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Reliability and Fault Analysis of a Stochastic Model of a Standby System with Cost Depended Repair/Replacement of Substandard Unit and Correlated Life Time

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Abstract

The present paper deals with the reliability and fault analysis of a stochastic model for two non-identical units, in which the first unit is kept as operative and the other sub-standard one. The sub-standard unit may be repaired or may be replaced by another sub-standard unit on its failure, depending on the cost of repairing/replacement. Failure and repair times are considered correlated using bivariate point exponential distribution. Analysis of a system is done to find various reliability measures, which gives the system's effectiveness. The conclusion about these reliability measures is carried out by graphical studies. The main emphasis is on the correlation between repair time and failure time.

Keywords: Stochastic process; Regenerative point; Semi-Markov process.

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1. Introduction

Many researchers have contributed many folds in the area of Reliability modeling. Many researchers have discussed many models under various assumptions, including the assumption that if a unit fails, it has to be repaired/replaced [1,2]. Various concepts under different situations have been analyzed. In the present investigation study, a stochastic model of two dissimilar units is good quality, and the other is substandard. The substandard unit may be repaired or replaced depending on if the replacement cost is higher or less than repair [3]. It will work well if a unit becomes operative [4,5]. On the failure of a unit, a repairman comes immediately to repair or replace it. This model works with the assumption that failure and repair /replacement are correlated to each other, and joint distribution of failure and repair/replacement vs. times is taken as bivariate exponential. This model is analyzed by using the semi-Markov process and regenerating point technique.

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Two-unit cold standby system: A two-unit cold standby system with two types of repair facilities has been investigated by many researchers [6,7]. The two-unit cold standby redundant system has been discussed subject to random checking and corrective maintenance [8,9]. This system describes the profit analysis of a single unit with programmable logic control [10]. This model involves a system's cost analysis where repair of the main unit depends on the sub-unit [11]. A comparison of a redundant system based on correlated lifetime can be carried out [12]. The two-unit complex system with correlated failure and repair time can be estimated [13-17]. The aim of this paper is to develop a more general and practically viable stochastic model of two dissimilar units, one is of good quality, and the other is a sub-standard one that may be repaired or replaced by the other sub-standard unit on its failure. Expressions for various system performance measures such as Mean Time to System Failure (MTSF), availability, a busy period of the repairman, etc., are derived for the model using the Markov process, regenerative point technique, and bivariate exponential distribution. Various conclusions regarding the reliability and profit of the system are drawn based on graphical studies.

Description of model and assumptions: The system consists of two dissimilar units. Initially, one unit is operative, and the other is cold standby. Upon failure of an operative unit, the cold standby unit became operative instantaneously, and failed unit went under repair. If a unit is under repair, it does not work for the system. When both the unit fails, the system becomes inoperable. The system is good as new, after each repair and replacement. A single serviceman facility is provided to the system for inspection, repair, and replacement of the components. Time distributions of various failures are exponential.

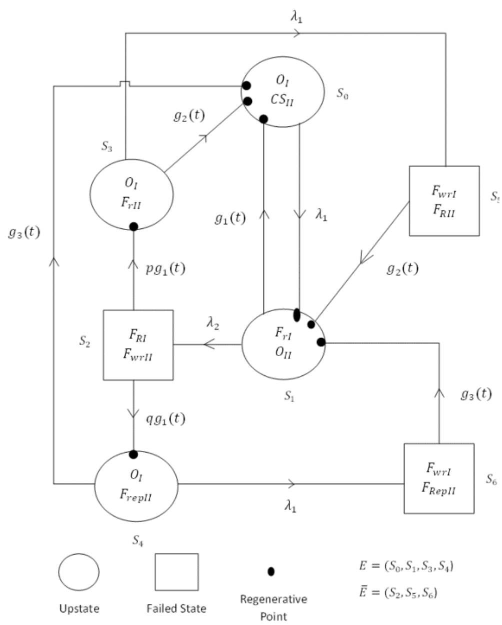

Plate 1. State transition diagram.

Notation:

λ_1 / λ_2 = constant failure rate of first and second unit

p = probability that unit 2 is repairable on failure

q = probability that unit 2 is not repairable on failure

G_1 (Y/X), g_1 (Y/X) = Conditional c. d. f and Conditional p. d .f of the repair time of unit 1

G_2 (Y/X), g_2 (Y/X) =Conditional c. d. f and Conditional p .d .f of the repair time of unit 2

G_3 (Y/X), g_3 (Y/X) = Conditional c. d .f and Conditional p. d .f of the replacement time of unit 2

$$F_I\,(x, y) = \lambda_i\lambda_j(1-r_i)e^{-\lambda_i x-\lambda_j y}I_0\left(2\sqrt{\lambda_i\lambda_j\, r_i xy}\right);\; X, Y, \lambda_i\,, \lambda_j > 0; 0 \le r_i$$

Where $I_0\left(2\sqrt{\lambda_i\lambda_j r_i xy}\right) = \sum_{j=0}^{\infty}\frac{(\lambda_i\lambda_j r_i xy)^j}{(j!)^2}$

r= Corr (x,y)

$g_i(Y/X) = \; conditional\; pdf$ of $Y_i\; given\; X_i = x\; is\; given\; by$

$$\beta_i\, e^{-\lambda_i x-\lambda_j y}I_0\left(2\sqrt{\lambda_i\lambda_j r_i xy}\right)$$

$G_i(Y/X) = \; conditional\; cdf$ of $Y_i\; given\; X_i = x\; is\; given\; by$

$$= \int_0^\infty \overline{G_i(Y/X)}\; dy = \int_0^\infty y\,\beta_i\, e^{-\lambda_i x-\lambda_j y}I_0\left(2\sqrt{\lambda_i\lambda_j r_i xy}\right)$$

$Marginal\; pdf\; of\; X_i = \lambda_i(1-r_i)e^{-\lambda_i\,(1-r_i)x}$

$Marginal\; pdf\; of\; Y_i = \lambda_j\,(1-r_i)e^{-\lambda_j\,(1-r_i)y}$

Symbols for the states of the system

S_i = State number i, i = 1, 2, 3, 4, 5, 6

O_I, O_{II} = Operating state of a first and second unit, respectively.

C_{SI}, C_{SII} = Cold standby state of a first and second unit, respectively.

F_{rI}, F_{rII} = Repair state of a first and second unit, respectively.

F_{RI}, F_{RII} = repair is continuing from the previous state of the first and second unit, respectively.

$F_{w\,rI}$, $F_{w\,rII}$ = waiting for the repair of a first and second unit, respectively.

F_{repII} = replacement of the second unit

F_{RepII} = replacement of the second unit from the previous state model.

Transition probabilities

$$Q_{0,1} = \int_0^t \lambda_1(1-r_1)\, e^{-\lambda_1(1-r_1)u}\, du$$

$$Q_{1,0/x} = \int_0^t e^{-\lambda_2(1-r_2)u}\, g_1(u/x)du$$

$$= g_1 e^{-\lambda_1 r_1 x}\sum_{J=0}^{\infty}\frac{(\lambda_1 g_1 r_1 x)^j}{(j!)^2}\int_0^t e^{-[g_1+\lambda_2(1-r_2)]u}\, u^j du$$

$$Q_{1,2/x} = \int_0^t \lambda_2(1-r_2)e^{-\lambda_2(1-r_2)u}\,\bar{G}_1(u/x)du$$

$$= g_1 e^{-\lambda_1 r_1 x} \sum_{j=0}^{\infty} \frac{(\lambda_1 g_1 r_1 x)^j}{(j!)^2} \int_0^t e^{-g_1 v}(1 - e^{-\lambda_2(1-r_2)v})\, v^j dv$$

$$Q_{3,0/x} = \int_0^t e^{-\lambda_1(1-r_1)u}\, G_2(u/x)du$$

$$= g_2 e^{-\lambda_2 r_2 x} \sum_{j=0}^{\infty} \frac{(\lambda_2 g_2 r_2 x)^j}{(j!)^2} \int_0^t e^{-[g_2+\lambda_1(1-r_1)]u}\, u^j du$$

$$Q_{3,5/x} = \int_0^t \lambda_1(1-r_1)e^{-\lambda_1(1-r_1)u}\, \bar{G}_2(u/x)du$$

$$= g_2 e^{-\lambda_2 r_2 x} \sum_{j=0}^{\infty} \frac{(\lambda_2 g_2 r_2 x)^j}{(j!)^2} \int_0^t e^{-g_2 v}(1 - e^{-\lambda_1(1-r_1)v})\, v^j dv$$

$$Q_{4,0/x} = \int_0^t e^{-\lambda_1(1-r_3)u}\, G_3(u/x)du$$

$$= g_3 e^{-\lambda_1 r_3 x} \sum_{j=0}^{\infty} \frac{(\lambda_3 g_3 r_3 x)^j}{(j!)^2} \int_0^t e^{-[g_3+\lambda_1(1-r_3)]u}\, u^j du$$

$$Q_{4,6/x} = \int_0^t \lambda_1(1-r_3)e^{-\lambda_1(1-r_3)u}\, \bar{G}_3(u/x)du$$

$$= g_3 e^{-\lambda_1 r_3 x} \sum_{j=0}^{\infty} \frac{(\lambda_1 g_3 r_3 x)^j}{(j!)^2} \int_0^t e^{-g_3 v}(1 - e^{-\lambda_1(1-r_3)v})\, v^j dv$$

$$Q_{1,3/x}^2 = \int_0^t \lambda_2(1-r_2)e^{-\lambda_2(1-r_2)u}\, p\bar{G}_1(u/x)du \int_u^t \frac{g_1(v/x)}{\bar{G}_1(u/x)}dv$$

$$= p g_1 e^{-\lambda_1 r_1 x} \sum_{j=0}^{\infty} \frac{(\lambda_1 g_1 r_1 x)^j}{(j!)^2} \int_0^t e^{-g_1 v}(1 - e^{-\lambda_2(1-r_2)v})\, v^j dv$$

$$Q_{1,4/x}^2 = \int_0^t \lambda_2(1-r_2)e^{-\lambda_2(1-r_2)u}\, q\bar{G}_1(u/x)du \int_u^t \frac{g_1(v/x)}{\bar{G}_1(u/x)}dv$$

$$= q g_1 e^{-\lambda_1 r_1 x} \sum_{j=0}^{\infty} \frac{(\lambda_1 g_1 r_1 x)^j}{(j!)^2} \int_0^t e^{-g_1 v}(1 - e^{-\lambda_2(1-r_2)v})\, v^j dv$$

$$Q_{3,1/x}^5 = \int_0^t \lambda_1(1-r_1)e^{-\lambda_1(1-r_1)u}\, \bar{G}_2(u/x)du \int_u^t \frac{g_2(v/x)}{\bar{G}_2(u/x)}dv$$

$$= g_2 e^{-\lambda_2 r_2 x} \sum_{j=0}^{\infty} \frac{(\lambda_2 r_2 v x)^j}{(j!)^2} \int_0^t e^{-g_2 v}(1 - e^{-\lambda_1(1-r_1)v})\, v^j dv$$

$$Q_{4,1/x}^6 = \int_0^t \lambda_1(1-r_3)e^{-\lambda_1(1-r_3)u}\, \bar{G}_3(u/x)du \int_u^t \frac{g_3(v/x)}{\bar{G}_3(u/x)}dv$$

$$= g_3 e^{-\lambda_1 r_3 x} \sum_{j=0}^{\infty} \frac{(\lambda_1 g_3 r_3 x)^j}{(j!)^2} \int_0^t e^{-g_3 v}(1 - e^{-\lambda_1(1-r_3)v})\, v^j dv$$

Conditional transitional probability

$P_{i,j} = \lim_{t\to\infty} q_{i,j}(t)$

So that $P_{0,1} = 1$

$P_{1,0/x} = g_1' e^{-r_1\lambda_1(1-g_1')x}$

$P_{1,2/x} = 1 - g_1' e^{-r_1\lambda_1(1-g_1')x}$

$P_{3,0/x} = g_2' e^{-r_2\lambda_2(1-g_2')x}$

$P_{3,5/x} = 1 - g_2' e^{-r_2\lambda_2(1-g_2')x}$

$P_{4,0/x} = g_3' e^{-r_3\lambda_1(1-g_3')x}$

$P_{4,6/x} = 1 - g_3' e^{-r_3\lambda_1(1-g_3')x}$

$P_{1,3/x}^2 = \text{p}\,[1 - g_1' \, e^{-\lambda_1 r_1(1-g_1')x}]$

$P_{1,4/x}^2 = q[1 - g_1' \, e^{-\lambda_1 r_1(1-g_1')x}]$

$P_{3,1/x}^5 = 1 - g_2' e^{-r_2\lambda_2(1-g_2')x}$

$P_{4,1/x}^6 = 1 - g_3' e^{-r_3\lambda_1(1-g_3')x}$

Unconditional transition probabilities with correlated coefficients are

$P_{0,1} = 1$

$$P_{1,0} = \int_0^\infty P_{1,0/x}\, g_1(x)dx = \frac{(1-r_1)g_1'}{1-g_1'r_1}$$

$$P_{1,2} = \int_0^\infty P_{1,2/x}\, g_1(x)dx = 1 - \frac{(1-r_1)g_1'}{1-g_1'r_1}$$

$$P_{1,3}^2 = \int_0^\infty P_{1,3/x}^2\, g_1(x)dx = p[\,1 - \frac{(1-r_1)g_1'}{1-g_1'r_1}\,]$$

$$P_{1,4}^2 = \int_0^\infty P_{1,4/x}^2\, g_1(x)dx = q[\,1 - \frac{(1-r_1)g_1'}{1-g_1'r_1}\,]$$

$$P_{3,0} = \int_0^\infty P_{3,0/x}\, g_2(x)dx = \frac{(1-r_2)g_2'}{1-g_2'r_2}$$

$$P_{3,1}^5 = \int_0^\infty P_{3,1/x}^5\, g_2(x)dx = 1 - \frac{(1-r_2)g_2'}{1-g_2'r_2}$$

$$P_{4,0} = \int_0^\infty P_{4,0/x}\, g_3(x)dx = \frac{(1-r_3)g_3'}{1-g_3'r_3}$$

$$P_{4,1}^6 = \int_0^\infty P_{4,1/x}^6\, g_3(x)dx = 1 - \frac{(1-r_3)g_3'}{1-g_3'r_3}$$

Now

$P_{0,1} = 1$

$P_{1,0} + P_{1,2} = 1$

$P_{1,0} + P_{1,3}^2 + P_{1,4}^2 = 1$

$P_{3,0} + P_{3,1}^5 = 1$

$P_{4,0} + P_{4,1}^{6} = 1$

Also μ_i, the mean sojourn times in the state S_i are

$$\mu_0 = \frac{1}{\lambda_1(1-r_1)}$$

$$\mu_1 = \frac{1-g_1^*\lambda_2(1-r_2)}{\lambda_2(1-r_2)}$$

$$\mu_3 = \frac{1-g_2^*\lambda_1(1-r_1)}{\lambda_1(1-r_1)}$$

$$\mu_4 = \frac{1-g_3^*\lambda_1(1-r_3)}{\lambda_1(1-r_3)}$$

Thus, the unconditional mean times in state S_j are

$m_{0,1} = \mu_0$

$m_{1,0} + m_{1,2} = \mu_1$

$m_{1,0} + m_{1,3}^{2} + m_{1,4}^{2} = k_1(say)$

$m_{3,0} + m_{3,1}^{5} = k_2(say)$

$m_{4,0} + m_{4,1}^{6} = k_3(say)$

MTSF (mean time to system failure)

To determine the MTSF of the system, the failed state of the system is regarded as an absorbing state by probabilistic argument.

$\phi_0(t) = Q_{01}(t) © \phi_1(t)$

$\phi_1(t) = Q_{10}(t) © \phi_0(t) + Q_{12}(t)$

Taking Laplace Stieltjes Transforms (L.S.T.) of these relations and solving for $\phi_0^{**}(s)$, following was obtained:

$$\phi_0^{**}(s) = \frac{N(s)}{D(s)}$$

Where N(s) = $Q_{0,1}^{**}(s)Q_{1,2}^{**}(s)$

D(s) = $1 - Q_{0,1}^{**}(s)Q_{1,0}^{**}(s)$

In the steady-state

$$T_0 = \lim_{s\to 0}\frac{1-\phi_0^{**}(s)}{s}$$

This led to, $T_0 = \frac{N}{D}$

Where N= $\mu_0 + \mu_1, D = 1 - p_{10}$

Availability analysis

Let A_i (t) be the probability that the system is in upstate at instant t given that the system entered regenerative state i at t=0. Using the arguments of the theory of a regenerative process, the pointwise availability A_i (t) is seen to satisfy the following recursive relations:

$A_0(t) = M_0(t) + q_{01}(t)©A_1(t)$

$A_1(t) = M_1(t) + q_{10}(t)©A_0(t) + q_{13}^{2}(t)©A_3(t) + q_{14}^{2}(t)©A_4(t)$

$A_3(t) = M_3(t) + q_{30}(t)©A_0(t) + q_{31}^5(t)©A_1(t)$
$A_4(t) = M_4(t) + q_{40}(t)©A_0(t) + q_{41}^6(t)©A_1(t)$

Where

$M_0(t) = e^{-\lambda_1(1-r_1)t}$ $M_1(t) = e^{-\lambda_2(1-r_2)t}\,\bar{G}_1(t/x)$
$M_3(t) = e^{-\lambda_1(1-r_1)t}\bar{G}_2(t/x)$
$M_4(t) = e^{-\lambda_1(1-r_3)t}\,\bar{G}_3(t/x)$

Now taking Laplace Transform of these equations and solving for $A_0^*(s)$, which lead to

$A_0^*(s) = \frac{N_1(s)}{D_1(s)}$

The steady-state availability is $A_0 = \lim_{s\to 0}[sA_0^*(s)] = \frac{N_1}{D_1}$

Where $N_1 = \mu_0[1 - p_{13}^2 p_{31}^5 - p_{14}^2 p_{41}^6] + \mu_1 + \mu_3 p_{13}^2 + \mu_4 p_{14}^2$
$D_1 = \mu_0[p_{10} + p_{13}^2 p_{30} + p_{14}^2 p_{40}] + K_1 + K_2 p_{13}^2 + K_3 p_{14}^2$

Busy period analysis of the repairman

Let $B_i(t)$ be the probability that the repairman is busy at instant t, given that the system entered in regenerative state i at t=0. By probabilistic arguments, the following recursive relations for $B_i(t)$ was obtained.

$B_0(t) = q_{01}(t)\,©B_1(t)$
$B_1(t) = W_1(t) + q_{10}(t)©B_0(t) + q_{13}^2(t)©B_3(t) + q_{14}^2(t)©B_4(t)$
$B_3(t) = W_3(t) + q_{30}(t)©B_0(t) + q_{31}^5(t)©B_1(t)$
$B_4(t) = q_{40}(t)©B_0(t) + q_{41}^6(t)©B_1(t)$

Where

$W_1(t) =$
$e^{-\lambda_1(1-r_1)t}\,\bar{G}_1(t/x) + p\left[1 - e^{-\lambda_1(1-r_1)t}\right]\bar{G}_1(t/x) + q\left[1 - e^{-\lambda_1(1-r_1)t}\right]\bar{G}_1(t/x) =$
$\bar{G}_1(t/x)$

$W_3(t) = e^{-\lambda_1(1-r_2)t}\,\bar{G}_2(t/x) + \left[1 - e^{-\lambda_1(1-r_2)t}\right]\bar{G}_2(t/x) = \bar{G}_2(t/x)$

Taking Laplace Transform of the equations of busy period analysis and solving them for $B_0^*(s), the$ following statement is obtained:

$B_0^*(s) = \frac{N_2(s)}{D_1(s)}$

where $N_2(s) = W_1^*(s)q_{01}^*(s) + W_3^*(s)q_{01}^*(s)q_{13}^{2*}(s)$

In steady-state

$B_0 = \lim_{s\to 0}[sB_0^*(s)] = \frac{N_2}{D_1}$

Where $N_2 = K_1 + K_2 p_{13}^2 + K_3 p_{14}^2$

D_1 is already specified.

Expected numbers of visits by the repairman

By probabilistic arguments, the following recursive relation for $V_i(t)$ will be obtained

$V_0(t) = Q_{01}(t)©[1 + V_1(t)]$
$V_1(t) = Q_{10}(t)©V_0(t) + Q_{13}^2(t)©V_3(t) + Q_{14}^2(t)©V_4(t)$

$V_3(t) = Q_{30}(t)©V_0(t) + Q_{31}^5(t)©V_1(t)$
$V_4(t) = Q_{40}(t)©V_0(t) + Q_{41}^6(t)©V_1(t)$

Taking Laplace stieltjes Transform of these equations of the expected number of visits and solving them for $V_0^{**}(s)$, the following item is obtained:

$$V_0 = \lim_{s\to 0}[sV_0^*(s)] = \frac{N_3}{D_1}$$

Where $N_3(s) = q_{10}^*(s)[\ 1 - q_{13}^{2*}(s)q_{31}^{5*}(s) - q_{14}^{2*}(s)q_{41}^{6*}(s)$

And D_1 is already specified.

In steady-state

$$V_0 = \lim_{s\to 0}[sV_0^*(s)] = \frac{N_3}{D_1}$$

Where $N_3 = p_{10}(1 - p_{13}^2 p_{31}^5 - p_{14}^2 p_{41}^6)$

And D_1 is already specified.

Expected number of replacements in the system

Let $R_i(t)$ be the expected number of replacements in (0,t), given that the system started from the regenerative state i at t=0. By probabilistic arguments, the following recursive statement is concluded:

$R_0(t) = Q_{01}(t)©R_1(t)$
$R_1(t) = Q_{10}(t)©R_0(t) + Q_{13}^2(t)©R_3(t) + Q_{14}^2(t)©[1 + R_4(t)]$
$R_3(t) = Q_{30}(t)©R_0(t) + Q_{31}^5(t)©R_1(t)$
$R_4(t) = Q_{40}(t)©R_0(t) + Q_{41}^6(t)©R_1(t)$

Taking Laplace stieltjes Transform of these equations and solving them for$R_0^{**}(s)$, which resulted as

$$R_0^{**}(s) = \frac{N_4(s)}{D_1(s)}$$

Where $N_4(s) = q_{14}^{2*}(s)q_{01}^*(s)$

D_1 is already specified.

In steady-state

$$R_0 = \lim_{s\to 0}[sR_0^*(s)] = \frac{N_4}{D_1}$$

Where $N_4 = p_{14}^2$ and D_1 is already specified.

Cost-Benefit analysis

The expected total profit incurred to the system in a steady state is given by

$$P_2 = C_0A_0 - C_1B_0 - C_2V_0 - C_3R_0$$

C_0 = Revenue per unit uptime of the system.
C_1 = cost per unit for which the repairman is busy.
C_2 = cost per visit of the repairman.
C_3 = cost per replacement in the system.

Graphical analysis

Let $g_1(t) = \alpha e^{-\alpha t}$, $g_2(t) = \beta e^{-\beta t}$, $g_3(t) = \gamma e^{-\gamma t}$,

$\boldsymbol{\alpha}$=0.1, β=0.1, γ=0.1

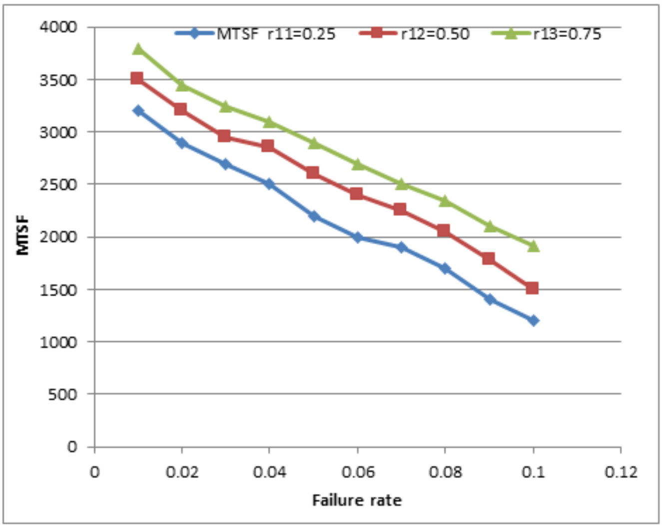

Fig. 1. MTSF vs. failure rate.

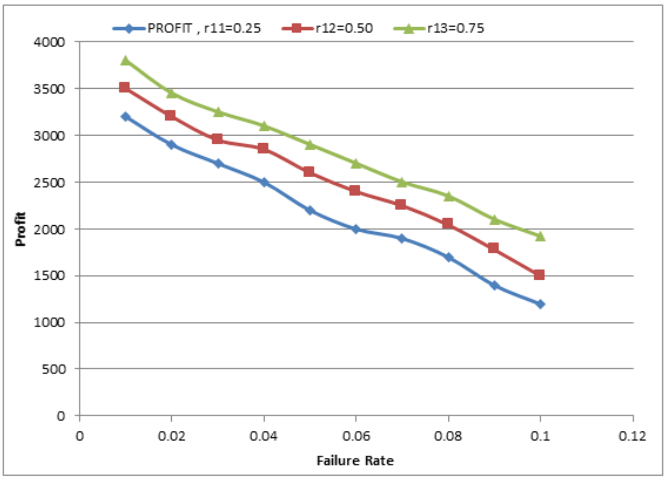

Fig. 2. Profit vs. failure rate.

2. Conclusion

This study reveals that MTSF decreases with an increase in the failure rate value and increases for higher correlation coefficient values (Fig. 1). Profit decreases with an increase in the failure rate value and increases for a higher correlation coefficient value (Fig. 2). It can also be concluded that a higher correlation between failure and repair of the system yields better system performance.

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