varphi=\varphi_0 a^{\alpha}=\varphi_0 V^{\frac{\alpha}{3}} \tag{24}$$

A characterized scale factor is assumed as

$$a(t)=c_2 e^{\frac{t}{c_1}}(c_1 t+c_1-1)^{\frac{1}{c_1^2}} \tag{25}$$

where$c_1$and c_2 are constants.

Using this equation (25) in (15), the deceleration parameter is obtained as

$$q=-1+\frac{1}{(t+1)^2} \tag{26}$$

Using (16), (23) and (25) the metric functions are obtained as

$$A=k_3 e^{\frac{3\mathrm{lt}}{c_1(l+2)}}(c_1 t+c_1-1)^{\frac{3l}{c_1^2(l+2)}} \tag{27}$$

$$B=k_1 e^{\frac{3t}{c_1(l+2)}}(c_1 t+c_1-1)^{\frac{3}{c_1^2(l+2)}} \tag{28}$$

$$C=k_2 e^{\frac{3t}{c_1(l+2)}}(c_1 t+c_1-1)^{\frac{3}{c_1^2(l+2)}} \tag{29}$$

where, $k_1=m^{\frac{-1}{l+2}}c_2^{\frac{3}{(l+2)}}, k_2=mk_1, k_3=k_1^l$.

Hence, the metric (1) becomes

$$ds^2= -dt^2+k_3^2 e^{\frac{6\mathrm{lt}}{c_1(l+2)}}(c_1 t+c_1-1)^{\frac{6l}{c_1^2(l+2)}}dx^2+k_1^2 e^{\frac{6t}{c_1(l+2)}}(c_1 t+c_1-1)^{\frac{6}{c_1^2(l+2)}}e^{2x}dy^2+ k_2^2 e^{\frac{6t}{c_1(l+2)}}(c_1 t+c_1-1)^{\frac{6}{c_1^2(l+2)}}e^{-2x}dz^2 \tag{30}$$

4. Physical and Geometrical Properties of the Model

The spatial volume (V), Hubble's parameter (H), expansion scalar (θ), shear scalar (σ) and Anisotropic parameter (A_m) are obtained as

$$V=c_2^3 e^{\frac{3t}{c_1}}(c_1 t+c_1-1)^{\frac{3}{c_1^2}} \tag{31}$$

$$H=\frac{1}{c_1}+\frac{1}{c_1}\frac{1}{(c_1 t+c_1-1)} \tag{32}$$

$$\varphi=\varphi_0 c_2^{\alpha} e^{\frac{\alpha t}{c_1}}(c_1 t+c_1-1)^{\frac{\alpha}{c_1^2}} \tag{33}$$

$$\theta = \frac{3(t+1)}{(c_1t+c_1-1)} \tag{34}$$

$$\sigma^2 = 3\left(\frac{l-1}{l+2}\right)^2\left(\frac{t+1}{c_1t+c_1-1}\right)^2 \tag{35}$$

$$A_m = 2\left(\frac{l-1}{l+2}\right)^2 \tag{36}$$

From eqs. (19), (27), (28), and (33) the energy density, the EoS parameter and the skewness parameter are obtained as

$$\rho = \varphi_0^2 c_2^{2\alpha} e^{\frac{2\alpha t}{c_1}}(c_1t+c_1-1)^{\frac{2\alpha}{c_1^2}}\left[\frac{9(2l+1)(t+1)^2}{(l+2)^2(c_1t+c_1-1)^2} - k_4\frac{1}{e^{\frac{6lt}{c_1(l+2)}}}\frac{1}{(c_1t+c_1-1)^{\frac{6l}{c_1^2(l+2)}}} + \frac{\alpha\omega}{2}\left(\frac{1}{c_1}+\frac{1}{c_1^2(c_1t+c_1-1)}\right)^2\right] \tag{37}$$

where $k_4 = \frac{1}{k_1^{2l}}$

$$w = -\frac{\frac{27(t+2)^2-6}{(l+2)^2(c_1t+c_1-1)^2}+k_4\frac{1}{e^{\frac{6lt}{c_1(l+2)}}}\frac{1}{(c_1t+c_1-1)^{\frac{6l}{c_1^2(l+2)}}}+\frac{\alpha\omega}{2}\left(\frac{1}{c_1}+\frac{1}{c_1^2(c_1t+c_1-1)}\right)^2}{\frac{9(2l+1)(t+1)^2}{(l+2)^2(c_1t+c_1-1)^2}-k_4\frac{1}{e^{\frac{6lt}{c_1(l+2)}}}\frac{1}{(c_1t+c_1-1)^{\frac{6l}{c_1^2(l+2)}}}+\frac{\alpha\omega}{2}\left(\frac{1}{c_1}+\frac{1}{c_1^2(c_1t+c_1-1)}\right)^2} \tag{38}$$

$$\gamma = -\frac{\frac{9(t+1)^2(l^2+l+1)}{(l+2)^2(c_1t+c_1-1)^2}+\frac{3-3l(c_1t+c_1-1)}{(l+2)(c_1t+c_1-1)^2}-k_4\frac{1}{e^{\frac{6lt}{c_1(l+2)}}}\frac{1}{(c_1t+c_1-1)^{\frac{6l}{c_1^2(l+2)}}}+\frac{\alpha\omega}{2}\left(\frac{1}{c_1}+\frac{1}{c_1^2(c_1t+c_1-1)}\right)^2}{\frac{9(2l+1)(t+1)^2}{(l+2)^2(c_1t+c_1-1)^2}-k_4\frac{1}{e^{\frac{6lt}{c_1(l+2)}}}\frac{1}{(c_1t+c_1-1)^{\frac{6l}{c_1^2(l+2)}}}+\frac{\alpha\omega}{2}\left(\frac{1}{c_1}+\frac{1}{c_1^2(c_1t+c_1-1)}\right)^2} \tag{39}$$

5. The Jerk parameter (j) and Statefinder Parameters $\{r, s\}$

The jerk parameter (j) is a dimensionless and third derivative of the scale factor with respect to cosmic time t [29,30]. It is defined as

$$j(t) = \frac{\dddot{a}}{aH^3} \tag{40}$$

Equation (40) can be written as

$$j(t) = q + 2q^2 - \frac{\dot{q}}{H} \tag{41}$$

From equation (26), (32) and (41), yields

$$j(t) = 1 - \frac{3}{(t+1)^2} + \frac{2c_1}{(t+1)^2} \tag{42}$$

For flat ΛCDM model, jerk parameters (j) has the value $j = 1$. Sahni *et al.* [30] introduced the statefinder $\{r, s\}$, defined as

$$r = \frac{\dddot{a}}{aH^3} = 1 + 3\frac{\dot{H}}{H^2} + \frac{\ddot{H}}{H^3} \tag{43}$$

and $s = \frac{r-1}{3\left(q-\frac{1}{2}\right)}$ (44)

From the above eqs. (26), (32), (43) and (44) the results obtained are

$$r = 1 - \frac{3}{(t+1)^2} + \frac{2c_1}{(t+1)^3} \tag{45}$$

and $$s = \frac{-12(t+1)-4c_1}{3(t+1)\{2-3(t+1)^2\}} \tag{46}$$

From the above results we see that as $t \to \infty, \{r, s\} \to \{1,0\}$ which draw the idea that the model of the universe starts from Einstein static era to the ΛCDM model as in recent observation by Ahmad and Pradhan [31].

6. Conclusion

In this paper, a Bianchi Type-VI_0 cosmological model with a special form of scale factor in Sen-Dunn theory has been studied and solved for exact solutions. Spatial volume increases as time tends to infinity. Gauge function φ is also increasing function of time and increases as timepasses. The Hubble parameter decreases and approaches to zero as time tend to infinity and our models anisotropic and homogeneous since $\frac{\sigma^2}{\theta^2} = \text{constant}$. Also the EoS parameter w is negative throughout the time t representing the DE model dynamics which in good agreement with recent observation. The jerk parameter (j) has the value $j = 1$, deceleration parameter $q = -1$. A deceleration to acceleration transition occurs for models as time passes. This solution gives a scenario of the dark energy model which tends to a ΛCDM model.

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