Publications

Available Online

J. Sci. Res. **13** (2), 455-465 (2021)

JOURNAL OF SCIENTIFIC RESEARCH

www.banglajol.info/index.php/JSR

Two Warehouse Inventory Model for Deteriorating Items with Ramp Type Demand and Price Discount on Backorders

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Received 31 October 2020, accepted in final revised form 5 February 2021

Abstract

In this paper, a two warehouse inventory model for deteriorating items is studied with ramp type demand rate. Holding cost of rented warehouse has higher than the owned warehouse due to better preservation facilities in rented warehouse. Due to the improved services offer in rented warehouse, the deterioration rate in rented warehouse is less than deterioration rate in owned warehouse. When stock on hand is zero, the inventory manager offers a price discount to customers who are willing to backorder their demand. The study includes some features that are likely to be associated with certain types of inventory, like inventory of seasonal fruits and vegetables, newly launched fashion items, etc. The optimum ordering policy and the optimum discount offered for each backorder are determined by minimizing the total cost in a replenishment interval.

Keywords: Two warehouses; Ramp type demand; Deteriorating item; Shortage; Price discount on backorder.

doi: http://dx.doi.org/10.3329/jsr.v13i2.50000 J. Sci. Res. **13** (2), 455-465 (2021)

1. Introduction

In classical inventory models it is assumed that organization have a single warehouse with the facility of unlimited storage capacity. But in reality, when suppliers provide attractive price discount for bulk purchase at a time, the inventory manager may purchase more goods. These large amounts of goods can not to be stored in its own warehouse (OW) due to its limited capacity. For these excess quantities, additional warehouse is required and items are stored in rented warehouse (RW). Due to different preservation facilities the inventory costs in RW are assumed to be higher than those in OW. So, it will be economical for the inventory manager to store items in OW before RW, but the items of RW are consumed first and the items of OW are the next to reduce the inventory cost. Two warehouse inventory model was first discussed by Hartley. Sarma [1] developed two warehouse inventory model for deteriorating items with an infinite replenishment rate and shortage. Pakkala and Achary [2] extended the two warehouse inventory model for deteriorating items with finite rate of replenishment and shortages, taking time as discrete

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and continuous variable, respectively. Bhunia and Maiti [3] considered a two warehouse inventory model for deteriorating items with linearly increasing demand and shortages. Zhou [4] studied two warehouse inventory models with time varying demand. Wee et al. [5] presented a two warehouse model with constant demand and Weibull distribution deterioration under inflation. Shaikh *et al*. [6] developed a two-warehouse inventory model with advanced payment, partial backlogged shortages. Subsequently, the ideas of two warehouse modelling were considered by some other authors [7-10].

In traditional inventory models, it is generally assumed that the demand rate is independent of factors like stock availability, price of items, etc. However, in actual practice, the demand of newly launched products such as fashionable garments, electronic items, mobile phones etc. increases with time and later it becomes constant. This phenomenon is termed as 'ramp type demand'. It is commonly observed in seasonable products, new brand of consumer goods. The demand for these items increases in its growth stage and then remains stable in its maturity stage. The inventory model with ramp type demand rate was proposed by Hill [11] for the first time. He considered the inventory models for increasing demand followed by a constant demand. Mandal and Pal [12] developed an order level inventory model for deteriorating items with ramp type demand. Wu *et al*. [13] derived an EOQ model with Weibull deterioration rate, and the demand rate with a ramp type function of time. Giri *et al*. [14] developed an economic order quantity model with Weibull deterioration distribution, shortages and ramp type demand. Deng *et al*. [15] studied the inventory model for deteriorating items with ramp type demand rate. Skouri *et al*. [16] developed an economic order quantity model with general ramp type demand rate, time dependent deterioration rate, and partial backlogging rate. Ahmed *et al*. [17] proposed a new method for finding the EOQ policy, for an inventory model with ramp type demand rate, partial backlogging and general deterioration rate. Chandra [18] studied a periodic review inventory model in ramp type demand environment.

In classical inventory models with shortages, it is generally assumed that the unmet demand is either completely lost or completely backlogged. But in cases of many products of famous brands or fashionable commodities, customers prefer their demands to be backordered. Some customers may be willing to wait till the stock is replenished (i.e., backorder case), while some may be impatient and satisfy their demand immediately from some other source (i.e. lost sales case). To hold his customer when a stock-out occurs, the inventory manager may offer a discount on backorders and/or a reduction in waiting time to tempt the customers to wait. Through controlling a price discount, inventory manager could generate high customer loyalty. This means that he could reduce cost of lost-sales and also reduce holding cost. The larger the backorder discount is, the larger the backorder rate is likely to be. Thus, the backorder rate is dependent on the amount of shortages and backorder price discounts. It is, therefore, an interesting problem to find the optimal backorder price discount so as to minimize the total expected cost or maximize the total expected profit of the organization. Pan and Hsiao [19] proposed a continuous review inventory model considering the order quantity and with negotiable backorders as decision variables. Ouyang *et al*. [20] developed a periodic review inventory model with backorder discounts to accommodate more practical features of the real inventory systems. Chuang *et al*. [21] discussed a distribution free procedure for mixed inventory model with backorder discount and variable lead time. Uthayakumar and Parvati [22] considered a model with only first two moments of the lead time demand known, and obtained the optimum backorder price discount and order quantity in that situation. Pal and Chandra [23] studied a deterministic inventory model with shortages. They considered only a fraction of the unmet demand is backlogged, and the inventory manager offers a discount on it. Chandra [24] studied an inventory model where holding cost is linearly increasing function of time and demand rate is a ramp type function of time with price discount on backorders. Salas-Navarro *et al*. [25] developed an EPQ inventory model considering an imperfect production system with probabilistic demand and collaborative approach. Gupta *et al*. [26] discussed firm investment decisions for information security under a fuzzy environment through game-theoretic approach. Vandana and Sana [27] developed a two-echelon inventory model for ameliorating / deteriorating items with single vendor and multi-buyers scenario. Mashud *et al*. [28] considered a two-level trade-credit approach to an integrated price-sensitive inventory model with shortages. Udayakumar *et al*. [29] studied an economic ordering policy for non- instantaneous deteriorating items with price and advertisement dependent demand and permissible delay in payment under inflation. Taleizadeh *et al.* [30] developed an inventory model for complementary and substitutable products. Sana [31] discussed price competition between green and non-green products under corporate social responsible firm. Moghdani *et al*. [32] developed a fuzzy EPQ model for multi-item with multiple deliveries. Roy and Sana [33] developed production rate and lot size-dependent lead time reduction strategies in a supply chain model with stochastic demand, controllable setup cost and trade-credit financing.

Table 1. Major contribution of the proposed model.

Literature	Warehouse facility	Type of price discount on backorders	Demand rate
Sarma [1]	two	Shortage allowed	Deterministic
Panda *et al*. [10]	two	partial backlogging	stock-dependent
Mandal and Pal [12]	One	No shortage	Ramp Type
Giri *et al.* [14]	One	shortage	Ramp Type
Ahmed *et al*. [17]	One	partial backlogging	Ramp Type
Chandra [24]	One	discount on backorders	Ramp Type
Mashud *et al*. [28]	Two	shortage	price-sensitive
This paper	Two	Fractional backorders price discount	Ramp Type

In this paper, a two warehouse inventory model for deteriorating items is considered with ramp type demand. It is assumed that the items of rented warehouse are consumed first and then the items of owned warehouse are consumed because rented warehouse has higher unit holding cost than the owned warehouse. The manager offers his customer a discount in case he is willing to backorder his demand when there is a stock-out. Through controlling a price discount, inventory manager could generate high customer loyalty. The objective of this model is to find the best replenishment policies and optimal price discount on backorders for minimizing the total appropriate inventory cost. A two warehouse inventory model is developed with considering the above scenario.

2. Notations and Assumptions

To develop the model, the following notations and assumptions have been used.

2.1 Notations

$I_0(t)$ = inventory level in owned warehouse (OW) at time point t
$I_r(t)$ = inventory level in rented warehouse (RW) at time point t
K = ordering cost per order
b = fraction of the demand backordered during stock out
b_0 = upper bound of backorder ratio
h_r = inventory holding cost per unit per unit time in RW
h_0 = inventory holding cost per unit per unit time in OW
θ_1 = deterioration rate in RW, $0 < \theta_1 < 1$
θ_2 = deterioration rate in OW, $0 < \theta_2 < 1$, $\theta_2 > \theta_1$
T = length of a replenishment cycle
T_1 = time taken for stock on hand to be exhausted at RW, $0 < T_1 < T$
T_2 = time taken for stock on hand to be exhausted at OW, $0 < T_1 < T_2 < T$
S = maximum stock height in a replenishment cycle at OW
s_1 = backorder cost per unit backordered per unit time
s_2 = cost of a lost sale
π = price discount on unit backorder offered
π_0 = marginal profit per unit

2.2 Assumptions

1. The model considers only one item in inventory.
2. Replenishment of inventory occurs instantaneously on ordering i.e., lead time is zero.
3. The OW has the limited capacity of storage (S) and RW has unlimited capacity.
4. Items of RW are consumed first and then the items of OW are consumed due to the more holding cost in RW than in OW ($h_r > h_0$).
5. Due to the improved services offer in RW, the deterioration rate in RW is less than deterioration rate in OW ($\theta_2 > \theta_1$).
6. Shortages are allowed, and a fraction b of unmet demands during stock-out is backlogged.

7. The demand rate $R(t)$ is assumed to be a ramp type function of time t
   $R(t) = D_0[t-(t-\mu)H(t-\mu)]$
   where D_0 and μ are positive constants and $H(t-\mu)$ is the Heaviside's function defined as follows: $H(t-\mu) = \begin{cases} 1 & for\ t \geq \mu \\ 0 & for\ t < \mu \end{cases}$

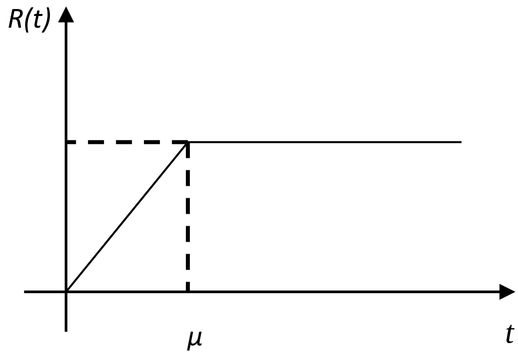

Fig. 1. The ramp type demand rate.

8. The time taken for stock on hand to be exhausted (T_1) is greater than μ.
9. During the stock-out period, the backorder fraction b is directly proportional to the price discount π offered by the inventory manager. Thus,
   $b = \frac{b_0}{\pi_0}\pi$, where $0 \leq b_0 \leq 1,\ 0 \leq \pi \leq \pi_0$

3. Model Formulation

The planning period is divided into reorder intervals, each of length T units. Orders are placed at time points *0, T, 2T, 3T, ….* At the beginning of the reorder interval order quantity being just sufficient to bring the stock height at OW to a certain maximum level S and the remaining order quantity in RW. Due to different preservation facilities the inventory costs (including holding cost and deterioration cost) in RW are assumed to be higher than those in OW. So, it will be economical for the inventory manager to store items in OW before RW, but the items of RW are consumed first and the items of OW are the next to reduce the inventory cost. Stocks on hand of RW and OW are exhausted at time point T_1 and T_2 respectively.

Depletion of inventory at RW occurs due to demand and deterioration during the period (0, T_1). Hence, the variation in inventory level at RW with respect to time is given by

$$\frac{d}{dt}I_r(t) + \theta_1 I_r(t) = -D_0 t, \quad \text{if } 0 < t < \mu$$

$$\frac{d}{dt}I_r(t) + \theta_1 I_r(t) = -D_0 \mu, \quad \text{if } \mu < t < T_1$$

Since $I_r(T_1)=0$, and considering the continuity condition of $I_r(t)$ at $t=\mu$, it follows that

$$I_r(t)=\frac{D_0}{\theta_1^2}\left(\mu\theta_1 e^{\theta_1(T_1-t)}-e^{\theta_1(\mu-t)}-\left(t\theta_1-1\right)\right),\quad \text{if } 0<t<\mu$$

$$=\frac{D_0\mu}{\theta_1}\left(e^{\theta_1(T_1-t)}-1\right),\quad \text{if } \mu<t<T_1$$

Depletion of inventory at OW occurs due to deterioration during the period $(0, T_1)$, and due to demand and deterioration both during the period (T_1, T_2), $T_1 < T_2$. In the interval (T_2, T), $T_2 < T$ shortage occurs, of which a fraction b is backlogged. Hence, the variation in inventory level at OW with respect to time is given by

$$\frac{d}{dt}I_0(t)+\theta_2 I_0(t)=0,\qquad \text{if } 0<t<T_1$$

$$\frac{d}{dt}I_0(t)+\theta_2 I_0(t)=-D_0\mu,\quad \text{if } T_1<t<T_2$$

$$\frac{d}{dt}I_0(t)=-bD_0\mu,\qquad \text{if } T_2<t<T$$

Since $I_0(0) = S$ and $I_0(T_2) = 0$, we get

$$I_0(t)=Se^{-\theta_2 t},\qquad \text{if } 0<t<T_1$$

$$=\frac{D_0\mu}{\theta_2}\left(e^{\theta_2(T_2-t)}-1\right),\ \text{if } T_1<t<T_2$$

$$=bD_0\mu\left(T_2-t\right),\qquad \text{if } T_2<t<T$$

Considering the continuity of $I_0(t)$ at $t=T_1$, it follows that

$$I_0(T_1)=Se^{-\theta_2 T_1}=\frac{D_0\mu}{\theta_2}\left(e^{\theta_2(T_2-T_1)}-1\right)$$

Hence, $S=\dfrac{D_0\mu}{\theta_2}\left(e^{\theta_2 T_2}-e^{\theta_2 T_1}\right)$

Then,

Ordering cost during a cycle (OC) = K

Holding cost of inventories at RW during a cycle (HC_r)

$$=h_r\int_0^{T_1} I_r(t)dt=h_r\left(\int_0^{\mu} I_r(t)dt+\int_{\mu}^{T_1} I_r(t)dt\right)$$

$$=\frac{h_r D_0\mu}{\theta_1^2}\left(\frac{1}{\mu\theta_1}\left(1-e^{\mu\theta_1}\right)+\frac{\mu\theta_1}{2}+e^{\theta_1 T_1}-\theta_1 T_1\right)$$

Holding cost of inventories at OW during a cycle (HC_0)

$$=h_0\int_0^{T_2} I_0(t)dt=h_0\left(\int_0^{T_1} I_0(t)dt+\int_{T_1}^{T_2} I_0(t)dt\right)$$

$$=h_0\left(\frac{S}{\theta_2}\left(1-e^{-\theta_2 T_1}\right)+\frac{D_0\mu}{\theta_2^2}\left(e^{\theta_2(T_2-T_1)}-\theta_2(T_2-T_1)-1\right)\right)$$

Deterioration cost of inventories at RW during a cycle (DC_r)

$$=\theta_1\int_0^{T_1} I_r(t)dt$$

$$=\frac{D_0\mu}{\theta_1}\left(\frac{1}{\mu\theta_1}\left(1-e^{\mu\theta_1}\right)+\frac{\mu\theta_1}{2}+e^{\theta_1 T_1}-\theta_1 T_1\right)$$

Deterioration cost of inventories at OW during a cycle (DC_0)

$$=\theta_2\int_0^{T_2} I_0(t)dt=\theta_2\left(\int_0^{T_1} I_0(t)dt+\int_{T_1}^{T_2} I_0(t)dt\right)$$

$$=S\left(1-e^{-\theta_2 T_1}\right)+\frac{D_0\mu}{\theta_2}\left(e^{\theta_2(T_2-T_1)}-\theta_2(T_2-T_1)-1\right)$$

Backorder cost during a cycle (BC) $=-s_1\int_{T_2}^{T} I_0(t)dt=\frac{s_1 b D_0\mu}{2}\left(T-T_2\right)^2$

Lost sales cost during a cycle (LC) $=s_2\left(1-b\right)D_0\mu(T-T_2)$

Hence, the cost per unit length of a replenishment cycle is given by

$$C(T_1,T_2,b)=\frac{1}{T}[\mathrm{OC}+\mathrm{HC_r}+\mathrm{HC_0}+\mathrm{DC_r}+\mathrm{DC_0}+\mathrm{BC}+\mathrm{LC}]$$

$$=\frac{1}{T}\left(\begin{array}{l} K+\frac{D_0\mu}{\theta_1}\left(\frac{h_r}{\theta_1}+1\right)\left(\frac{1}{\mu\theta_1}\left(1-e^{\mu\theta_1}\right)+\frac{\mu\theta_1}{2}+e^{\theta_1 T_1}-\theta_1 T_1\right)\\ +\frac{D_0\mu}{\theta_2}\left(\frac{h_0}{\theta_2}+1\right)\left(\left(e^{\theta_2 T_2}-e^{\theta_2 T_1}\right)\left(1-e^{-\theta_2 T_1}\right)+e^{\theta_2(T_2-T_1)}-\theta_2(T_2-T_1)-1\right)\\ +\frac{s_1 b D_0\mu}{2}\left(T-T_2\right)^2+s_2\left(1-b\right)D_0\mu(T-T_2)\end{array}\right)$$

$$=\frac{N_1(T_1,T_2,b)}{T}$$

The optimal values of T_1, T_2 and b, which minimize $C(T_1, T_2, b)$, must satisfy the following equations:

$$\theta_1\left(h_0+\theta_2\right)\left(1-e^{\theta_2 T_1}\right)=\theta_2\left(h_r+\theta_1\right)\left(1-e^{\theta_1 T_1}\right) \tag{1}$$

$$\left(h_0+\theta_2\right)\left(e^{\theta_2 T_2}-1\right)=\theta_2\left(s_1 b(T-T_2)+s_2\left(1-b\right)\right) \tag{2}$$

$$T-T_2=\frac{2s_2}{s_1} \tag{3}$$

4. Numerical Illustration and Sensitivity Analysis

Since it is difficult to find closed form solutions to the sets of Eq. (1) – Eq. (3), we numerically find solutions to the equations for given sets of costs using the statistical software MATLAB. The following tables show the change in optimal inventory policy with change in a model parameter, when the other parameters remain fixed.

Table 2. The optimal inventory policy for different values of s_1, when $T = 5$, $K = 500$, $h_r = 15$, $h_0 = 8$, $\theta_1 = 0.1$, $\theta_2 = 0.25$, $D_0 = 100$, $\mu = 0.015$ and $s_2 = 17$.

s_1	T_1	T_2	b	$C(T_1,T,b)$
11	0.0150	1.9091	0.3335	121.09
13	0.0150	2.3846	0.3330	122.01
15	0.0151	2.7333	0.3320	123.33
17	0.0150	3.0000	0.1717	124.73
19	0.0150	3.2105	0.3027	126.11

Table 3. The optimal inventory policy for different values of s_2, when $T = 5$, $K = 500$, $h_r = 15$, $h_0 = 8$, $\theta_1 = 0.1$, $\theta_2 = 0.25$, $D_0 = 100$, $\mu = 0.015$ and $s_1 = 13$.

s_2	T_1	T_2	b	$C(T_1,T,b)$
13	0.0150	3.0000	0.1908	122.33
15	0.0150	2.6923	0.3327	121.76
17	0.0150	2.3846	0.3330	122.01
19	0.0150	2.0769	0.3335	123.06
23	0.0151	1.4615	0.3337	127.41
25	0.0150	1.1538	0.3606	130.66
27	0.0150	0.8462	0.4044	134.60

Table 4. The optimal inventory policy for different values of h_r, when $T = 5$, $K = 500$, $h_0 = 8$, $\theta_1 = 0.1$, $\theta_2 = 0.25$, $D_0 = 100$, $\mu = 0.015$, $s_1 = 13$ and $s_2 = 17$.

h_r	T_1	T_2	b	$C(T_1,T,b)$
3	2.3846	2.3846	0.2622	116.21
5	2.3846	2.3846	0.2994	118.06
9	2.3846	2.3846	0.3350	121.76
15	0.0150	2.3846	0.3330	122.01
25	0.0150	2.3846	0.3194	122.01

Table 5. The optimal inventory policy for different values of h_0, when $T = 5$, $K = 500$, $h_r = 15$, $\theta_1 = 0.1$, $\theta_2 = 0.25$, $D_0 = 100$, $\mu = 0.015$, $s_1 = 13$ and $s_2 = 17$.

h_0	T_1	T_2	b	$C(T_1,T,b)$
1	0.0150	2.3846	0.2241	114.65
3	0.0150	2.3846	0.3651	116.75
5	0.0150	2.3846	0.3342	118.86
7	0.0150	2.3846	0.3333	120.96
9	0.0150	2.3846	0.3322	123.06
11	0.0150	2.3846	0.1943	125.16
13	0.0150	2.3846	0.3558	127.26

Table 6. The optimal inventory policy for different values of T, when $K = 500$, $h_r = 15$, $h_0 = 8$, $\theta_1 = 0.1$, $\theta_2 = 0.25$, $D_0 = 100$, $\mu = 0.015$, $s_1 = 13$ and $s_2 = 17$.

T	T_1	T_2	b	$C(T_1,T,b)$
3	0.0150	0.3846	0.3333	189.21
3.5	0.0150	0.8846	0.3334	163.40
4	0.0150	1.3846	0.3304	145.01
4.5	0.0150	1.8846	0.3335	131.68

5	0.0150	2.3846	0.3330	122.01
5.5	0.0150	2.8846	0.3342	115.12
6	0.0150	3.3846	0.1797	110.44

Tables 2-6 show that, for other parameters remaining constant,
(a) b, and hence π, decreases with increase in s_1, but increases with h, s_2 and P;
(b) T_2 is increasing in s_1 and T while T_2 is decreasing in s_2.

The above observations indicate that, with the aim to minimizing total cost, the policy should be to maintain high inventory level for longer length of replenishment cycle and for low holding costs. Also, higher the backorder cost, lower should be the price discount offered and for higher lost sales cost, higher price discount should be offered.

5. Conclusion

This paper studies two warehouse inventory model for deteriorating items under ramp type demand environment. The study includes some features that are likely to be associated with certain types of inventory, like inventory of seasonal fruits and vegetables, newly launched fashion items, etc. A fraction of the demand is backlogged, and the inventory manager offers a discount to each customer who is ready to wait till fulfilment of his demand. Some customers may be willing to wait till the stock is replenished (i.e., backorder case), while some may be impatient and satisfy their demand immediately from some other source (i.e., lost sales case). To hold his customer when a stock-out occurs, the inventory manager may offer a discount on backorders and/or a reduction in waiting time to tempt the customers to wait. Through controlling a price discount, inventory manager could generate high customer loyalty. This means that inventory manager could reduce cost of lost-sales and also reduce holding cost. The optimum ordering policies and the optimum discount offered for each backorder are determined by minimizing the total cost in a replenishment interval. Through numerical study, it is observed that the policy should be to maintain high inventory level for longer length of replenishment cycle and for low holding costs. It is also observed that for low backorder cost, it is beneficial to the inventory manager to offer the customers high discount on backorders.

A natural extension of the model would be to study the case of the permissible delay period. The permissible delay in payments produces a benefit to the supplier is that the policy should attract new customers who consider it to be a type of price reduction and increase sales. On the other hand, during the permitted time period, the inventory manager is free to sell his goods, accumulate revenue and earn interest. It would also be interesting to study optimal order quantity in the situation when permissible delay period is fixed or order quantity dependent.

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