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Squeezing in the Difference of the Fields in Degenerate Four-and Five-wave Interaction Processes

N. Sahu[1,2], D. K. Giri[3*]

[1]Department of Education, Bokaro Steel City College (A Constituent Unit of Binod Bihari Mahto Koyalanchal University), Bokaro, Jharkhand, India

[2]Department of Physics, Vinoba Bhave University, Hazaribag, Jharkhand, India

[3]Department of Physics, Binod Bihari Mahto Koyalanchal University, Dhanbad, India

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Abstract

Squeezing in the difference of the fields in degenerate four- and five-wave interaction processes is studied. It is shown that the difference squeezing can be led to normal squeezing of the field for uncorrelated modes. It is found that the amplitude-squared squeezing of the fundamental mode directly converted into the normal squeezing of the signal mode in the four-wave interaction process and amplitude-cubed of the fundamental directly converted into the normal squeezing of the signal mode in the five-wave interaction process. Detection of higher-order squeezing in these processes is also studied. It is observed that difference squeezing responds nonlinearly to the number of pump photons and found greater in the stimulated process than in spontaneous one. Difference squeezing exists only in certain domain value of pump photons. It is inferred that the multi-photon absorption process is more suitable for the generation of optimum squeezed light.

Keywords: Squeezed states; Amplitude-squared and -cubed squeezing; Difference squeezing; Multi-wave mixing; Photon number operator.

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1. Introduction

Squeezed states of light [1-6] is one of the examples of nonclassical light. They can be expressed by complex amplitude, which describes both the magnitude and the phase of the field. The amplitude of the electric field of a mode of the electromagnetic field is not a fixed quantity; there are always quantum residual fluctuations, called zero-point fluctuation. In a coherent state, the fluctuations in the quadrature components are equal and are randomly distributed in the field quadrature components. On the other hand, it is possible to reduce fluctuations in one quadrature component than a coherent state at the expense of increased fluctuations in the other quadrature component. These states are called squeezed states. It has drawn greater attention of the community due to its low-noise fluctuation in any quantum state [7,8] with an application in optical

* *Corresponding author*: dilipkumargiri@gmail.com

telecommunication [9], quantum cryptography [10,11], an interferometric technique [12], amplification of signals [13], computing [14] and development of techniques for making higher-order correlation measurements in quantum optics [15,16]. Recently several workers have obtained theoretical as well as experimental evidence of squeezed states in various nonlinear optical processes such as parametric amplification [17,18], harmonic generations [19-23], multi-photon processes [24-29], and others [30-34]. More recently, higher-order squeezing has been studied by Prakash and Mishra in some other optical processes [35-37] for improving the performance of many optical devices and optical communication networks. Garcia Fernandez *et al*. [38] and Mishra *et al.* [39, 40] have worked on higher-order nonclassical states in single-mode and their use in detecting nonclassical light. Further, another type of higher-order squeezing, called sum and difference squeezing was proposed by Hillery [41] for the two modes and generalized to include three modes [42] as well as an arbitrary number of modes for sum and difference squeezing [43-45]. Prakash *et al*. [46] studied enhancement and generation of sum squeezing in two-mode light in mixing with coherent light using a beam splitter. Truong *et al.* [47] and Wang *et al*. [48] have recently given the concept of higher-order nonclassical properties and intermodal entanglement in the two-mode photon added squeezed state in a lucid manner. More recently Mukherjee *et al*. [49] have reported the idea that sum-and-difference squeezing is possible in harmonic generation processes. Giri *et al.* [50] have pointed out sum squeezing in frequency up conversion process and Mishra *et al*. [51] have given the concept of the generation of sum-and difference-squeezing by the beam splitter having third-order nonlinear material.

The present paper is to extend our theoretical study on the concept of squeezing in the difference of the fields in degenerate four- and five-wave interaction processes. The paper is organized as follows: Section 2 gives the definition of normal and higher-order squeezing. We establish the analytic expression in the difference of the fields in degenerate four-wave and five-wave mixing processes in sections 3 and 4 respectively. Section 5 incorporates results and discussion. Finally, we conclude the paper in section 6.

2. Definition of Normal and Higher-Order Squeezing

2.1. *Amplitude squeezing of single mode*

The squeezing is the reduction of quantum fluctuations in one quadrature at the expense of other one. It may be characterized by its real and imaginary parts as

$$\hat{X}_1 = \left(\frac{1}{2}\right)\left(\hat{A} + \hat{A}^\dagger\right) \tag{1}$$

and $$\hat{X}_2 = \left(\frac{1}{2i}\right)\left(\hat{A} - \hat{A}^\dagger\right). \tag{2}$$

where $\hat{A}(t) = \hat{a}(t)\, exp(\, i\omega t)$ and $\hat{A}^\dagger(t) = \hat{a}^\dagger(t)\, exp(\, - i\omega t)$ are the slowly varying operators with time t and $\hat{a}^\dagger(\hat{a})$ are creation (annihilation) operators of the electromagnetic field with frequency ω.

Equations (1) and (2) obey the commutation relation as

$$\left[\hat{X}_1, \hat{X}_2\right] = \frac{i}{2}. \tag{3}$$

and the uncertainty relation (ħ = 1) is

$$\Delta\hat{X}_1 \, \Delta\hat{X}_2 \geq \frac{1}{4}. \tag{4}$$

where $\Delta\hat{X}_1$ and $\Delta\hat{X}_2$ are the uncertainties in the quadrature operators $\hat{X}_1$ and $\hat{X}_2$ respectively.

A state is squeezed in the $\hat{X}_1$ direction if $\Delta\hat{X}_1 < {}^1/_2$ and is squeezed in the $\hat{X}_2$ direction if $\Delta\hat{X}_2 < {}^1/_2$.

2.2. *Amplitude-squared and amplitude-cubed squeezing of single mode*

Amplitude-squared squeezing of single mode

Amplitude-squared and amplitude-cubed squeezing of single mode are the type of higher-order squeezing. It is the higher powers of the field amplitude [20,21].

The amplitude-squared squeezing of the field may be defined as

$$\hat{Y}_1 = \left(\frac{1}{2}\right)\left(\hat{A}^2 + \hat{A}^{\dagger 2}\right) \tag{5}$$

and $$\hat{Y}_2 = \left(\frac{1}{2i}\right)\left(\hat{A}^2 - \hat{A}^{\dagger 2}\right). \tag{6}$$

where the symbols have as usual meanings.

These operators follow the commutation relation

$$\left[\hat{Y}_1, \hat{Y}_2\right] = i\left(2\hat{N}_{\hat{A}} + 1\right). \tag{7}$$

where $\hat{N}_{\hat{A}} = \hat{A}^{\dagger}\hat{A}$ is the number operator in pump mode.

The equation (7) leads to the uncertainty relation (ħ = 1)

$$\Delta\hat{Y}_1 \, \Delta\hat{Y}_2 \geq \left\langle \hat{N}_{\hat{A}} + \frac{1}{2} \right\rangle. \tag{8}$$

where $\Delta\hat{Y}_1$ and $\Delta\hat{Y}_2$ are the uncertainties in the quadrature operators $\hat{Y}_1$ and $\hat{Y}_2$ respectively.

Amplitude-squared squeezing exist if

$$(\Delta\hat{Y}_i)^2 < \left\langle \hat{N}_{\hat{A}} + \frac{1}{2} \right\rangle, \text{ where i = 1 or 2.} \tag{9}$$

Amplitude-cubed squeezing of single mode

The amplitude-cubed squeezing of the field may be defined as

$$\hat{Z}_1 = \left(\frac{1}{2}\right)\left(\hat{A}^3 + \hat{A}^{\dagger 3}\right) \quad (10)$$

and $$\hat{Z}_2 = \left(\frac{1}{2i}\right)\left(\hat{A}^3 - \hat{A}^{\dagger 3}\right). \quad (11)$$

Equations (10) and (11) satisfy the commutation relation

$$\left[\hat{Z}_1, \hat{Z}_2\right] = \frac{i}{2}\left(9\hat{N}_{\hat{A}}^2 + 9\hat{N}_{\hat{A}} + 6\right). \quad (12)$$

and equation (12) leads to the uncertainty relation (ħ = 1)

$$\Delta\hat{Z}_1\,\Delta\hat{Z}_2 \geq \frac{1}{4}\left\langle 9\hat{N}_{\hat{A}}^2 + 9\hat{N}_{\hat{A}} + 6\right\rangle \quad (13)$$

where $\Delta\hat{Z}_1$ and $\Delta\hat{Z}_2$ are the uncertainties in the quadrature operators $\hat{Z}_1$ and $\hat{Z}_2$ respectively.

Amplitude-cubed squeezing exist if

$$(\Delta\hat{Z}_i)^2 < \frac{1}{4}\left\langle 9\hat{N}_{\hat{A}}^2 + 9\hat{N}_{\hat{A}} + 6\right\rangle, \text{ where i = 1 or 2.} \quad (14)$$

The state when satisfies equations (9) and (14) exhibits non-classical features.

3. Squeezing in the Difference of the Fields in Degenerate Four-Wave Interaction Process

The degenerate four-wave energy level model is shown in Fig. 1, in which the process involving absorption of two pump photons of frequency ω_1 each and emission of one Stokes photon of frequency ω_2 and one signal photon at frequency ω_3 to the initial state.

Let us define difference of the fields having frequency ω_1 and ω_2 with creation (annihilation) operators $\hat{a}^\dagger(\hat{a})$ and $\hat{b}^\dagger(\hat{b})$ respectively, through variables $\hat{V}_1$ and $\hat{V}_2$ as

$$\hat{V}_1 = \left(\frac{1}{2}\right)\left(\hat{A}^2\hat{B}^\dagger + \hat{A}^{\dagger 2}\hat{B}\right) \quad (15)$$

and $$\hat{V}_2 = \left(\frac{1}{2i}\right)\left(\hat{A}^2\hat{B}^\dagger - \hat{A}^{\dagger 2}\hat{B}\right). \quad (16)$$

The operators $\hat{V}_1$ and $\hat{V}_2$ satisfy the commutation relation as

$$\left[\hat{V}_1, \hat{V}_2\right] = \frac{i}{2}\left[4\hat{N}_{\hat{A}}\hat{N}_{\hat{B}} + 2\hat{N}_{\hat{B}} + \hat{N}_{\hat{A}} - \hat{N}_{\hat{A}}^2\right]. \quad (17)$$

and the uncertainty relation (ħ = 1)

$$\Delta\hat{V}_1\Delta\hat{V}_2 \geq \frac{1}{4}\left\langle 4\hat{N}_{\hat{A}}\hat{N}_{\hat{B}} + 2\hat{N}_{\hat{B}} + \hat{N}_{\hat{A}} - \hat{N}_{\hat{A}}^2\right\rangle. \quad (18)$$

where $\hat{N}_A = \hat{A}^\dagger \hat{A}$ and $\hat{N}_B = \hat{B}^\dagger \hat{B}$ are the photon number operator.

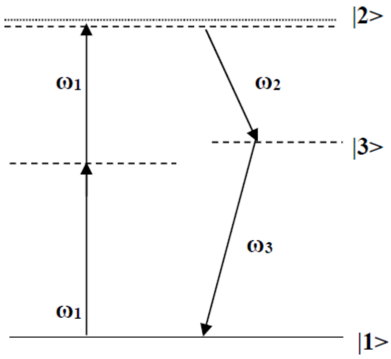

Fig. 1. Four-wave energy level diagram.

Difference squeezing in $\hat{V}_j$ direction exists if

$$\left(\Delta \hat{V}_j\right)^2 < \frac{1}{4}\left\langle 4\hat{N}_{\hat{A}}\hat{N}_{\hat{B}} + 2\hat{N}_{\hat{B}} + \hat{N}_{\hat{A}} - \hat{N}^2_{\hat{A}} \right\rangle, \text{ where j = 1 or 2.} \quad (19)$$

From Fig. 1, the interaction Hamiltonian can be written as

$$\hat{H} = \omega_1 \hat{a}^\dagger \hat{a} + \omega_2 \hat{b}^\dagger \hat{b} + \omega_3 \hat{c}^\dagger \hat{c} + g(\hat{a}^2 \hat{b}^\dagger \hat{c}^\dagger + \hat{a}^{\dagger 2} \hat{b} \hat{c}). \quad (20)$$

where $\hat{a}^\dagger(\hat{a})$, $\hat{b}^\dagger(\hat{b})$ and $\hat{c}^\dagger(\hat{c})$ are the creation (annihilation) operators of the pump field ($\hat{A}$-mode), Stokes field ($\hat{B}$-mode) and signal field ($\hat{C}$-mode) respectively and g is the coupling constant per second. The field operators can be expressed as $\hat{A} = \hat{a}\, exp(i\omega_1 t)$, $\hat{B} = \hat{b}\, exp(i\omega_2 t)$ and $\hat{C} = \hat{c}\, exp(i\omega_3 t)$ with the relation $2\omega_1 - \omega_2 = \omega_3$ under short interaction time 't' and gt<<1.

Using interaction Hamiltonian of equation (20) in coupled Heisenberg equation of motion

$$\dot{\hat{A}} = \frac{\partial \hat{A}}{\partial t} + i\left[\hat{H}, \hat{A}\right], \ (\hbar = 1). \quad (21)$$

we obtain $\dot{\hat{A}} = -2ig\hat{A}^\dagger \hat{B}\hat{C}$. (22)

Similarly $\dot{\hat{B}} = -ig\hat{A}^2 \hat{C}^\dagger$. (23)

and $\dot{\hat{C}} = -ig\hat{A}^2 \hat{B}^\dagger$ (24)

Using equations (22) and (23) in equation (24), we obtain

$$\ddot{\hat{C}} = -|g|^2\left[4\hat{N}_{\hat{A}}\hat{N}_{\hat{B}} + 2\hat{N}_{\hat{B}} + \hat{N}_{\hat{A}} - \hat{N}^2_{\hat{A}}\right]\hat{C} \ . \quad (25)$$

In the interaction Hamiltonian the coupling constant is used $|g|^2$ in place of g^2 for real value. Using short interaction time and keep terms up to second-order in 'gt' in the Taylor's expansion, we get

$$\hat{C}(t) = \hat{C}(0) + t\dot{\hat{C}}(0) + \left(t^2 \big/ 2!\right)\ddot{\hat{C}}(0) + \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \tag{26}$$

Use of equations (24) and (25) in equation (26), gives

$$\hat{C}(t) = \hat{C} - igt\hat{A}^2\hat{B}^\dagger - \left(\frac{|g|^2t^2}{2}\right)\left(4\hat{N}_{\hat{A}}\hat{N}_{\hat{B}} + 2\hat{N}_{\hat{B}} + \hat{N}_{\hat{A}} - \hat{N}_{\hat{A}}^2\right)\hat{C} \tag{27}$$

and $$C^\dagger(t) = C^\dagger + igt\hat{A}^{\dagger 2}\hat{B} - \left(\frac{|g|^2t^2}{2}\right)\left(4\hat{N}_{\hat{A}}\hat{N}_{\hat{B}} + 2\hat{N}_{\hat{B}} + \hat{N}_{\hat{A}} - \hat{N}_{\hat{A}}^2\right)\hat{C}^\dagger \tag{28}$$

where the operators at $t = 0$ represents $\hat{C}(0) = \hat{C}$ throughout the paper.
In order to examine the existence of squeezing in the signal mode, we define

$$\hat{X}_{1\hat{C}}(t) = \left(\frac{1}{2}\right)[\hat{C}(t) + \hat{C}^\dagger(t)] \tag{29}$$

and $$\hat{X}_{2\hat{C}}(t) = \left(\frac{1}{2i}\right)[\hat{C}(t) - \hat{C}^\dagger(t)] \tag{30}$$

Using equations (27) and (28) in equations (29) and (30) we obtain

$$\hat{X}_{1\hat{C}}(t) = \hat{X}_{1\hat{C}} + |g|t(\hat{V}_2) - \left(\frac{|g|^2t^2}{2}\right)\left(4\hat{N}_{\hat{A}}\hat{N}_{\hat{B}} + 2\hat{N}_{\hat{B}} + \hat{N}_{\hat{A}} - \hat{N}_{\hat{A}}^2\right)\hat{X}_{1\hat{C}} \tag{31}$$

and $$\hat{X}_{2\hat{C}}(t) = \hat{X}_{2\hat{C}} - |g|t(\hat{V}_1) - \left(\frac{|g|^2t^2}{2}\right)\left(4\hat{N}_{\hat{A}}\hat{N}_{\hat{B}} + 2\hat{N}_{\hat{B}} + \hat{N}_{\hat{A}} - \hat{N}_{\hat{A}}^2\right)\hat{X}_{2\hat{C}}\,. \tag{32}$$

At $t = 0$ the modes $\hat{A}$ and $\hat{B}$ are uncorrelated, then equations (31) and (32) may be written as,

$$\left[\Delta\hat{X}_{1\hat{C}}(t)\right]^2 = \left(\Delta\hat{X}_{1\hat{C}}\right)^2 + |g|^2t^2\left(\Delta\hat{V}_2\right)^2 - |g|^2t^2\left\langle 4\hat{N}_{\hat{A}}\hat{N}_{\hat{B}} + 2\hat{N}_{\hat{B}} + \hat{N}_{\hat{A}} - \hat{N}_{\hat{A}}^2\right\rangle\left(\Delta\hat{X}_{1\hat{C}}\right)^2 \tag{33}$$

and $$\left[\Delta\hat{X}_{2\hat{C}}(t)\right]^2 = \left(\Delta\hat{X}_{2\hat{C}}\right)^2 + |g|^2t^2\left(\Delta\hat{V}_1\right)^2 - |g|^2t^2\left\langle 4\hat{N}_{\hat{A}}\hat{N}_{\hat{B}} + 2\hat{N}_{\hat{B}} + \hat{N}_{\hat{A}} - \hat{N}_{\hat{A}}^2\right\rangle\left(\Delta\hat{X}_{2\hat{C}}\right)^2 \tag{34}$$

If the $\hat{C}$ mode is initially in a coherent state, then

$$\left(\Delta\hat{X}_{1\hat{C}}\right)^2 = \left(\Delta\hat{X}_{2\hat{C}}\right)^2 = \left(\frac{1}{4}\right). \tag{35}$$

Equations (33) and (34) reduce to

$$\left[\Delta\hat{X}_{1\hat{C}}(t)\right]^2 = \left(\frac{1}{4}\right) + |g|^2t^2\left[\left(\Delta\hat{V}_2\right)^2 - \left(\frac{1}{4}\right)\left\langle 4\hat{N}_{\hat{A}}\hat{N}_{\hat{B}} + 2\hat{N}_{\hat{B}} + \hat{N}_{\hat{A}} - \hat{N}_{\hat{A}}^2\right\rangle\right] \tag{36}$$

and $$\left[\Delta\hat{X}_{2\hat{C}}(t)\right]^2 = \left(\frac{1}{4}\right) + |g|^2 t^2 \left[\left(\Delta\hat{V}_1\right)^2 - \left(\frac{1}{4}\right)\left\langle 4\hat{N}_{\hat{A}}\hat{N}_{\hat{B}} + 2\hat{N}_{\hat{B}} + \hat{N}_{\hat{A}} - \hat{N}^2_{\hat{A}}\right\rangle\right]. \quad (37)$$

We rewrite equations (36) and (37) as follows

$$\left[\Delta\hat{X}_{1\hat{C}}(t)\right]^2 - \left(\frac{1}{4}\right) = |g|^2 t^2 \left[\left(\Delta\hat{V}_2\right)^2 - \left(\frac{1}{4}\right)\left\langle 4\hat{N}_{\hat{A}}\hat{N}_{\hat{B}} + 2\hat{N}_{\hat{B}} + \hat{N}_{\hat{A}} - \hat{N}^2_{\hat{A}}\right\rangle\right] \quad (38)$$

and $$\left[\Delta\hat{X}_{2\hat{C}}(t)\right]^2 - \left(\frac{1}{4}\right) = |g|^2 t^2 \left[\left(\Delta\hat{V}_1\right)^2 - \left(\frac{1}{4}\right)\left\langle 4\hat{N}_{\hat{A}}\hat{N}_{\hat{B}} + 2\hat{N}_{\hat{B}} + \hat{N}_{\hat{A}} - \hat{N}^2_{\hat{A}}\right\rangle\right] \quad (39)$$

These equations (38) and (39) establish the relation between difference squeezing and normal squeezing of the signal mode in the degenerate four-wave interaction process. It may be inferred that if the input state is difference squeezed in the $\hat{V}_2$ or $\hat{V}_1$ direction, then difference-frequency generation will produce an output, and will lead to normal squeezing in $\hat{X}_{1C}$ or $\hat{X}_{2C}$ respectively. It is shown that difference squeezing can be turned into normal squeezing.

Further, to study the squeezing in signal mode, let us assume Stokes mode as a constant and represent a constant term *m* for $\hat{B}$ and $\hat{B}^\dagger$ so that the change in the $\hat{B}$-mode is negligible.

Hence equation (24) becomes

$$\dot{\hat{C}} = -ig\hat{A}^2 m \quad (40)$$

and $$\ddot{\hat{C}} = -2|g|^2 m^2 \left(2\hat{N}_{\hat{A}} + 1\right)\hat{C}. \quad (41)$$

Using equation (26), we have

$$\hat{C}(t) = \hat{C} - i|g|mt\hat{A}^2 - |g|^2 t^2 m^2 \left(2\hat{N}_{\hat{A}} + 1\right)\hat{C} \quad (42)$$

and $$\hat{C}^\dagger(t) = \hat{C}^\dagger + i|g|mt\hat{A}^{\dagger 2} - |g|^2 t^2 m^2 \left(2\hat{N}_{\hat{A}} + 1\right)\hat{C}^\dagger. \quad (43)$$

Using equations (42) and (43) in equations (29) and (30), we get

$$\hat{X}_{1\hat{C}}(t) = \hat{X}_{1\hat{C}} + |g|mt\hat{Y}_{2\hat{A}} - |g|^2 t^2 m^2 \left(2\hat{N}_{\hat{A}} + 1\right)\hat{X}_{1\hat{C}} \quad (44)$$

and $$\hat{X}_{2\hat{C}}(t) = \hat{X}_{2\hat{C}} - |g|mt\hat{Y}_{1\hat{A}} - |g|^2 t^2 m^2 \left(2\hat{N}_{\hat{A}} + 1\right)\hat{X}_{2\hat{C}}. \quad (45)$$

where $\hat{Y}_{1\hat{A}}$ and $\hat{Y}_{2\hat{A}}$ define in equations (5) and (6).

At *t* = 0, the modes are uncorrelated, then equations (44) and (45) becomes

$$\left[\Delta\hat{X}_{1\hat{C}}(t)\right]^2 = \left(\Delta\hat{X}_{1\hat{C}}\right)^2 + |g|^2 m^2 t^2 \left[\left(\Delta\hat{Y}_{2\hat{A}}\right)^2 - 2\left\langle 2\hat{N}_{\hat{A}} + 1\right\rangle\left(\Delta\hat{X}_{1\hat{C}}\right)^2\right] \quad (46)$$

and $$\left[\Delta\hat{X}_{2\hat{C}}(t)\right]^2 = \left(\Delta\hat{X}_{2\hat{C}}\right)^2 + |g|^2 m^2 t^2 \left[\left(\Delta\hat{Y}_{1\hat{A}}\right)^2 - 2\left\langle 2\hat{N}_{\hat{A}} + 1\right\rangle\left(\Delta\hat{X}_{2\hat{C}}\right)^2\right] \tag{47}$$

Using equation (35), then we obtain

$$\left[\Delta\hat{X}_{1\hat{C}}(t)\right]^2 - \left(\frac{1}{4}\right) = |g|^2 m^2 t^2 \left[\left(\Delta\hat{Y}_{2\hat{A}}\right)^2 - \left\langle \hat{N}_{\hat{A}} + \frac{1}{2}\right\rangle\right] \tag{48}$$

and $$\left[\Delta\hat{X}_{2\hat{C}}(t)\right]^2 - \left(\frac{1}{4}\right) = |g|^2 m^2 t^2 \left[\left(\Delta\hat{Y}_{1\hat{A}}\right)^2 - \left\langle \hat{N}_{\hat{A}} + \frac{1}{2}\right\rangle\right] \tag{49}$$

Equations (48) and (49) establish the relation between amplitude-squared squeezing and normal squeezing of the signal mode in degenerate four-wave interaction process when the Stokes mode taken as constant term. That means the signal $\hat{C}$ mode is squeezed in the $\hat{X}_{1C}$ direction if the $\hat{A}$ mode is amplitude-squared squeezed in the $\hat{Y}_{2\hat{A}}$ direction and the signal $\hat{C}$ mode is squeezed in the $\hat{X}_{2C}$ direction if the $\hat{A}$ mode is amplitude-squared squeezing in the $\hat{Y}_{1\hat{A}}$ direction. That is, if a fundamental mode with amplitude-squared squeezing propagates through a nonlinear medium then normal squeezing will generate in the signal mode. It is shown that amplitude-squared squeezing can be changed into normal squeezing. These results suggest a method for the detection of difference squeezing and amplitude-squared squeezing by degenerate four-wave interaction process.

4. Squeezing in the Difference of the Fields in Degenerate Five-Wave Interaction Process

In degenerate five-wave energy level model, shown in Fig. 2, the process involving absorption of three pump photons of frequency ω_1 each and emission of one Stokes photon of frequency ω_2 and one signal photon at frequency ω_3 to the initial state.

Let us define difference of the fields having frequency ω_1 and ω_2 with creation (annihilation) operators $\hat{a}^\dagger(\hat{a})$ and $\hat{b}^\dagger(\hat{b})$ respectively, through variables $\widehat{W}_1$ and $\widehat{W}_2$ as

$$\hat{W}_1 = \left(\frac{1}{2}\right)\left(\hat{A}^3\hat{B}^\dagger + \hat{A}^{\dagger 3}\hat{B}\right) \tag{50}$$

and $$\hat{W}_2 = \left(\frac{1}{2i}\right)\left(\hat{A}^3\hat{B}^\dagger - \hat{A}^{\dagger 3}\hat{B}\right). \tag{51}$$

The operators $\widehat{W}_1$ and $\widehat{W}_2$ follow the commutation relation as

$$\left[\hat{W}_1, \hat{W}_2\right] = \frac{i}{2}\left[9\hat{N}_{\hat{A}}^2\hat{N}_{\hat{B}} + 9\hat{N}_{\hat{A}}\hat{N}_{\hat{B}} + 6\hat{N}_{\hat{B}} + 3\hat{N}_{\hat{A}}^2 - 2\hat{N}_{\hat{A}} - \hat{N}_{\hat{A}}^3\right]. \tag{52}$$

and leads to the uncertainty relation (ħ = 1)

$$\Delta\hat{W}_1\Delta\hat{W}_2 \geq \frac{1}{4}\left\langle 9\hat{N}_{\hat{A}}^2\hat{N}_{\hat{B}} + 9\hat{N}_{\hat{A}}\hat{N}_{\hat{B}} + 6\hat{N}_{\hat{B}} + 3\hat{N}_{\hat{A}}^2 - 2\hat{N}_{\hat{A}} - \hat{N}_{\hat{A}}^3\right\rangle. \tag{53}$$

where $\widehat{N}_A = \hat{A}^\dagger\hat{A}$ and $\widehat{N}_B = \hat{B}^\dagger\hat{B}$ are the number operator.
Difference squeezing in $\widehat{W}_j$ direction exists if the condition follows

$$\left(\Delta\hat{W}_j\right)^2 < \frac{1}{4}\left\langle 9\hat{N}_{\hat{A}}^2\hat{N}_{\hat{B}} + 9\hat{N}_{\hat{A}}\hat{N}_{\hat{B}} + 6\hat{N}_{\hat{B}} + 3\hat{N}_{\hat{A}}^2 - 2\hat{N}_{\hat{A}} - \hat{N}_{\hat{A}}^3\right\rangle, \text{ where j=1or 2.} \qquad (54)$$

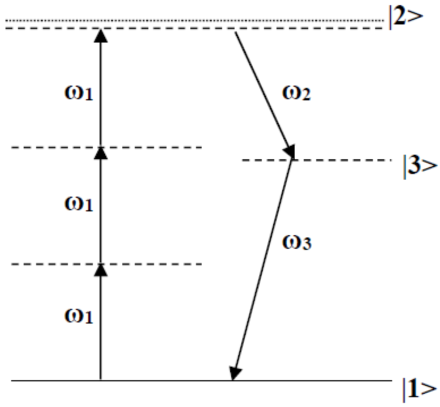

Fig. 2. Five-wave energy level diagram.

From Fig. 2, the interaction Hamiltonian can be written as

$$\hat{H} = \omega_1\hat{a}^\dagger\hat{a} + \omega_2\hat{b}^\dagger\hat{b} + \omega_3\hat{c}^\dagger\hat{c} + g(\hat{a}^3\hat{b}^\dagger\hat{c}^\dagger + \hat{a}^{\dagger 3}\hat{b}\hat{c}) . \qquad (55)$$

Here $\hat{A}$, $\hat{B}$ and $\hat{C}$ are slowly varying operators defined as $\hat{A} = \hat{a}\,exp(\,i\omega_1 t)$, $\hat{B} = \hat{b}\,exp(\,i\omega_2 t)$ and $\hat{C} = \hat{c}\,exp(\,i\omega_3 t)$ with the relation $3\omega_1$ - ω_2 = ω_3.
Using interaction Hamiltonian of equation (55) in coupled Heisenberg equation of motion (21),
we obtain

$$\dot{\hat{A}} = -3ig\hat{A}^{\dagger 2}\hat{B}\hat{C} . \qquad (56)$$

Similarly $\dot{\hat{B}} = -ig\hat{A}^3\hat{C}^\dagger$ (57)

and $\dot{\hat{B}} = -ig\hat{A}^3\hat{C}^\dagger$ (58)

Using equations (56) and (57) in equation (58), we obtain

$$\ddot{\hat{C}} = -|g|^2\left[9\hat{N}_{\hat{A}}^2\hat{N}_{\hat{B}} + 9\hat{N}_{\hat{A}}\hat{N}_{\hat{B}} + 6\hat{N}_{\hat{B}} + 3\hat{N}_{\hat{A}}^2 - 2\hat{N}_{\hat{A}} - \hat{N}_{\hat{A}}^3\right]\hat{C} \; . \qquad (59)$$

Use of equations (58) and (59) in equation (26), gives

$$\hat{C}(t) = \hat{C} - igt\hat{A}^3\hat{B}^\dagger - \left(\frac{|g|^2 t^2}{2}\right)\left(9\hat{N}_{\hat{A}}^2\hat{N}_{\hat{B}} + 9\hat{N}_{\hat{A}}\hat{N}_{\hat{B}} + 6\hat{N}_{\hat{B}} + 3\hat{N}_{\hat{A}}^2 - 2\hat{N}_{\hat{A}} - \hat{N}_{\hat{A}}^3\right)\hat{C} \qquad (60)$$

and $$C^{\dagger}(t) = C^{\dagger} + igt\hat{A}^{\dagger 3}\hat{B} - \left(\frac{|g|^2 t^2}{2}\right)\left(9\hat{N}_{\hat{A}}^2\hat{N}_{\hat{B}} + 9\hat{N}_{\hat{A}}\hat{N}_{\hat{B}} + 6\hat{N}_{\hat{B}} + 3\hat{N}_{\hat{A}}^2 - 2\hat{N}_{\hat{A}} - \hat{N}_{\hat{A}}^3\right)\hat{C}^{\dagger}. \quad (61)$$

Using equations (60) and (61) in equations (29) and (30) we obtain

$$\hat{X}_{1\hat{C}}(t) = \hat{X}_{1\hat{C}} + |g|t(\hat{W}_2) - \left(\frac{|g|^2 t^2}{2}\right)\left(9\hat{N}_{\hat{A}}^2\hat{N}_{\hat{B}} + 9\hat{N}_{\hat{A}}\hat{N}_{\hat{B}} + 6\hat{N}_{\hat{B}} + 3\hat{N}_{\hat{A}}^2 - 2\hat{N}_{\hat{A}} - \hat{N}_{\hat{A}}^3\right)\hat{X}_{1\hat{C}} \quad (62)$$

and $$\hat{X}_{2\hat{C}}(t) = \hat{X}_{2\hat{C}} - |g|t(\hat{W}_1) - \left(\frac{|g|^2 t^2}{2}\right)\left(9\hat{N}_{\hat{A}}^2\hat{N}_{\hat{B}} + 9\hat{N}_{\hat{A}}\hat{N}_{\hat{B}} + 6\hat{N}_{\hat{B}} + 3\hat{N}_{\hat{A}}^2 - 2\hat{N}_{\hat{A}} - \hat{N}_{\hat{A}}^3\right)\hat{X}_{2\hat{C}} \quad (63)$$

Since at $t = 0$ the modes $\hat{A}$ and $\hat{B}$ are uncorrelated, then equations (62) and (63) becomes as,

$$\left[\Delta\hat{X}_{1\hat{C}}(t)\right]^2 = \left(\Delta\hat{X}_{1\hat{C}}\right)^2 + |g|^2 t^2\left(\Delta\hat{W}_2\right)^2 - |g|^2 t^2\left\langle 9\hat{N}_{\hat{A}}^2\hat{N}_{\hat{B}} + 9\hat{N}_{\hat{A}}\hat{N}_{\hat{B}} + 6\hat{N}_{\hat{B}} + 3\hat{N}_{\hat{A}}^2 - 2\hat{N}_{\hat{A}} - \hat{N}_{\hat{A}}^3\right\rangle\left(\Delta\hat{X}_{1\hat{C}}\right)^2 \quad (64)$$

and

$$\left[\Delta\hat{X}_{2\hat{C}}(t)\right]^2 = \left(\Delta\hat{X}_{2\hat{C}}\right)^2 + |g|^2 t^2\left(\Delta\hat{W}_1\right)^2 - |g|^2 t^2\left\langle 9\hat{N}_{\hat{A}}^2\hat{N}_{\hat{B}} + 9\hat{N}_{\hat{A}}\hat{N}_{\hat{B}} + 6\hat{N}_{\hat{B}} + 3\hat{N}_{\hat{A}}^2 - 2\hat{N}_{\hat{A}} - \hat{N}_{\hat{A}}^3\right\rangle\left(\Delta\hat{X}_{2\hat{C}}\right)^2 \quad (65)$$

Using equation (35) then equations (64) and (65) reduces to

$$\left[\Delta\hat{X}_{1\hat{C}}(t)\right]^2 = \left(\frac{1}{4}\right) + |g|^2 t^2\left[\left(\Delta\hat{W}_2\right)^2 - \left(\frac{1}{4}\right)\left\langle 9\hat{N}_{\hat{A}}^2\hat{N}_{\hat{B}} + 9\hat{N}_{\hat{A}}\hat{N}_{\hat{B}} + 6\hat{N}_{\hat{B}} + 3\hat{N}_{\hat{A}}^2 - 2\hat{N}_{\hat{A}} - \hat{N}_{\hat{A}}^3\right\rangle\right] \quad (66)$$

and $$\left[\Delta\hat{X}_{2\hat{C}}(t)\right]^2 = \left(\frac{1}{4}\right) + |g|^2 t^2\left[\left(\Delta\hat{W}_1\right)^2 - \left(\frac{1}{4}\right)\left\langle 9\hat{N}_{\hat{A}}^2\hat{N}_{\hat{B}} + 9\hat{N}_{\hat{A}}\hat{N}_{\hat{B}} + 6\hat{N}_{\hat{B}} + 3\hat{N}_{\hat{A}}^2 - 2\hat{N}_{\hat{A}} - \hat{N}_{\hat{A}}^3\right\rangle\right] \quad 67)$$

We rewrite equations (32) and (33) as follows

$$\left[\Delta\hat{X}_{1\hat{C}}(t)\right]^2 - \left(\frac{1}{4}\right) = |g|^2 t^2\left[\left(\Delta\hat{W}_2\right)^2 - \left(\frac{1}{4}\right)\left\langle 9\hat{N}_{\hat{A}}^2\hat{N}_{\hat{B}} + 9\hat{N}_{\hat{A}}\hat{N}_{\hat{B}} + 6\hat{N}_{\hat{B}} + 3\hat{N}_{\hat{A}}^2 - 2\hat{N}_{\hat{A}} - \hat{N}_{\hat{A}}^3\right\rangle\right] \quad (68)$$

and $$\left[\Delta\hat{X}_{2\hat{C}}(t)\right]^2 - \left(\frac{1}{4}\right) = |g|^2 t^2\left[\left(\Delta\hat{W}_1\right)^2 - \left(\frac{1}{4}\right)\left\langle 9\hat{N}_{\hat{A}}^2\hat{N}_{\hat{B}} + 9\hat{N}_{\hat{A}}\hat{N}_{\hat{B}} + 6\hat{N}_{\hat{B}} + 3\hat{N}_{\hat{A}}^2 - 2\hat{N}_{\hat{A}} - \hat{N}_{\hat{A}}^3\right\rangle\right] \quad (69)$$

These equations (68) and (69) give the relation between difference squeezing and normal squeezing of the signal mode in the degenerate five-wave interaction process. In other words, it may be stated that if the input state is difference squeezed in the $\widehat{W}_2$ or $\widehat{W}_1$ direction, then difference-frequency generation will produce an output, and will lead to normal squeezing in $\hat{X}_{1C}$ or $\hat{X}_{2C}$ respectively. It is found that difference squeezing can be converted into normal squeezing.

To study the squeezing in signal mode, let us assume Stokes mode as a constant and represent a constant term m for $\hat{B}$ and $\hat{B}^{\dagger}$so that the change in the $\hat{B}$ mode is negligible. Hence equation (58) becomes

$$\dot{\hat{C}} = -ig\hat{A}^3 m \tag{70}$$

and $$\ddot{\hat{C}} = -|g|^2 m^2\left(9\hat{N}_{\hat{A}}^2 + 9\hat{N}_{\hat{A}} + 6\right)\hat{C}\cdot \tag{71}$$

Use of equations (70) and (71) in equation (26), gives

$$\hat{C}(t) = \hat{C} - i|g|mt\hat{A}^3 - \left(\frac{|g|^2 t^2}{2}\right)m^2\left(9\hat{N}_{\hat{A}}^2 + 9\hat{N}_{\hat{A}} + 6\right)\hat{C} \tag{72}$$

and $$\hat{C}^{\dagger}(t) = \hat{C}^{\dagger} + i|g|mt\hat{A}^{\dagger 3} - \left(\frac{|g|^2 t^2}{2}\right)m^2\left(9\hat{N}_{\hat{A}}^2 + 9\hat{N}_{\hat{A}} + 6\right)\hat{C}^{\dagger}\cdot \tag{73}$$

Using equations (72) and (73) in equations (29) and (30) we get

$$\hat{X}_{1\hat{C}}(t) = \hat{X}_{1\hat{C}} + |g|mt\hat{Z}_{2\hat{A}} - \left(\frac{|g|^2 t^2}{2}\right)m^2\left(9\hat{N}_{\hat{A}}^2 + 9\hat{N}_{\hat{A}} + 6\right)\hat{X}_{1\hat{C}} \tag{74}$$

and $$\hat{X}_{2\hat{C}}(t) = \hat{X}_{2\hat{C}} - |g|mt\hat{Z}_{1\hat{A}} - \left(\frac{|g|^2 t^2}{2}\right)m^2\left(9\hat{N}_{\hat{A}}^2 + 9\hat{N}_{\hat{A}} + 6\right)\hat{X}_{2\hat{C}}\cdot \tag{75}$$

where $\hat{Z}_{1\hat{A}}$ and $\hat{Z}_{2\hat{A}}$ define in equations (10) and (11).
Since the modes are uncorrelated at t = 0, then equations (74) and (75) becomes

$$\left[\Delta\hat{X}_{1\hat{C}}(t)\right]^2 = \left(\Delta\hat{X}_{1\hat{C}}\right)^2 + |g|^2 m^2 t^2\left[\left(\Delta\hat{Z}_{2\hat{A}}\right)^2 - \left\langle 9\hat{N}_{\hat{A}}^2 + 9\hat{N}_{\hat{A}} + 6\right\rangle\left(\Delta\hat{X}_{1\hat{C}}\right)^2\right] \tag{76}$$

and $$\left[\Delta\hat{X}_{2\hat{C}}(t)\right]^2 = \left(\Delta\hat{X}_{2\hat{C}}\right)^2 + |g|^2 m^2 t^2\left[\left(\Delta\hat{Z}_{1\hat{A}}\right)^2 - \left\langle 9\hat{N}_{\hat{A}}^2 + 9\hat{N}_{\hat{A}} + 6\right\rangle\left(\Delta\hat{X}_{2\hat{C}}\right)^2\right] \tag{77}$$

Using equation (35), then we obtain

$$\left[\Delta\hat{X}_{1\hat{C}}(t)\right]^2 - \left(\frac{1}{4}\right) = |g|^2 m^2 t^2\left[\left(\Delta\hat{Z}_{2\hat{A}}\right)^2 - \left(\frac{1}{4}\right)\left\langle 9\hat{N}_{\hat{A}}^2 + 9\hat{N}_{\hat{A}} + 6\right\rangle\right] \tag{78}$$

and $$\left[\Delta\hat{X}_{2\hat{C}}(t)\right]^2 - \left(\frac{1}{4}\right) = |g|^2 m^2 t^2\left[\left(\Delta\hat{Z}_{1\hat{A}}\right)^2 - \left(\frac{1}{4}\right)\left\langle 9\hat{N}_{\hat{A}}^2 + 9\hat{N}_{\hat{A}} + 6\right\rangle\right]. \tag{79}$$

Equations (78) and (79) establish the relation between amplitude-cubed squeezing and normal squeezing of the signal mode in degenerate five-wave interaction process when the Stokes mode taken as constant term. That means the signal $\hat{C}$ mode is squeezed in the $\hat{X}_{1C}$ direction if the $\hat{A}$ mode is amplitude-cubed squeezed in the $\hat{Z}_{2\hat{A}}$ direction and the signal $\hat{C}$ mode is squeezed in the $\hat{X}_{2C}$ direction if the $\hat{A}$ mode is amplitude-cubed squeezing in the $\hat{Z}_{1\hat{A}}$ direction. That is, if a fundamental mode with amplitude-cubed squeezing propagates through a nonlinear medium then normal squeezing will generate in the signal mode. It is shown that amplitude-cubed squeezing can be converted into normal squeezing. These findings suggest a method of detection for difference squeezing as well as amplitude-cubed squeezing by degenerate five-wave interaction process.

5. Results and Discussion

We plotted a graph (Fig. 3) between left hand side of equation (38) or (39) say D_{SV} and $|\alpha|^2$ having different values of $|\beta|^2$ with typical values $(\Delta\hat{V}_2)^2 = (\Delta\hat{V}_1)^2 = \left(\frac{1}{4}\right)$ so that it could satisfy the equation (19).

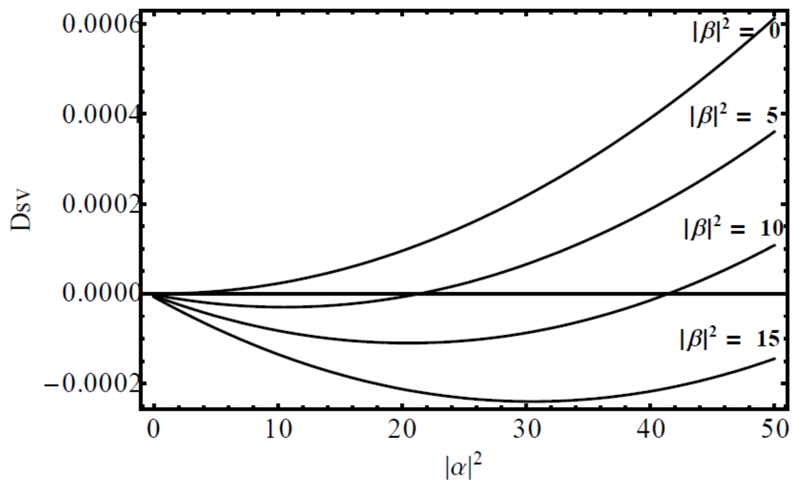

Fig. 3. Variation of difference squeezing D_{SV} with $|\alpha|^2$in degenerate four-wave interaction process ($|gt|^2 = 10^{-6}$ and $|\beta|^2$= 0, 5, 10, 15).

The curves infer that the difference squeezing exists and responds nonlinearly to the number of pump photons. It shows that when $|\alpha|^2$is increasing the degree of difference squeezing is first increasing i.e. D_{SV} is getting more negative until a critical value of $|\alpha|^2$ and then it is decreasing and finally disappears i.e. D_{SV} turns out to be positive. These findings agree with the result of Truong *et al*. [47].

Let us denote left hand side of equation (68) or (69) by D_{SW} and plot a graph (Fig. 4) with $|\alpha|^2$ having different values of $|\beta|^2$ and typical values $(\Delta\hat{W}_2)^2 = (\Delta\hat{W}_1)^2 = \left(\frac{1}{4}\right)$ so that it could satisfy the equation (54).

The plot (Fig. 4) shows that the difference squeezing responds nonlinearly to the pump photons. It is seen that when $|\alpha|^2$ is increasing the degree of difference squeezing D_{SW} is getting more negative until a critical value of $|\alpha|^2$; but subsequently, it is decreasing and finally disappears i.e. D_{SW} turns out to be positive. These results agree with the result of Wang *et al*. [48]. Hence, difference squeezing exists only in certain domain value of pump photons. Comparing Figs. 3 and 4, we inferred that the depth of non-classicality is increasing with an increase of $|\beta|^2$. Hence in stimulated interaction squeezing is more pronounced than spontaneous one. It is also seen that difference squeezing is more in the five-wave interaction process than the corresponding squeezing in the four-wave interaction process. Hence it is inferred that higher multi-photon absorption process is suitable for the generation of optimum squeezed light.

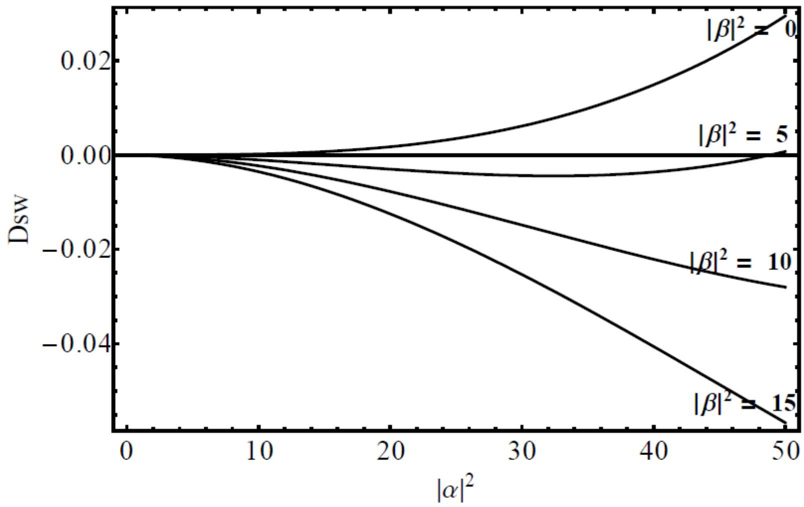

Fig. 4. Variation of difference squeezing D_{SW} with $|\alpha|^2$in degenerate five-wave interaction process ($|gt|^2 = 10^{-6}$ and $|\beta|^2 = 0, 5, 10, 15$).

To study higher–order squeezing, we denote the right hand side of equations (48) and (78) respectively by $D_{SV'}$ and $D_{SW'}$ and plots with $|gt|^2$ as shown in Figs. 5 and 6.

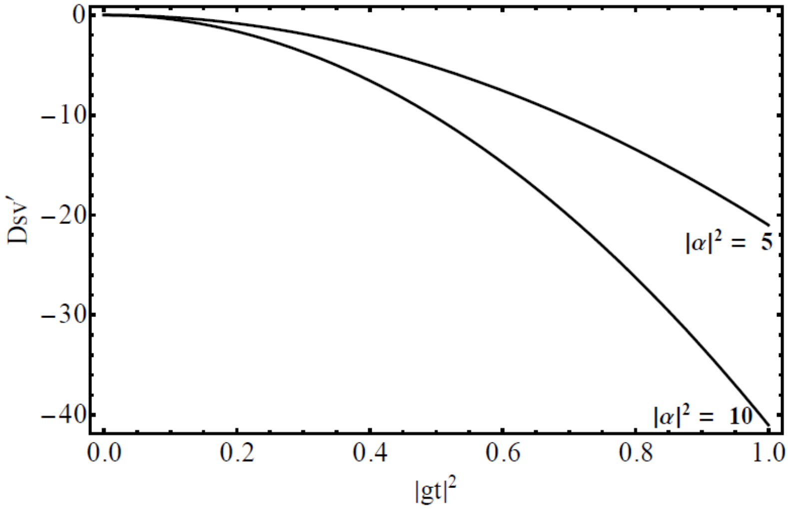

Fig. 5. Variation of the squeezing $D_{SV'}$ with $|gt|^2$ (when $m^2 = 4$ = Constant) in degenerate four-wave interaction process.

The steady decrease of the curves (Figs. 5 and 6) show that the degree of squeezing increases nonlinearly with the increase of the number of photons ($|\alpha|^2$). This confirms that the squeezed states are associated with large number of pump photons. It also confirmed that the higher order squeezing is directly associated with the coupling of the field and interaction time.

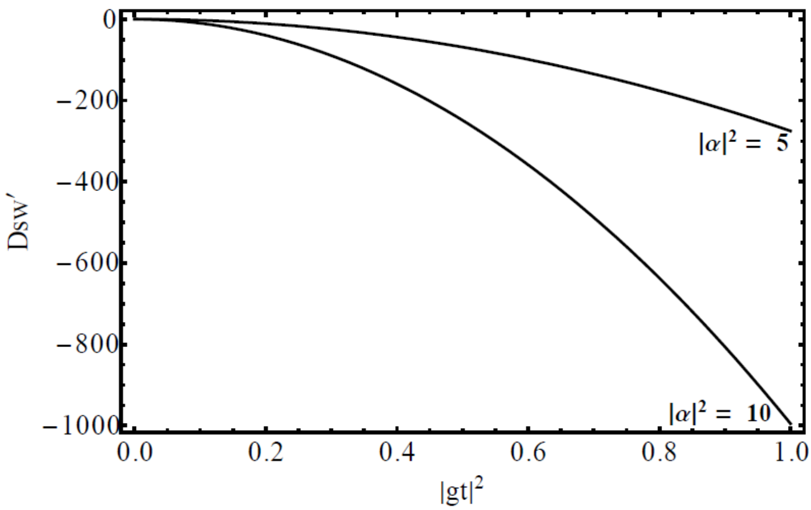

Fig. 6. Variation of the squeezing $D_{SW'}$ with $|gt|^2$ (when $m^2 = 4$ = Constant) in degenerate five-wave interaction process.

A comparison between Figs. (5) and (6) shows greater noise reduction in amplitude-cubed (i.e. in degenerate five-wave interaction process) than in amplitude-squared (i.e. in degenerate four-wave interaction process), having same number of photons. Hence the third-order will give more squeezed laser light than the second-order.

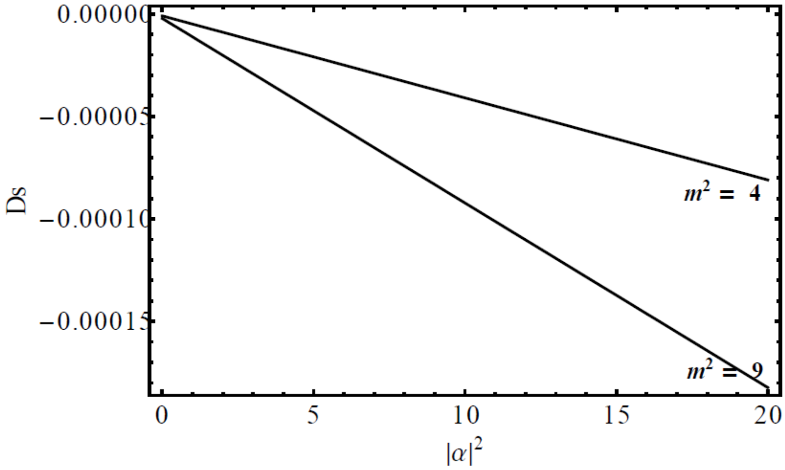

Fig. 7. Variation of the squeezing D_S in signal mode with $|\alpha|^2$ in degenerate four-wave mixing process.

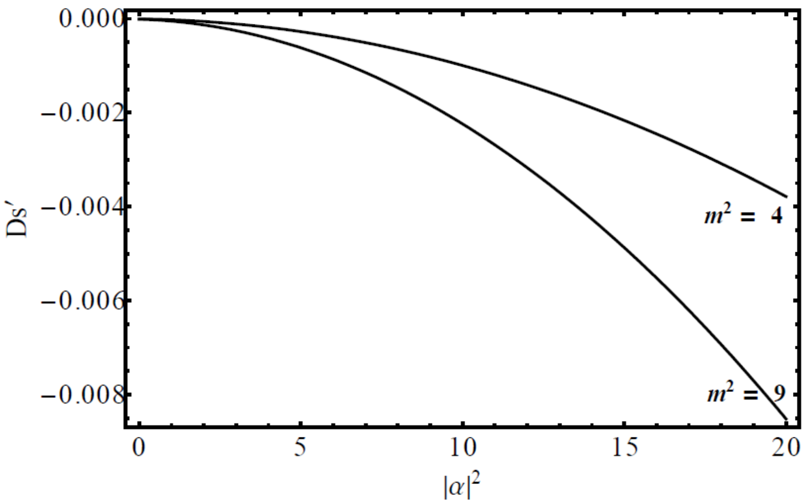

Fig. 8. Variation of the squeezing $D_{S'}$ in signal mode with $|\alpha|^2$ in degenerate five-wave mixing process.

Now taking $|gt|^2 = 10^{-6}$ and plot a graph of equations (48) and (78) respectively by D_S and $D_{S'}$ with $|\alpha|^2$ having Stokes mode as constant value $m^2 = 4$ and 9 (arbitrary values).
Figs. 7 and 8 show that the squeezing increases with the increase of number of pump photons $|\alpha|^2$ as well as with increase of value of constant m^2 in Stokes mode. It is also

inferred that squeezing is more in five-wave interaction process than the corresponding squeezing in four-wave interaction process.

6. Conclusion

In this paper, we observed that difference squeezing of the optical fields can be converted into normal squeezing by degenerate multi-wave (four & five) interaction process.

It is shown that the difference squeezing responds nonlinearly to the pump photons and is found to be dependent on the coupling of the field amplitude and interaction time. It is found that the squeezing increases with the increase of number of pump photons $|\alpha|^2$ as well as with increase of value of constant m^2 in Stokes mode. It is shown that squeezing is more pronounced in stimulated process than the corresponding squeezing in spontaneous process having same number of photons. This also confirms that the squeezed states are associated with large number of photons.

It is found that when number of photons is increasing the degree of difference squeezing is getting more negative until a critical value of $|\alpha|^2$; but subsequently, it is decreasing and finally disappears. Hence, difference squeezing exists only in certain domain value of pump photons.

When an amplitude-squared squeezing of the fundamental mode propagates through a nonlinear medium then normal squeezing will generate in the signal mode of the degenerate four-wave interaction process. Similarly, amplitude-cubed squeezing of the fundamental mode generates normal squeezing in the signal mode of the degenerate five-wave interaction process. Thus, the nonlinear interaction (signal mode) converts higher-order squeezing into normal squeezing. It suggests a method for the detection of higher-order squeezing in the degenerate multi-wave interaction process. It is shown that greater noise reduction in amplitude-cubed (i. e. in degenerate five-wave interaction process) than in amplitude-squared (i. e. in degenerate four-wave interaction process), having the same number of photons. Hence third-order will give more squeezed laser light than the second-order. Therefore, the difference squeezing is more in the five-wave interaction process than the corresponding difference squeezing in the four-wave interaction process. Hence it is inferred that the higher photon absorption process is suitable for the generation of optimum squeezed light.

The above findings stated that the process with higher-order nonlinearity i. e. multi-photon absorption in pump mode is more suitable for the generation of optimum squeezed light. These results also suggest ways in selecting a suitable process for obtaining greater noise reduction in optical systems and can be useful in high-quality optical telecommunication [52].

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