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Review Article

Lepton Flavor Violating μ^- - e^- Conversion: An Overview

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Abstract

The search for lepton flavor violation in charged lepton decays is a highly sensitive tool to look for physics beyond the Standard Model. Among the possible processes, μ-decays are considered to have the largest discovery potential in most of the standard model extensions. Many searches have been performed in the past, but no evidence has been found so far. In this paper, we have reviewed the current theoretical and experimental status of the field of muon to electron decay and its potential to search for new physics beyond the Standard Model. Future prospects of experiments for further progress in this field are also discussed.

Keywords: Lepton flavour violation; Muon to electron conversion; Coherent matrix elements; Photonic process; Muon sources.

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1. Introduction

All currently known experimental data are consistent with the standard model (SM) of weak and electromagnetic interactions. Within the framework of the SM, baryon and lepton quantum numbers are separately conserved. In fact, one can associate an additive lepton flavor quantum number with each lepton generation which appears to be conserved. There are thus three such conserved quantum numbers L_e, L_μ and L_τ, each one associated with the lepton generations (e^-, ν_e), (μ^-, ν_μ) and (τ^-, ν_τ), with their antiparticles having opposite lepton flavor. These quantum numbers distinguish between the three neutrino species and lepton number is an exactly conserved quantity if they are exactly massless. However, the experiments with solar, atmospheric, reactor and accelerator neutrinos have provided compelling evidences for oscillations of neutrinos caused by nonzero neutrino masses and neutrino mixing.

Most theorists, however, view the SM not as the ultimate theory of nature but as a successful low energy approximation. In possible extensions of the SM, it is legitimate to ask whether lepton flavor conservation still holds. In fact, in such gauge models (grand unified theories, super-symmetric extensions of the SM, superstring inspired models) such

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quantum numbers are associated with global (non-local) symmetries and their conservation must be broken at some level.

Motivated in part by this belief, the search for lepton flavor violation, which began more than half a century ago [1-4], has been revived in recent years and is expected to continue in the near future. In the meantime, the number of possible reactions for testing lepton flavor has been increased. The most prominent such reactions are:

$$\mu \to e\gamma \tag{1}$$

$$\tau \to e\gamma\,, \qquad \tau \to \mu\gamma \tag{2}$$

$$\mu \to ee^+e^- \tag{3}$$

$$\tau \to ee^+e^-\,, \qquad \tau \to \mu e^+e^- \tag{4}$$

$$\tau \to e\mu^+\mu^-\,, \qquad \tau \to \mu\mu^+\mu^- \tag{5}$$

$$K_L \to \mu^\pm e^\mp\,, \qquad K^+ \to \pi^+\mu e \tag{6}$$

$$\left(\mu^+e^-\right) \leftrightarrow \left(\mu^-e^+\right) \quad \text{(muonium-antimuonium oscillations)} \tag{7}$$

$$\mu^- + (A,Z) \to e^- + (A,Z) \quad \text{(muon electron conversion)} \tag{8}$$

One could also have both lepton number and lepton flavor violating processes like

$$(A,Z) \to (A,Z\pm 2) + e^\mp e^\mp \quad (0\nu\beta\beta \text{ decay}) \tag{9}$$

$$\mu^- + (A,Z) \to e^+ + (A,Z-2) \quad \text{(muon positron conversion)} \tag{10}$$

From an experimental point of view, the most interesting reactions are (1), (3), (8), (9) and (10). Flavor violating decay processes of muon have been intensively studied and have been probed to high precision due to the fact that muon decay processes are simple and easy to detect and also due to the intense muon source available in experiment. They are expected to be further probed to a very high precision by future experiments with muon source improved by orders of magnitude.

From a theoretical physics point of view the problem of lepton flavor non-conservation is connected with family mixing in the leptonic sector. Almost in all models, the above process can proceed at the one loop level via the neutrino mixing. However, due to the GIM (Glashow–Iliopoulos–Maiani) mechanism in the leptonic sector, the amplitude vanishes in the limit in which the neutrinos are massless. In some special cases the GIM mechanism may not be completely operative even if one considers the part of the amplitude which is independent of the neutrino mass [5]. Even then, however, the process is suppressed if the neutrinos are degenerate. It should be mentioned that processes (1)-(8) cannot distinguish between Dirac and Majorana neutrinos. Processes (9) and (10) can proceed only if the neutrinos are Majorana particles. In more elaborate models one may encounter additional mechanism for lepton flavor violation. In grand unified theories (GUT's) one may have additional Higgs scalars which can serve as intermediate particles at the one or two loop level leading to processes (1-8). In super symmetric extensions of the standard model, one may encounter as intermediate particles, the super partners of the above. Lepton flavor violation can also occur in composite models, e.g. Technicolour [6], but such models have already been ruled out by the present experimental bounds. The question of lepton flavor non-conservation has been the subject of several review papers

[7-27]. The observation of any of the processes (1-10) would definitely signal new physics beyond the standard model. It will severely restrict most models. It may take, however, even then much more experimental effort to unravel specific mechanisms responsible for lepton flavor violation or fix the parameters of the models. From a nuclear physics point of view the most interesting muon number violating process is the μ^--e^- conversion in Eq. (8) and below we discuss it in more detail.

2. Muon Electron ($\mu^- - e^-$) Conversion

A prominent process concerning lepton flavor violation is $\mu^- - e^-$ conversion in a muonic atom. When a negative muon is stopped in some material, it is trapped by an atom, and forms a muonic atom. After it cascades down in energy levels in the muonic atom, a muon is bound in its 1s ground state. The fate of the muon is then either decay in orbit $\mu^- \to e^- \nu_\mu \bar{\nu}_e$ or capture by a nucleus of mass number *A* and atomic number *Z*, namely

$$\mu^- + (A,Z) \to \nu_\mu + (A,Z-1) \tag{11}$$

However, in the context of physics beyond the Standard Model, the exotic process of neutrinoless muon capture, such as

$$\mu^- + (A,Z) \to e^- + (A,Z) \tag{12}$$

is also expected. This process is called $\mu^- - e^-$ conversion in a muonic atom. It violates the conservation of the lepton flavor numbers L_e and L_μ by one unit, but conserves the total lepton number $L = L_e + L_\mu + L_\tau$.

The branching ratio of $\mu^- - e^-$ conversion is defined as

$$B\left[\mu^- + (A,Z) \to e^- + (A,Z)\right] \equiv \frac{\Gamma\left[\mu^- + (A,Z) \to e^- + (A,Z)\right]}{\Gamma\left[\mu^- + (A,Z) \to capture\right]}, \tag{13}$$

where Γ is the corresponding decay width. The final state of the nucleus (*A*, *Z*) could be either the ground state or excited states. In general, the transition process to the ground state, which is called coherent capture, is dominant. The rate of the coherent capture process over noncoherent ones is enhanced by a factor approximately equal to the number of nucleons in the nucleus, since all of the nucleons participate in the process.

The possible contributions to $(\mu^- - e^-)$ conversion in a muonic atom can be grouped into two parts, which are the photonic contribution and the nonphotonic contribution. Therefore, in principle, this process is theoretically interesting, since it does occur by mechanisms which only marginally contribute to the $\mu^+ \to e^+\gamma$ process. The study of the photonic contribution was initiated by S. Weinberg *et al.* [28]. The nonphotonic contribution was studied later, for instance by W. J. Marciano *et al*. [29].

From an experimental point of view, $(\mu^- - e^-)$ conversion is a very attractive process in comparison to $\mu \to e\gamma$, $\mu \to 3e$ due to following reasons [18]:

1. The detection of only one particle is sufficient. No coincidence is needed.

2. For electrons with the highest possible energy, i.e. $E_e \approx m_\mu c^2 - \in_b$ with $\in_b$ the muon binding energy, the reaction is almost background free. Indeed, the sources of background are:
(i) Muon decay in orbit. There is a tiny tail in the region of interest which is proportional to $\left(E_{bg}^{\max} - E_e\right)^5$, i.e. very small ($E_{bg}^{\max}$ denotes the maximum energy of the background electrons). But in this region the shape is known and it can be subtracted out.
(ii) Radiative muon capture. Indeed, this can be a source of background since the photon can decay to e^+e^- pairs as

$$\mu^- + (A,Z) \rightarrow (A,Z-1) + \nu_\mu + \gamma \qquad \text{and} \qquad \gamma \rightarrow e^+ + e^- \tag{14}$$

If the neutrino and the positron carry away zero kinetic energy, the background electron can be confused with the interesting electron. But the maximum electron energy for process (14) is

$$E_{bg}^{\max} = m_\mu c^2 - \in_b - m_e c^2 - \Delta = E_e - \Delta - m_e c^2 \tag{15}$$

where Δ is the difference in the binding energy of the two nuclei involved in Eq. (14). By a judicious choice of the target nucleus Δ can be quite large (Δ= 2.5 MeV for ^{12}C. Also, the half-life of muon in carbon muonic atom is aprox. 1.93 μs which is smaller than 2.2 μs value for its free decay). Thus, one has a background free region if one restricts himself to the coherent mode.

3. Expression for the Amplitude of (μ⁻- e⁻) Conversion

The amplitude for the (μ^-, e^-) conversion can be cast in the form [30]

$$M = \frac{4\pi\alpha}{q^2} J_\lambda^{(1)} J_{(1)}^\lambda + \frac{\zeta}{m_\mu^2} J_\lambda^{(2)} J_{(2)}^\lambda \tag{16}$$

where the first term is the photonic and the second the non-photonic contribution. q is the momentum transfer and ζ takes the form

$$\zeta = \begin{cases} \dfrac{G_F m_\mu^2}{\sqrt{2}} & \text{W boson mediated models} \\ \dfrac{m_{3/2}^2}{m_{\tilde{a}}^2} & \text{Supersymmetric models} \end{cases} \tag{17}$$

$m_{3/2}$ is the gravitino mass and $m_{\tilde{a}}$ is the relevant s-quark (supersymmetric partner of quark) mass.
The hadronic currents are

$$J_\lambda^{(1)} = \overline{N}\gamma_\lambda \frac{1+\tau_3}{2} N \qquad \text{(photonic)}, \tag{18}$$

$$J_\lambda^{(2)} = \overline{N}\gamma_\lambda \frac{1}{2}[(3 + \beta f_v \tau_3) + (f_v + f_A \beta \tau_3)\gamma_5]N \qquad \text{(non-photonic)} \tag{19}$$

(N = nucleon) while the leptonic currents are

$$J_{(1)}^\lambda = \overline{u}(p_1)(f_{M1} + \gamma_5 f_{E1}) i\sigma^{\lambda\nu} \frac{q_\nu}{m_\mu} + (f_{E1} + \gamma_5 f_{M1})\gamma^\nu (g_{\lambda\nu} - \frac{q^\lambda q^\nu}{q^2}) \tag{20}$$

$$J_{(2)}^\lambda = \overline{u}\frac{1}{2}(\tilde{f}_v + \tilde{f}_A \gamma_5) u(p_\mu) \tag{21}$$

where $\beta = \beta_1/\beta_0$, is the ratio of the isovector to the isoscalar component of the hadronic current at the quark level. The form factors $f_{M1}, f_{E1}, f_{E0}, f_{M0}, \tilde{f}_v$ and $\tilde{f}_A$ as well as the parameter β depend on the assumed gauge model and the mechanism adopted (the isoscalar parameter β_0 is absorbed in the definition of the leptonic form factors). For purely left-handed theories the number of independent form factors is reduced in half since

$$f_{E1} = -f_{M0}, \qquad f_{E1} = -f_{M1}, \qquad \tilde{f}_A = -\tilde{f}_v, \tag{22}$$

In some models involving the W-boson one has

$$\frac{f_{E0}}{q^2} = -\frac{f_{E1}}{m_\mu^2} \tag{23}$$

while in the supersymmetric models, one finds

$$f_{E0} = -f_{M0} = -\frac{1}{2}\tilde{\eta}\alpha^2 g(x)\frac{m_\mu^2}{m_{3/2}^2} \tag{24}$$

$$4\pi\alpha\, f_{E1} = -4\pi\alpha\, f_{M1} = -\frac{1}{2}\tilde{\eta}\alpha^2 f(x)\frac{m_\mu^2}{m_{3/2}^2} \tag{25}$$

$$\tilde{f}_V = -\tilde{f}_A = -\frac{\beta_0}{2}\tilde{\eta}\alpha^2 f_b(x)\frac{m_\mu^2}{m_{3/2}^2} \tag{26}$$

and

$$\beta_0 = \frac{4}{9} + \frac{1}{9}\frac{m_{\tilde{u}}^2}{m_{\tilde{d}}^2}, \qquad \beta_1 = \frac{4}{9} - \frac{1}{9}\frac{m_{\tilde{u}}^2}{m_{\tilde{d}}^2} \tag{27}$$

The functions $g(x), f(x), f_b(x)$ depend on the ratio x = $m_{\tilde{\gamma}}/m_{\tilde{a}}$ where $m_{\tilde{a}}$ is the relevant *s* quark mass and $m_{\tilde{\gamma}}$ is the photino mass. However, since this quantity is much smaller than unity, we get

$$f(x) \approx \frac{1}{2}, \qquad g(x) \approx \frac{1}{18}, \qquad f_b(x) \approx \frac{1}{8} \tag{28}$$

4. Effective Nuclear Transition Operator and Nuclear Matrix Elements

The first step in constructing the effective transition operator is to take the non-relativistic limits of the hadronic currents in Eqs. (18), (19). This leads to the operators [30,31]

$$\boldsymbol{\Omega}_0 = \tilde{g}_V \sum_{j=1}^{A} (3 + f_V \beta \tau_{3j}) e^{-iq.r_j}, \quad \boldsymbol{\Omega} = -\tilde{g}_A \sum_{j=1}^{A} (\xi + \beta\tau_{3j}) \frac{\sigma_j}{\sqrt{3}} e^{-iq.r_j} \tag{29}$$

with $\xi = f_V / f_A$ and

$$\tilde{g}_V = \frac{1}{6}, \qquad \tilde{g}_A = 0, \qquad f_V = 1, \qquad \beta = 3 \qquad \text{(photonic case)}, \tag{30}$$

$$\tilde{g}_V = \tilde{g}_A = \frac{1}{2}, \qquad f_V = 1, \qquad f_A = 1.24 \quad \text{(non-photonic case)}, \tag{31}$$

For neutrino mediated processes

$$\beta_0 = \begin{array}{c} 30 \\ 1 \end{array} \qquad \beta_1 = \begin{array}{cc} 25 & \text{lightneutrinos} \\ 5/6 & \text{heavyneutrinos} \end{array} \tag{32}$$

i.e. β = 5/6 $\approx$ 0.8 in both cases. For the supersymmetric models,

$$\beta_0 = \frac{5}{9}, \qquad \beta_1 = \frac{1}{3}, \qquad \text{i.e.} \quad \beta = 0.6 \tag{33}$$

The factor $1/\sqrt{3}$ in Ω was introduced for convenience. Thus, one has

$$|ME|^2 = f_V^2 |\langle f|\boldsymbol{\Omega}_0|i,\mu\rangle|^2 + 3f_A^2 |\langle f|\boldsymbol{\Omega}|i,\mu\rangle|^2 \tag{34}$$

The second step is to factor out the muon 1s wave function [32,33], i.e.

$$|\langle f|\boldsymbol{\Omega}\phi_\mu(r)|i\rangle|^2 \approx \langle \phi_{1s}^2\rangle |\langle f|\boldsymbol{\Omega}|i\rangle|^2 \tag{35}$$

$$\langle \phi_{1s}^2\rangle \equiv \frac{\int d^3r |\phi_\mu(r)|^2 \rho(r)}{\int d^3r \rho(r)} \tag{36}$$

with $\phi_\mu(\mathrm{r})$, the muon wave function and $\rho(\mathrm{r})$, the nuclear density. The above approximation was found to underestimate the width in heavy nuclei by as much as 40 %. With the above operators one can easily proceed with the evaluation of the relevant nuclear matrix elements. The following two cases can be discussed as-

4.1. *The coherent (μ⁻- e⁻) conversion matrix elements*

For $0^+ \to 0^+$ transitions only the operator Ω_0 of Eq. (34) contributes. One finds [30]

$$\langle i|\boldsymbol{\Omega}_0|i\rangle = \tilde{g}_V(3 + f_V\beta) ZF(q^2) \tag{37}$$

where

$$F(q^2) = F_Z(q^2) + \frac{3 - f_V\beta}{3 + f_V\beta}\frac{N}{Z} F_N(q^2) \tag{38}$$

$$F_Z(q^2) = \frac{1}{Z}\int d^3r \rho_p(r) e^{-iq.r} \tag{39}$$

$$F_N(q^2) = \frac{1}{N}\int d^3r \rho_n(r) e^{-iq.r} \tag{40}$$

F_Z and F_N are the proton, neutron nuclear form factors with $\rho_p(r), \rho_n(r)$,the corresponding densities normalized to Z and N, respectively.

Then, the branching ratio R_{e^-N} takes the form

$$R_{e^-N} = \frac{1}{(G_F m_\mu^2)^2}\left\{\left|\frac{m_\mu^2}{q^2} f_{M1} + f_{E0} + \frac{1}{2}\kappa\tilde{f}_V\right|^2 + \left|\frac{m_\mu^2}{q^2} f_{E1} + f_{M0} + \frac{1}{2}\kappa\tilde{f}_A\right|^2\right\}\gamma_{ph} \tag{41}$$

where κ and γ_{ph} carry all the dependence on the nuclear physics, i.e.

$$\gamma_{ph} = \frac{Z\left|F_Z(q^2)\right|^2}{G^2 f_{PR}(A,Z)} \tag{42}$$

and

$$\kappa = \left(1 + \frac{N}{Z}\frac{3-\beta}{3+\beta}\frac{F_N(q^2)}{F_N(q^2)}\right)\zeta \tag{43}$$

In Eq. (42), G^2 is a combination of the coupling constants entering the ordinary muon capture, $G^2 = 6$ and f_{PR} is the well-known Primakoff function [32,33] which adequately describes the ordinary muon capture throughout the periodic table. It is approximately given by [33]

$$f_{PR} \approx 1.6\frac{Z}{A} - 0.62 \tag{44}$$

It is sometimes convenient to factor out the nuclear dependence from the dependence on the rest of the parameters of the theory [34], i.e. we write

$$R_{e^-N} = \rho\gamma \tag{45}$$

where the quantity ρ is independent of nuclear physics. The quantity γ takes the form

$$\gamma = \frac{\left|ME\right|^2}{G^2 Z f_{PR}(A,Z)} \tag{46}$$

In the case of the supersymmetric model, γ takes the form

$$\gamma = \left(\frac{1}{3} + \frac{3\kappa}{4}\right)^2 \gamma_{ph} \tag{47}$$

4.2. *Total (μ^-- e^-) conversion matrix elements*

As it was mentioned in the introduction, only the coherent rate is of experimental interest. It is, however, important to know what portion of the sum rule is exhausted by the coherent mode. Then, the total matrix element to be evaluated is,

$$M_{tot}^2 = f_V^2 \sum_f \left(\frac{q_f}{m_\mu}\right)^2 \left|\langle f|\boldsymbol{\Omega}_0|i\rangle\right|^2 + 3 f_A^2 \sum_f \left(\frac{q_f}{m_\mu}\right)^2 \left|\langle f|\boldsymbol{\Omega}|i\rangle\right|^2 \tag{48}$$

Where, $q_f = m_\mu - \in_b - (E_f - E_{gs})$ (49)

E_f, E_{gs} are the energies of the final and ground state of the nucleus. This evaluation clearly can be done in a model in which the final state can be explicitly constructed. This is a formidable task, however, and only in simple models this can easily be done, e.g., RPA [33]. The other alternative is to use some approximation scheme. The first is the so-called "closure approximation" [31,35]. In this approximation one first replaces the momentum q_f by a suitable average, i.e. $q_f \rightarrow k = \langle q_f \rangle$. Thus [30],

$$\boldsymbol{\Omega}_0(q_f) \approx \sum_j \omega_0(j) e^{-ik.r_j} \equiv \boldsymbol{\Omega}_0(k) \tag{50}$$

$$\boldsymbol{\Omega}(q_f) \approx \sum_j \omega(j) e^{-ik.r_j} \equiv \boldsymbol{\Omega}(k) \tag{51}$$

where

$$\omega_0(j) = 3 + f_V \tau_{3j}, \qquad \omega(j) = (\xi + \beta\tau_{3j})\sigma / \sqrt{3} \tag{52}$$

Then, we write

$$S_A = \sum_f \left(\frac{q_f}{m_\mu} \right)^2 \left| \langle f | \boldsymbol{\Omega}(q_f) | i \rangle \right|^2$$

$$\approx \frac{k^2}{m_\mu^2} \sum_f \langle i | \boldsymbol{\Omega}^+(k) | f \rangle . \langle f | \boldsymbol{\Omega}(k) | i \rangle \tag{53}$$

or, using closure over the final states, we get

$$S_A = k^2 m_\mu^2 \langle i | \boldsymbol{\Omega}^+(k) . \boldsymbol{\Omega}(k) i | \rangle$$

$$= \frac{k^2}{m_\mu^2} \left\{ \langle i | \boldsymbol{\Omega}_{1b} | i \rangle + \langle i | \boldsymbol{\Omega}_{2b} | i \rangle \right\} \tag{54}$$

where

$$\boldsymbol{\Omega}_{1b} = \sum_j \boldsymbol{\omega}^+(j) . \boldsymbol{\omega}(j) \tag{55}$$

$$\boldsymbol{\Omega}_{2b} = \sum_{i \neq j} \boldsymbol{\omega}^+(i) . \boldsymbol{\omega}(j) e^{-ik.(r_i - r_j)} \tag{56}$$

The computation of S_V is analogous with

$$\boldsymbol{\Omega}_{1b} = \sum_j \omega_0^+(j) \omega_0(j) \tag{57}$$

$$\boldsymbol{\Omega}_{2b} = \sum_{i \neq j} \omega_0^+(i) \omega_0(j) e^{-ik.(r_i - r_j)} \tag{58}$$

we thus find

$$M_{tot}^2 = f_V^2 S_V + 3 f_A^2 S_A \tag{59}$$

which is the total $(\mu^- - e^-)$ conversion matrix element.

5. Summary and Discussion of Calculated Matrix Elements

Below in Table 1, we have shown the calculated coherent (M_{coh}^2) and incoherent (M_{inc}^2) nuclear transition matrix elements (NTMEs) for photonic mechanism in case of ^{27}Al, ^{48}Ti, ^{60}Ni, ^{72}Ge, ^{112}Cd, ^{162}Yb and ^{208}Pb nuclei. The corresponding coherent and incoherent NTMEs for non-photonic mechanism are shown in Table 2 for the same nuclei. The available experimental values of coherent matrix elements M_{coh}^2 are also given for comparison in both Tables.

Table 1. Coherent (M_{coh}^2) and incoherent (M_{inc}^2) nuclear transition matrix elements (NTMEs) for photonic mechanism in case of ^{27}Al, ^{48}Ti, ^{60}Ni, ^{72}Ge, ^{112}Cd, ^{162}Yb and ^{208}Pb nuclei.

Nuclei	Exp.		Theory [Photonic mechanism (γ exchange)]				
	M_{coh}^2	Ref.	M_{coh}^2	M_{inc}^2	M_{tot}^2	Model	Ref.
^{27}Al	068.8	[36]	77.4	0.26	77.66	SM	[36]
			69.66	0.02	69.68	SM	[40]
^{48}Ti	1137.0	[36]	179.3	0.51	179.8	SM	[36]
			144.6	44.2	188.8	SM	[38]
			135.0	11.2	146.2	QRPA	[38]
			139.6			LDA	[38]
			117.7	5.51	123.21	QRPA	[39]
			127.2	4.60	131.8	RQRPA	[39]
^{60}Ni			187.5	101.5	289	SM	[38]
			187.8			QRPA	[38]
			198.7			LDA	[38]
			149.4	4.48	153.88	QRPA	[39]
			171.1	3.54	174.94	RQRPA	[39]
^{72}Ge	200.9	[37]	212.9	143.7	356.6	SM	[38]
			212.7			QRPA	[38]
			227.8			LDA	[38]
			189.9			QRPA	[39]
			199.1			RQRPA	[39]
			206.0	1.2	207.2	DHF	[37]
^{112}Cd			274.0			SM	[38]
			280.0			QRPA	[38]
			346.7			LDA	[38]
			222.6	8.14	230.74	QRPA	[39]
			256.7	6.48	292.18	RQRPA	[39]
^{162}Yb			313.6	537	850.6	SM	[38]
			311.0			QRPA	[38]
			283.8	13.52	297.32	QRPA	[39]
			393.3	9.63	402.93	RQRPA	[39]
^{208}Pb			240.2	701.3	941.5	SM	[38]
			287.5			QRPA	[38]
			582.9			LDA	[38]
			379.4	8.97	388.37	QRPA	[39]
			415.0	7.26	422.86	RQRPA	[39]

The photonic exchange includes exchange of photons while non photonic mechanism includes the exchange of *W* bosons and SUSY particles. For the coherent mode one essentially needs only to calculate the proton and neutron nuclear form factors while the incoherent conversion involves the matrix elements of all the excited states of the participating nucleus.

Table 2. Coherent (M^2_{coh}) and incoherent (M^2_{inc}) nuclear transition matrix elements (NTMEs) for non- photonic mechanism in case of ^{27}Al, ^{48}Ti, ^{60}Ni, ^{72}Ge, ^{112}Cd, ^{162}Yb and ^{208}Pb nuclei.

Nuclei	Exp.		Theory [Non-photonic mechanism]							
	M^2_{coh}	Ref.	W exchange			SUSY exchange			Model	Ref.
			M^2_{coh}	M^2_{inc}	M^2_{tot}	M^2_{coh}	M^2_{inc}	M^2_{tot}		
^{27}Al	068.8	[36]	663.0	3.2	666.2	607.6	60.1	667.7	SM	[36]
			664.9	1.3	666.2	743.5	20.7	764.2	SM	[40]
^{48}Ti	1137.0	[36]	1478	4.97	1482	1237	50.5	1287.5	SM	[36]
			374.3	93.7	468.0				SM	[38]
			363.0	23.4	386.4				QRPA	[38]
			375.7	34.8	410.5				LDA	[38]
			316.3	10.9	327.2				QRPA	[39]
			341.8	8.7	350.5				RQRPA	[39]
^{60}Ni			499.6	167	666.6				SM	[38]
			498.2						QRPA	[38]
			527.4						LDA	[38]
			396.6	10.2	406.8				QRPA	[39]
			454.1	8.5	462.6				RQRPA	[39]
^{72}Ge	200.9	[37]	595.8	251.2	847.0				SM	[38]
			596.2						QRPA	[38]
			639.5						LDA	[38]
			477.1						QRPA	[39]
			558.9						RQRPA	[39]
			623.2	7.0	630.2				DHF	[37]
^{112}Cd			769.4						SM	[38]
			785.8						QRPA	[38]
			983.3						LDA	[38]
			631.3	18.0	649.3				QRPA	[39]
			810.2	13.8	824.0				RQRPA	[39]
^{162}Yb			796.0	1187.3	1983.3				SM	[38]
			840.3						QRPA	[38]
			1412.1						LDA	[38]
			785.9	28.4	814.3				QRPA	[39]
			1089.1	19.4	1108.52				RQRPA	[39]
^{208}Pb			631.4	1609.6	2241.0				SM	[38]
			767.5						QRPA	[38]
			1674.9						LDA	[38]
			1077.8	20.8	1098.6				QRPA	[39]
			1180.4	17.1	1197.5				RQRPA	[39]

Siiskonen *et al*. [36] have evaluated the matrix elements for ^{27}Al and ^{48}Ti nuclei in the coherent as well as incoherent mode in shell model using USD interaction and OXBASH code. The coherent reaction channel, which is free from the background and is the measurable part of the total rate, is clearly dominant in both nuclei. The shell-model matrix elements overestimate the photonic matrix element by 25–30 % while non photonic matrix elements are overestimated by a factor of 10 nearly compared to experiment. Recently, the calculation for ^{27}Al was done by Kostensalo *et al*. [40], who extended the shell model calculations in the SD model space by including the p orbitals to see the effect of negative parity states on the conversion rate. The analysis shows dominance of coherent transitions mediated by isovector operators with practically null influence of excited positive or negative-parity states. The coherent matrix elements for photonic mechanism are smaller than SM calculation [36], but still larger by 10 % from experimental value. For ^{72}Ge, a deformed Hartree Fock (DHF) calculation, performed by T. S. Kosmas [37] and RQRPA calculation by Schwieger *et al.* [39] showed a close agreement with experimental value of M_{coh}^2 in case of photonic mechanism. From the Table it is seen that for all the nuclei except ^{27}Al, the coherent and incoherent matrix elements are evaluated by T. S. Kosmas and J. Schwieger *et al.* [38,39] using shell model (SM), Quasi particle RPA (QRPA) and renormalized QRPA(RQRPA) model.

Similar calculations have been performed Kostensalo *et al.* [41] employing the method of nuclear matter mapped into nuclei via a local density approximation (LDA) utilizing the relativistic Lindhard function and results are shown in Table. The coherent matrix elements are reduced for all the nuclei in both photonic and nonphotonic mechanism in QRPA calculation [42] but in case of ^{48}Ti, this reduces by a factor of approximately 70% as compared to SM calculation [36]. The total matrix elements in RQRPA [39] are slightly larger than those of ordinary QRPA although the incoherent RQRPA matrix elements are smaller than the corresponding QRPA ones [39] as seen from the table. This is due to the dominance of the coherent channel for which the trend of the matrix elements in the two methods is reversed.

6. Experimental Status of (μ^-, e^-) Conversion

The SINDRUM II collaboration at PSI carried out experiments to search for $(\mu^- - e^-)$ conversion in various nuclei [43]. It consisted of a set of concentric cylindrical drift chambers inside a superconducting solenoid magnet of 1.2 T. Negative muons with a momentum of about 90 MeV/c were stopped in a target located at the center of the apparatus, after passing a CH_2 moderator and a beam counter made of plastic scintillator. Charged particles with transverse momentum (with respect to the magnetic field direction) above 100 MeV/c, originating from the target, hit two layers of plastic scintillation arrays and then two layers of drift chambers, and eventually hit plexiglass Cherenkov hodoscopes placed at both ends. A momentum resolution of about 2.8 % (FWHM) for the energy region of conversion electrons was achieved. In a 1993 run with a titanium target, a total of 3×10^{13} stopped μ^-s were accumulated at a rate of 1.2×10^7 μ^-/sec from the μE_1 beam line at PSI and a 90 % C. L. upper limit of 6.1×10^{-13} was obtained [42]. The overall efficiency was about 13 %. Also, for a lead target, it gave$B(\mu^- Pb \rightarrow e^- Pb)$ < 4.6 x 10^{-11} [44]. A search for μ−e conversion in muonic gold was performed with the SINDRUM II spectrometer at PSI. The measurement resulted in $B(\mu^- Au \rightarrow e^- Au) < 7\times10^{-13}$ (90 % C. L.) [45]. Below in Table 3, we summarize the various $(\mu^- - e^-)$ conversion experiments performed for different nuclei.

Table 3. History and summary of (μ^--e^-) conversion in various nuclei

Process	90%-C.L.upper limit	Place	Year	Reference
$\mu^- + Cu \rightarrow e^- + Cu$	$<1.6\times10^{-8}$	SREL	1972	[46]
$\mu^- + {}^{32}S \rightarrow e^- + {}^{32}S$	$<7.0\times10^{-11}$	SIN	1982	[47]
$\mu^- + Ti \rightarrow e^- + Ti$	$<4.6\times10^{-12}$	TRIUMF	1988	[48]
$\mu^- + Ti \rightarrow e^- + Ti$	$<4.3\times10^{-12}$	PSI	1993	[49]
$\mu^- + Pb \rightarrow e^- + Pb$	$<4.6\times10^{-11}$	PSI	1996	[44]
$\mu^- + Ti \rightarrow e^- + Ti$	$<6.1\times10^{-13}$	PSI	1998	[42]
$\mu^- + Au \rightarrow e^- + Au$	$<7.0\times10^{-13}$	PSI	2006	[45]

A new experiment, E940, at Brookhaven National laboratory (BNL) AGS, the MECO (Muon Electron Conversion) experiment, was proposed [50] aiming to search for $\mu^- Al \rightarrow e^- Al$ at sensitivity below 10^{-16}. It used a new high intensity pulsed muon beam yielding about 10^{11} μ^-s /sec stopped in a target. The MECO apparatus consisted of a superconducting (SC) solenoid magnet to capture pions from the production target (production solenoid), a curved transport SC solenoid magnet system (transport solenoid), and a SC solenoid spectrometer, which was used to observe only the 105 MeV signal electrons (detector solenoid). In the experiment, curved transport solenoid captures muons from pion decays and selects the momentum and sign of charged particles by using collimators at three positions. A pulsed proton beam of about 1 MHz repetition with a pulse length of 30 nsec can be extracted at the AGS. It was expected to observe 6 signal events for $B(\mu^- Al \rightarrow e^- Al) \approx 10^{-16}$, during a one year run, with an expected background of 0.4 events. Unfortunately, the MECO experiment was canceled in 2005, owing to funding problems but several ideas of this experiment were adopted by the Mu2e experiment.

7. Future Prospects

The field of muon decay physics is presently very productive, even after its long history of over 60 years. Currently, there are several new experiments which are being either prepared or planned. Some of which are COMET, MEG II, Mu2e and Mu3e.

The J-PARC E21 experiment is an experiment to search for a CLFV process of neutrinoless muon-to electron conversion in a muonic atom, at a single-event sensitivity (SES) of 2.6×10^{-17} (or $< 6\times10^{-17}$, 90 % C.L. upper limit) at the Japanese Proton Accelerator Research Complex (J-PARC). Here, the SES is the experimental sensitivity to observe one event. This experiment is called COherent Muon to Electron Transition (COMET) [51]. The experiment will be carried out using a two-staged approach. COMET Phase-I aims at a single-event sensitivity (SES) of 3.1×10^{-15}, roughly a factor 100 better than the current experimental limit. The goal of the full experiment is a SES of 2.6×10^{-17}, which is referred to as Phase-II. This ultimate sensitivity goal is a factor of about 10,000 better than the current experimental limit of $B(\mu^{-}+Au \rightarrow e^{-}+Au) \leq 7\times10^{-13}$ from SINDRUM-II at PSI [45]. The experiment will be carried out in the Nuclear and Particle Physics Experimental Hall (NP Hall) at J-PARC using a bunched 8 GeV proton beam that is slow-extracted from the J-PARC main ring. Muons for the COMET experiment will be generated from the decay of pions produced by collisions of the 8 GeV proton beam on a production target. The yield of low-momentum muons transported to the experimental area is enhanced using a superconducting pion-capture solenoid surrounding the proton target in the pion-capture section. The signal electrons from the muon stopping target are then transported by additional curved solenoids to the main detector, a straw-tube tracker and electron calorimeter, called the StrECAL detector. For COMET Phase-I, the primary detector for the neutrinoless μ–e conversion signals consist of a cylindrical drift chamber and a set of trigger hodoscope counters, referred to as the CyDet detector. The experimental setup for Phase-I will be augmented with prototypes of the Phase-II StrECAL detector. The StrECAL and CyDet detectors will be used to characterize the beam and measure backgrounds to ensure that the Phase-II single-event sensitivity of 2.6×10^{-17} can be realized.

The Mu2e experiment [52,53] will search for the charged-lepton flavor violating (CLFV) neutrino-less conversion of a negative muon into an electron in the field of a nucleus. The conversion process results in a monochromatic electron with energy of 104.97 MeV, slightly below the muon rest mass. The goal of the experiment is to improve the previous upper limit by four orders of magnitude and reach a SES of 3×10^{-17} on the conversion rate, a 90% CL of 8×10^{-17}, and a 5σ discovery reach at 2×10^{-16}. The experiment will use an intense pulsed negative muon beam. The pulsed beam is essential to reducing backgrounds. The other essential element is a sophisticated magnetic system composed of three consecutive solenoids that form the muon beam. Mu2e will use an aluminum target and examine approximately 10^{18} stopped muons in 3 years of running. The Mu2e experiment is under design and construction at the Fermilab Muon Campus. The experiment will begin operations in 2022, and will require about 3 years of data-taking. The primary beam will start with the Fermilab Booster, supplying 8 GeV kinetic energy protons on target at 8 kW. Mu2e requires $\approx 3.6\times10^{20}$ protons-on-target to meet its goals.

COMET and Mu2e experiments are very similar in many respects. Both use curved solenoids that keep the low energy muons on helical trajectories while deflecting them through 90° in the horizontal plane. The curved transport line eliminates the direct line of sight from the production target to the muon stopping target, allowing the positioning of shielding to reduce the neutron background. The curved solenoids also have another, more specific, purpose. In deflecting the helical trajectories, a net drift is also created in the vertical plane, which depends on the charge sign and momentum of the particles in the beam. This removes positive and high-energy muons and other particles that contribute to the backgrounds, and the effect can be enhanced with a collimator. The exact arrangement of these components of these components is the most obvious difference between the two experiments. Mu2e uses two 90° transport sections arranged in an 'S' shape so that the largest beam dispersion is at the midpoint of the transport, the beam being returned to horizontal at the end of the second bend. A vertically offset collimator located at the midpoint is use to remove the unwanted positive and higher-momentum components. This collimator can be rotated for background studies. The capture solenoid of Mu2e uses a gradient field that has a maximum of 4.7 T at the forward end down to 2.5 T at the background end. This design reflects lower momentum particles from the forward direction to the backward direction, in order to increase the yield. On the other hand, COMET is designed to run in two phases, with different geometries. Phase-I of the experiments uses a 90° muon transport line which filters out less background. This is sufficient to make an intermediate sensitivity measurement on a shorter time-scale. More importantly, it can also be used to characterize the backward pion yield from the production target and improve the simulations used for use in Phase-II. The Phase –II experiment uses a longer transport line, which curves round 180° in a 'C' shape. This creates larger momentum dispersion in the vertical plane, improving the rejection of wrong-momentum muons. Unlike Mu2e there is no 'reverse turn' to level out the beam. COMET's beam transport instead uses a compensating dipole field to keep the desired lower-momentum negative muons vertically centered in the transport line. The pion capture solenoid in COMET has similar peak field strength of 5T, but unlike Mu2e it peaks at the target location, rather than at the end of the solenoid. As a result, there is no magnetic reflection to capture forward going pions. This configuration instead has wider solid angle of acceptance, for pions emitted at a larger angle from the solenoid axis. Thus, the two experiments basically differ in their transport line arrangement.

The MEG (Mu to Electron Gamma) experiment [54] has been running at the Paul Scherrer Institut (PSI), Switzerland since 2008 to search for the decay $\mu^+ \to e^+ \gamma$. The MEG experiment uses one of the world's most intense continuous surface muon beams, with maximum rate higher than 10^8 μ^+/s but the stopping intensity is limited to 3×10^7 μ^+/s. The muons are stopped in a thin polyethylene target, placed at the centre of the experimental set-up which includes a positron spectrometer and a photon detector. The positron spectrometer consists of a set of drift chambers and scintillating timing counters located inside a superconducting solenoid COBRA (COnstant Bending RAdius) with a gradient magnetic field along the beam axis, ranging from 1.27 T at the centre to 0.49 T at either end that guarantees a bending radius of positrons weakly dependent on the polar angle. The gradient field is also designed to remove quickly spiralling positrons sweeping them outside the spectrometer to reduce the track density inside the tracking volume. The photon detector, located outside of the solenoid, is a homogeneous volume of liquid xenon (LXe) viewed by photomultiplier tubes (PMTs) submerged in the liquid, that read the scintillating light from the LXe. The spectrometer measures the positron momentum vector and timing, while the LXe photon detector measures the photon energy as well as the position and time of its interaction in LXe. All the signals are individually digitised by in-house designed waveform digitisers (DRS4) [55]. The current most stringent limit, given by the MEG experiment on the $\mu^+ \rightarrow e^+\gamma$ decay branching ratio is $B(\mu^+ \rightarrow e^+\gamma) < 4.2 \times 10^{-13}$ at 90% confidence level (CL), based on the full data-set [54]. In order to increase the sensitivity, reach of the experiment by an order of magnitude to the level of 6×10^{-14}, a total upgrade, involving substantial changes to the experiment, has been undertaken, known as MEG II [56].

The Mu3e experiment to be realized at the Paul Scherrer Institute in Switzerland [25] aims at reaching a 10^{-16} sensitivity on the $\mu \rightarrow e^+e^-e^+$ decay in three successive phases of the experiment (called phase la, lb and Phase-II). The same muon beam presently used for the MEG and MEG II experiments will be transported to a thin (average thickness 85 μm) hollow double-cone Mylar target, with a total length of 10 cm. The target is surrounded by a 2 m long cylindrical detector located inside a 1.5 T solenoidal magnetic field which is segmented in five measuring stations. The central one consists of two double layers of silicon pixel detectors for charged particle tracking complemented by a scintillating fiber tracker for refined particle timing information. The four so-called "recurl" stations located at either side of the central one is made of two layers of pixel sensors surrounding a timing hodoscope made of thicker scintillator tiles. The detector has been designed in such a way as to exploit the fact that the effect of multiple scattering cancels out at first order after half a turn. In fact, a charged particle in a magnetic field follows a circular trajectory whose radius is proportional to its momentum. The experiment is planned in three stages with increasing sensitively, each stage corresponding to approximately an order of magnitude improvement. In the phase la, the experiment will run at a muon rate of a few $\times10^7$ μ^+/sec with no dedicated timing detector, since the 10 ns resolution of the tracker detector itself is sufficient to reject the accidental background to below 10^{-15}. In the phase lb, the addition of the scintillating fiber tracker in the central detector module will permit to withstand a rate of $\approx 10^8$ μ^+/sec, which is the maximum rate presently deliverable by the PSI beamline, while the two additional tracking station will almost double the acceptance for recurling tracks, improving significantly the momentum resolution. The final phase II, where the detector will be completed by two more recurling stations, will allow to reach an ultimate sensitivity of 10^{-16} at a muon decay rate of 2×10^9 μ^+/sec. To reach the intensity a new beam-line concept is required. The first stage of the experiment is currently under construction at PSI where beams with up to 10^8 muons per second are available [57].

Thus, to consider a next-generation experiment to search for CLFV, there are three important processes to be considered; namely, $\mu^+ \rightarrow e^+\gamma$, $\mu^+ \rightarrow e^+e^+e^-$, and $\mu^- - e^-$conversion. The three processes have different experimental issues that need to be solved to realize improved experimental sensitivities. The processes of $\mu^+ \rightarrow e^+\gamma$ and $\mu^+ \rightarrow e^+e^+e^-$ are detector-limited owing to accidental backgrounds. To consider and go beyond the present sensitivities, the resolutions of detection have to be improved, which requires innovative improvement of the detector technology. On the other hand, for $\mu^- - e^-$conversion, there are no accidental background events, and an experiment with higher rates can be performed. If a new muon source with a higher beam intensity and better beam quality for suppressing beam-associated background events can be constructed, measurements of the search for $\mu^- - e^-$conversion with a higher sensitivity can be performed. Furthermore, it is known that, in comparison with $\mu^+ \to e^+ \gamma$, there are more physical processes that $\mu^- - e^-$conversion and $\mu^+ \to e^+e^+e^-$ could contribute to. For instance, in SUSY models, photon-mediated diagrams can contribute to all three processes, but diagrams mediated by particles other than photons, such as Higgs-mediated diagrams, can contribute to only $\mu^- - e^-$conversion and $\mu^+ \to e^+e^+e^-$ [58]. In summary, with all the above considerations, we believe that a $\mu^- - e^-$conversion experiment would be the natural next step in the search for muon CLFV.

8. Conclusion

We have described the current theoretical and experimental status in the field of muon decay to search for physics beyond the standard model. Among many interesting topics of physics related to muons, we have discussed the muon to electron conversion process in detail. The physics motivation for LFV is extremely strong. LFV has recently attracted much attention from theorists and experimentalists, more than ever. We have presented the phenomenology of muon LFV process, the $\mu^- - e^-$ conversion in a muonic atom. Then, we have briefly mentioned the most recent experimental results, SINDRUM II, together with future experimental prospects. Muon physics becomes important with strong physics motivations. There are extraordinary opportunities which will allow us to explore discovery potentials of physics beyond the SM, with low energy muons.

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References

1. E. P. Hincks and B. Pontecorvo, Phys. Rev. **73**, 257 (1948). https://doi.org/10.1103/PhysRev.73.257
2. A. Lagarigue and C. R. Peyrou, Acad. Sc. (Paris) **234**, 1873 (1952).
3. S. Lokanathan and J. Steinberger, Phys. Rev. **98**, 240 (1955).
4. S. Frankel, In Muon Physics II, Weak Interactions, ed. V. W. Hughes et al. (Academic Press, New York 1975) pp. 83. https://doi.org/10.1016/B978-0-12-360602-0.50009-9
5. J. W. F. Valle, Prog. Part. Nucl. Phys. **26**, 91 (1991). https://doi.org/10.1016/0146-6410(91)90010-L
6. S. Dimopoulos, S. Raby, and G. L. Kane, Nucl. Phys. B **182**, 77 (1981). https://doi.org/10.1016/0550-3213(81)90459-4
7. F. Scheck, Phys. Rep. **44**, 187 (1978). https://doi.org/10.1016/0370-1573(78)90014-5
8. G. Costa and F Zwirner, Riv. Nuovo Cim, **9**, 1 (1986). https://doi.org/10.1007/BF02724479
9. R. Engfer and H. K. Walter, Ann. Rev. Nucl. Part. Sci. **36**, 327 (1986). https://doi.org/10.1146/annurev.ns.36.120186.001551
10. J. D. Vergados, Phys. Rep. **133**, 1 (1986). https://doi.org/10.1016/S0370-1573(01)00068-0

11. S. M. Bilenky and S. T. Petcov, Rev. Mod. Phys. **59**, 671 (1987). https://doi.org/10.1103/RevModPhys.59.671
12. P. L. Melese, Comments on Nucl. Part. Phys. **19**, 117 (1989).
13. C. A. Heusch, Nucl. Phys. (Proc. Suppl.) B **13**, 612 (1990). https://doi.org/10.1016/0920-5632(90)90141-G
14. P. Herczeg, Rare Decays - *Proc. 3rd Int. Symp. on weak and EM Int. in Nuclei*, Dubna, Russia, 1992, ed. T. D. Vylov (World Scientific, Singapore 1993) pp. 262.
15. A. Van der Schaaf, Prog. Part. Nucl. Phys. **31**, 1 (1993).
16. T. S. Kosmas, G. K. Leontaris, and J. D. Vergados, Prog. Part. Nucl. Phys. **33**, 397 (1994). https://doi.org/10.1016/0146-6410(94)90047-7
17. T. S. Kosmas, arXiv: nucl-th/0108045. https://arxiv.org/pdf/nucl-th/0108045.pdf
18. Y. Kuno and Y. Okada, Rev. Mod. Phys. **73**, 151 (2001). https://doi.org/10.1103/RevModPhys.73.151
19. A. Faessler, T, Gutsche, S. Kovalenko, V. E. Lyubovitskij, and I. Schmidt, Phys. Rev. D **72**, 075006 (2005). https://doi.org/10.1103/PhysRevD.72.075006
20. R. Bayes, J. F. Bueno, A. Hillairet, Y. I. Davydov, P. Depommier, W. Faszer, C. A. Gagliardi, A. Gaponenko, D. R. Gill, A. Grossheim, P. Gumplinger, M. D. Hasinoff, R. S. Henderson, J. Hu, D. D. Koetke, R. P. MacDonald, G. M. Marshall, E. L. Mathie, R. E. Mischke, K. Olchanski, A. Olin, R. Openshaw, J. M. Poutissou, R. Poutissou, V. Selivanov, G. Sheffer, B. Shin, T. D. S. Stanislaus, R. Tacik, and R. E. Tribble, Phys. Rev. Lett. **106**, ID 041804 (2011). https://doi.org/10.1103/PhysRevLett.106.041804
21. Y. Cai, X. G. He, M. R. Musolf, and L. H. Tsai, JHEP **12**, ID 054 (2011). https://doi.org/10.1007/JHEP12(2011)054
22. Y. Kuno, Prog. Theo. Exp. Phys. **022C01** (2013).
23. A. D. Gouvea and P. Vogel, Prog. Particle Nucl. Phys. **71**, 75 (2013). https://doi.org/10.1016/j.ppnp.2013.03.006
24. J. M. Cline, A. Diaz Furlong and J. Ren, Phys. Rev. D **93**, ID 036009 (2016). https://doi.org/10.1103/PhysRevD.93.036009
25. L. Calibbi and G. Signorelli, Rev. Nuovo Cim. **41**, 71 (2018).
26. M. Tanabashi, K. Hagiwara, K. Hikasa, K. Nakamura, Y. Sumino, F. Takahashi, and J. Tanaka, Phys. Rev. D **98**, ID 030001 (2018).
27. S. Zhao, Eur. Phys. J. C, **80**, 1167 (2020). https://doi.org/10.1140/epjc/s10052-020-8408-6
28. S. Weinberg and G. Feinberg, Phys. Rev. Lett. **3**, 244 (1959). https://doi.org/10.1103/PhysRevLett.3.244.2
29. W. J. Marciano and A. I. Sanda, Phys. Rev. Lett. **38**, 1512 (1977). https://doi.org/10.1103/PhysRevLett.38.1512
30. T. S. Kosmas, J. D. Vergados, O. Civitarese, and A. Faessler, Nucl. Phys. A **570**, 637 (1994). https://doi.org/10.1016/0375-9474(94)90077-9
31. T. S. Kosmas and J. D. Vergados, Nucl. Phys. A, **510**, 641 (1990). https://doi.org/10.1016/0375-9474(90)90353-N
32. H. Primakoff, Rev. Mod. Phys. **31**, 802 (1959). https://doi.org/10.1103/RevModPhys.31.802
33. B. Goulard and H. Primakoff, Phys. Rev. C **10**, 2034 (1974). https://doi.org/10.1103/PhysRevC.10.2034
34. T. S. Kosmas and J. D. Vergados, Phys. Lett. B, **215**, 460 (1988). https://doi.org/10.1016/0370-2693(88)91341-X
35. T. S. Kosmas, G. K. Leontaris, and J. D. Vergados. Phys. Lett. B, **219**, 419 (1989). https://doi.org/10.1016/0370-2693(89)91094-0
36. T. Siiskonen, J. Suhonen, and T.S. Kosmas, Phys. Rev. C **62**, ID 035502 (2000). https://doi.org/10.1103/PhysRevC.62.035502
37. T. S. Kosmas, A. Faessler, and R. Sahu, Phys. Rev. C **68**, ID 054315 (2003). https://doi.org/10.1103/PhysRevC.68.054315
38. T. S. Kosmas and J. D. Vergados, Phys. Reports. **264**, 251 (1996). https://doi.org/10.1016/0370-1573(95)00041-0

39. J. Schwieger, T. S. Kosmas, and A. Faessler, Phys. Lett. B **443**, 7 (1998). https://doi.org/10.1016/S0370-2693(98)01331-8
40. J. Kostensalo, J. Suhonen, and O. Civitarese, Phys. Rev. C **98**, ID 065504 (2018). https://doi.org/10.1103/PhysRevC.98.065504
41. H. C. Chiang, E. Oset, T. S. Kosmas, A. Faessler, and J. D. Vergados, Nucl. Phys. A **559**, 526 (1993). https://doi.org/10.1016/0375-9474(93)90259-Z
42. P. Wintz - *Proc. of the First Int. Symp. on Lepton and Baryon Number Violation*, ed. H. V. Klapdor-Kleingrothaus et al. (Institute of Physics Publishing, Bristol and Philadelphia, 1998) pp. 534.
43. A. Vander Schaaf, SINDRUM II - *3rd Int. Workshop on Neutrino Factories based on Muon Storage Rings, NuFACT'01* (Tsukuba, Japan, 2001).
44. W. Honecker, C. Dohmen, H. Haan, D. Junker, G. Otter, M. Starlinger, P. Wintz, J. Hofmann, W. Bertl, J. Egger, B. Krause, S. Eggli, R. Engfer, Ch. Findeisen, E. A. Hermes, T. Kozlowski, C. B. Niebuhr, H. S. Pruys, and A.V. Schaaf, Phys. Rev. Lett. **76**, 200 (1996). https://doi.org/10.1103/PhysRevLett.76.200
45. W. Bertl, R. Engfer, E. A. Hermes, G. Kurz, T. Kozlowski, J. Kuth, G. Otter, F. Rosenbaum, N.M. Ryskulov, A.V. Schaaf, P. Wintz, and I. Zychor, Eur. Phys. J. C **47**, 337 (2006). https://doi.org/10.1140/epjc/s2006-02582-x
46. D. A. Bryman, M. Blecher, K. Gotow, and R. J. Powers, Phys. Rev. Lett. **28**, 1469 (1972). https://doi.org/10.1103/PhysRevLett.28.1469
47. A. Badertscher, K. Borer, G. Czapek, A. Fluckiger, H. Hanni, B. Hahn, E. Hugentobler, A. Markees, T. Marti, U. Moser, E. Ramseyer, J. Schacher, H. Scheidiger, P. Schlatter, G. Viertel, and W. Zeller, Nucl. Phys. A **37**, 406 (1982). https://doi.org/10.1016/0375-9474(82)90049-5
48. S. Ahmad, G. Azuelos, M. Blecher, D.A. Bryman, R.A. Burnham, E.T.H. Clifford, P. Depommier, M.S. Dixit, K. Gotow, C.K. Hargrove, M. Hasinoff, M. Leitch, J.A. Macdonald, H. Mes, I. Navon, T. Numao, J.M. Poutissou, R. Poutissou, P. Schlatter, J. Spuller, and J. Summhammer, Phys. Rev. **38**, 2102 (1988). https://doi.org/10.1103/PhysRevD.38.2102
49. C. Dohmen, K. D. Groth, B. Heer, W. Honecker, G. Otter, B. Steinrücken, P. Wintz, V. Djordjadze, J. Hofmann, T. Kozlowski, S. Playfer, W. Bertl, J. Egger, W. Herold, B. Krause, H. K. Walter, R. Engfer, C. Findeisen, M. G. Handschin, E. A. Hermes, F. Muheim, C. B. Niebuhr, H. S. Pruys, L. Ricken, D. Vermeulen, and A. V. Schaaf, Phys. Lett. B **317**, 631 (1993). https://doi.org/10.1016/0370-2693(93)91383-X
50. M. Hebert, Nucl. Phys. A. **721**, 461 (2003).
51. R. Abramishvili, G. Adamov, R.R. Akhmetshin, A. Allin, J.C. Angelique, V. Anishchik, M. Aloki, D. Aznabayev and I. Bagaturia (The COMET Collaboration), Prog. Theo. Exp. Phys. **033C01** (2020).
52. R. H. Bernstein, Front. Phys. **7**, 1 (2019). https://doi.org/10.3389/fphy.2019.00001
53. M. Kargiantoulakis, arXiv: hep-ex/2003.12678. https://arxiv.org/pdf/2003.12678.pdf
54. A. M. Baldini, Y. Bao, E. Baracchini, C. Bemporad, F. Berg, M. Biasotti, G. Boca, and M. Cascella, Eur. Phys. J. C **76**, 434 (2016). https://doi.org/10.1140/epjc/s10052-016-4271-x
55. S. Ritt, R. Dinapoli, and U. Hartmann, Nucl. Instrum. Methods A **623**, 486 (2010). https://doi.org/10.1016/j.nima.2010.03.045
56. A. M. Baldini, E. Baracchini, C. Bemporad, F. Berg, M. Biasotti, G. Boca, and P. W. Cattaneo, Eur. Phys. J. C **78**, 380 (2018).
57. H. Augustin, N. Berger, A. Bravar, K. Briggl, H. Chen, S. Corrodi, S. Dittmeier, B. Gayther, L. Gerritzen, D. Gottschalk, U. Hartmann, G. Hesketh, M. Koppel, S. Kilani, A. Kozlinskiy, F. M. Aeschbacher, M. Muller, Y. Munwes, A.K. Perrevoort, S. Ritt, A. Schoning, H. C. S. Coulon, W. Shen, L. Vigani, D. Bruch, F. Wauters, and D. Wiedner, T. Zhong, arXiv: 2010.15648.
58. R. Kitano, M. Koike, S. Komine, and Y. Okada, Phys. Lett. B **575**, 300 (2003). https://doi.org/10.1016/j.physletb.2003.09.067