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# Bianchi Type-VI$_0$ Inflationary Model in Lyra Geometry

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**Abstract**

In this paper, we have studied Bianchi type metric in Lyra's geometry with scalar field and flat potential. The Einstein's field equations have been solved by taking the shear scalar in the model proportional to the expansion scalar, which leads to $A = B^n$, where *A* and *B* are metric functions and *n* is a positive constant. Also, we discuss some physical and geometrical features of the obtaining model.

*Keywords*: Bianchi type- VI$_0$; Scalar field; Flat potential; Lyra geometry.


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## 1. Introduction

Bianchi-type cosmological models are homogeneous, and anisotropic is a notable fact. On a temporal scale, the universe isotropization process may be investigated. Anisotropic universes are more general than isotropic cosmological universes from a theoretical standpoint. Bianchi spacetimes in constructing spatially homogeneous and anisotropic cosmological models are beneficial. Inflation refers to the accelerated expansion of the cosmos in its early stages. It enables spatial flatness and near large-scale homogeneity. The inclusion of inflation in linear cosmology gives the advantage of being the only known process that can explain the evolution of large-scale structures that are seen that formed under gravitational instability. Recently, the large-scale data from various cosmological surveys has attracted much attention in inflationary cosmology research. Despite the effectiveness in detecting inflation, the cause of the problem is not clear. Bali and Poonia [1] formed a Bianchi-type cosmological model in general relativity and discovered the model's inflationary solution. The models' anisotropic nature, which began with a decelerating phase and expanded later with acceleration, which corresponded to an inflationary situation, was also noticed. Some researchers have examined different aspects of the inflationary models [2-6].

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The field equations for Lyra's geometry are,

$$R_i^j+\frac{1}{2}Rg_i^j+\frac{3}{2}U_iU^j-\frac{3}{4}g_i^jU_kU^k=-8\pi T_i^j \tag{1}$$

Where $U_i$ are displacement fields and the other notations have the same meaning as in Riemannian Geometry
(here we have chosen $G=c=1$). Here,

$U_i=(0,0,0,\beta(t))$

Several researchers [7-17] have investigated many cosmological theories in different contexts in view of Lyra's geometry. Also, Basumatary *et al.* [18] have investigated Bianchi type -$VI_0$ Cosmological Model with a special form of scale factor in the Sen-Dunn Theory of Gravitation. The purpose of this study is to find a Bianchi type $VI_o$ inflationary model in the context of Lyra geometry. Our paper is organized as follows,

In section 2, we derive the field equations in Lyra's geometry with the aids of Bianchi-type $VI_0$ spacetime by using the scalar field as the source. The solution of field equations is found in section 3. In section 4, some physical and geometrical features have been examined, and the last section contains a conclusion.

## 2. Field Equations in Lyra's Geometry

Consider Bianchi Type-$VI_0$ metric in the form,

$$ds^2=-dt^2+A^2dx^2+B^2e^{2x}dy^2+C^2e^{-2x}dz^2 \tag{2}$$

Where, $A,B,C$ are functions of cosmic time $t$ only.
The energy-momentum tensor for scalar field [19] is given by,

$$T_{ij}=\partial_i\phi\partial_j\phi-\left[\frac{1}{2}\partial_l\phi\partial^l\phi+v(\phi)\right]g_{ij} \tag{3}$$

The law of conservation of energy-momentum tensor,

$$\frac{1}{\sqrt{-g}}\partial_i\left[\sqrt{-g}\,\partial_i\phi\right]=-\frac{dV}{d\phi} \tag{4}$$

Where,

$$\partial_i\phi=\frac{\partial\phi}{\partial x^i},\ \partial_j\phi=\frac{\partial\phi}{\partial x^j},$$

and $\partial^l\phi=g^{lv}\partial_v\phi=g^{lv}\dfrac{\partial\phi}{\partial x^v}$

The differential equations are obtained by combining the field Eqs. (1) and metric (2) with the components of the energy-momentum tensor (3).

$$\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}+\frac{\dot{B}\dot{C}}{BC}+\frac{1}{A^2}+\frac{3}{4}\beta^2=-8\pi\left(\frac{1}{2}\dot{\phi}^2-V(\phi)\right) \tag{5}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{C}}{C}+\frac{\dot{A}\dot{C}}{AC}-\frac{1}{A^2}+\frac{3}{4}\beta^2=-8\pi\left(\frac{1}{2}\dot{\phi}^2-V(\phi)\right) \tag{6}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}-\frac{1}{A^2}+\frac{3}{4}\beta^2=-8\pi\left(\frac{1}{2}\dot{\phi}^2-V(\phi)\right) \tag{7}$$

$$\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{A}\dot{C}}{AC}-\frac{1}{A^2}-\frac{3}{4}\beta^2=8\pi\left(\frac{1}{2}\dot{\phi}^2+V(\phi)\right) \tag{8}$$

$$\frac{\dot{B}}{B}-\frac{\dot{C}}{C}=0 \tag{9}$$

Equation (4) derives the law of conservation of energy-momentum tensor.

$$\ddot{\phi}+\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)\dot{\phi}=-\frac{dV}{d\phi} \tag{10}$$

where the above dot represents the ordinary derivative of *t*.

## 3. Solution of the Field Equation

From equation (9),

$$\frac{\dot{B}}{B}=\frac{\dot{C}}{C} \tag{11}$$

Which gives,

$$B=\mu C \tag{12}$$

For the simplicity, we consider $\mu=1$, $\mu$ is the constant of integration, such that

$$B=C \tag{13}$$

So the equations (5) to (8 ) can be rewritten as,

$$2\frac{\ddot{B}}{B}+\frac{\dot{B}^2}{B^2}+\frac{1}{A^2}+\frac{3}{4}\beta^2=-8\pi\left(\frac{1}{2}\dot{\phi}^2-V(\phi)\right) \tag{14}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}-\frac{1}{A^2}+\frac{3}{4}\beta^2=-8\pi\left(\frac{1}{2}\dot{\phi}^2-V(\phi)\right) \tag{15}$$

$$2\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}^2}{B^2}-\frac{1}{A^2}-\frac{3}{4}\beta^2=8\pi\left(\frac{1}{2}\dot{\phi}^2+V(\phi)\right) \tag{16}$$

$$\ddot{\phi}+\dot{\phi}\left(\frac{\dot{A}}{A}+2\frac{\dot{B}}{B}\right)=0 \tag{17}$$

(For the flat region, we assume potential $V$ = constant, resulting in $\frac{dV}{d\phi}=0$ )

From equation (17),

$$\dot{\phi} = \frac{K_1}{AB^2} \tag{18}$$

To solve field equations, we assume that the shear scalar is proportional to the scalar expansion of spacetime, as Thorne [20] and Collins *et al.* [21] have proposed. This leads to

$$A = B^n \tag{19}$$

By solving (14) to (16) and applying the condition specified in equation (19), the following is obtained:

$$\frac{\ddot{B}}{B} + \frac{n^2}{n-1}\frac{\dot{B}^2}{B^2} - \frac{2}{n-1}\frac{1}{B^{2n}} = 0 \tag{20}$$

Putting $\dot{B} = f, \ddot{B} = ff'$ where $f' = \frac{df}{dB}$ in equation (20), and then integrating

$$dt = \frac{dB}{\sqrt{\frac{4}{n-1}B^{\frac{3n-5-2n^2}{n-1}}}}$$

As a result, model (2) is simplified to:

$$ds^2 = \left(\frac{4}{n-1}B^{\frac{3n-5-2n^2}{n-1}}\right)^{-1} dB^2 - B^{2n}dx^2 + B^2\left(e^{2x}dy^2 + e^{-2x}dz^2\right) \tag{21}$$

By applying the appropriate transformation, $B = T$

And $dt = \frac{dT}{\left(\sqrt{\frac{4}{n-1}T^{3n-5-2n^2/n-1}}\right)}$

Equation (21) yields to,

$$ds^2 = \left(\frac{4}{n-1}T^{\frac{3n-5-2n^2}{n-1}}\right)^{-1} dT^2 - T^{2n}dx^2 + T^2\left(e^{2x}dy^2 + e^{-2x}dz^2\right) \tag{22}$$

If $n = 1$ singularity arises, so for the realistic model we take $n > 1$. In the present model $n = 1$ can not be taken for explaining the features of the universe.

## 4. Some Physical and Geometrical Features

In this section, we discussed some physical and geometrical features of the model obtained in Lyra's geometry.

Spatial Volume $v = T^{n+2}$ (23)

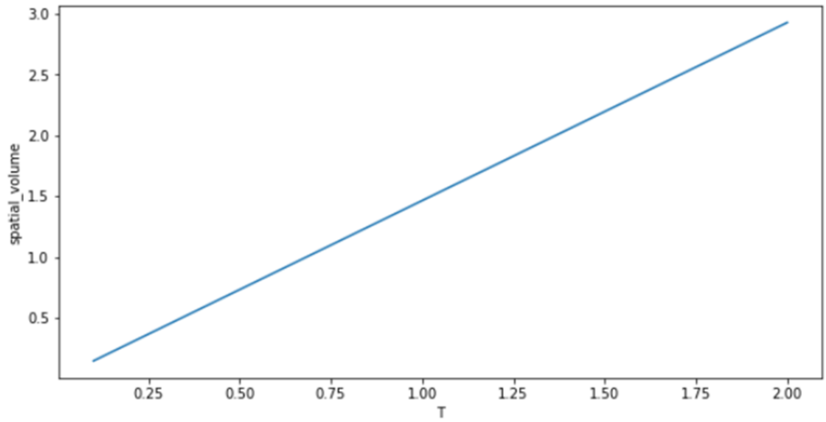


Fig. 1. Behavior of Spatial volume of the model versus time with the appropriate choice of constant.

The volume of the model appears to rise as time passes. As a result, the model begins to evolve with zero volume at the initial epoch with an infinite rate of expansion.
The Hubble parameter $(H)$, Scalar Expansion $(\theta)$, Shear Scalar ($\sigma$), Redshift, Decomposition of time like tidal tensor and Deceleration parameter are given by,

Hubble parameter $$H = \frac{n+2}{3}\left(\frac{4}{n-1}T^{3n-2n^2-5/n-1}\right)^{1/2} \quad (24)$$

Scalar Expansion $$\theta = n+2\left(\frac{4}{n-1}T^{3n-2n^2-5/n-1}\right)^{1/2} \quad (25)$$

Shear Scalar $$\sigma^2 = \frac{(n+2)^2}{6}\left(\frac{4}{n-1}T^{3n-2n^2-5/n-1}\right) \quad (26)$$

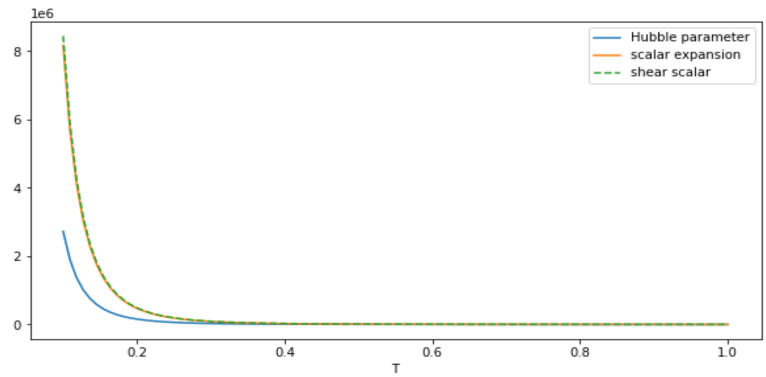


Fig. 2. Behavior of Hubble parameter, Expansion scalar, Shear scalar of the model versus time with the appropriate choice of constants.

Thus, from Hubble parameter and expansion scalar for the model (22) when T=0, both are infinite and steadily decrease as time increases. When $T \to \infty, H, \theta \to 0$ the model demonstrates that the cosmos expands with time. However, the rate of growth slows down and eventually ends. As a result, it has been discovered that the value of the shear scalar is initially positive. However, the value decreases until it becomes zero in the late universe as time passes. At late time shear tends to zero.

$$\beta^2 = \frac{4}{3}\left( (2n+1)(n+2)^2 \frac{4}{n-1} T^{3n-5-2n^2/n-1} - \frac{1}{T^{2n}} - K_2 + \frac{K_3}{T^{2(n+2)}} \right) \tag{27}$$

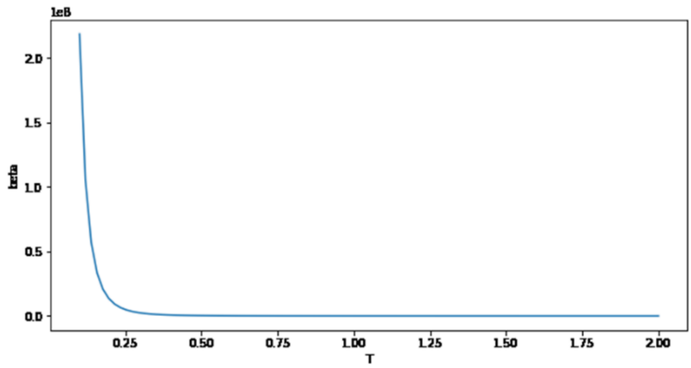


Fig. 3. Behavior of beta function of the model versus time with the appropriate choice of constants.

The (β(t)) defined by equation (27) is found to be infinite at the beginning epoch of time in this cosmological model, and it reduces with the progression of time. Finally, $\beta^2 \to 0$ *when* $T \to \infty$.

$$\phi = K_4 - \frac{K_1}{(n+1)} \frac{1}{T^{n+1}} \tag{28}$$

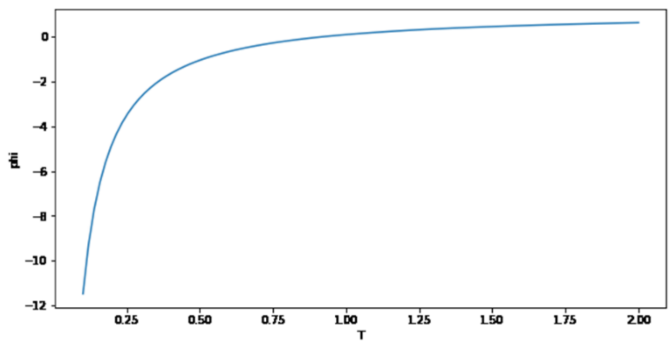


Fig. 4. Behavior of phi versus time with the appropriate choice of constant.

Shows that $\phi$ is a positive and expanding function of time throughout the model's history, and that finally reaches a constant value.

$$\text{Redshift } z = \left(\frac{1}{(T^{n+2})^{1/3}}\right) - 1 \tag{29}$$

Decomposition of time like tidal tensor.

$$u_{a;b} = -\left(\frac{4}{n-1} T^{3n-2n^2-5/n-1}\right)^{1/2} T^2 \left(nT^{2n-2} + e^{2x} + e^{-2x}\right) \tag{30}$$

Deceleration parameter,

$$q = -1 + \left(3 \frac{\left(5 - 3n + 2n^2\right)}{4(n-1)(n+2)T(t) \times (T^{(-2n^2+3n-5/(-1+n))}/(n-1))^{1/2}} T'\right) \tag{31}$$

It may be observed that as $T \to 0$ the volume $V \to 0$. So, at $T = 0$ the model starts evolving, and it gets expanded with cosmic time for $n > 0$. Thus we get that there is inflation in this model. When $T = 0;$ both $\theta$ and $H$ are infinite, and as the time increases gradually they decrease and when $T \to \infty$ both $\theta$ and $H$ tends to zero.

This analysis suggests that the cosmos expands with time but at a slower pace in the initial stage. The value of the expansions anisotropic parameter is constant, implying that the anisotropy will be preserved until the end of time.

## 5. Conclusion

Inflation and spacetime associated with them have cosmological interest due to their important applications in the structure formation of the universe. Also, it is well known that scalar fields have considerable effects in the early stages of the inflationary universe. Here inflationary Bianchi type-$VI_0$ cosmological model in the context of Lyra's geometry is obtained. Some physical and geometrical features of the obtaining model are also discussed.

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