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Unbiased Modified Two-Parameter Estimator for the Linear Regression Model

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Abstract

This study centers on estimating parameters in a linear regression model in the presence of multicollinearity. Multicollinearity poses a threat to the efficiency of the Ordinary Least Squares (OLS) estimator. Some alternative estimators have been developed as remedial measures to the earlier mentioned problem. This study introduces a new unbiased modified two-parameter estimator based on prior information. Its properties are also considered; the new estimator was compared with other estimators' Mean Square Error (MSE). A numerical example and Monte Carlo simulation were used to illustrate the performance of the new estimator.

Keywords: Linear regression model; Multicollinearity; Ordinary least squares; Unbiased modified two-parameter; Prior information.

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1. Introduction

The linear regression model is expressed as:

$$Y = X\beta + e, \quad (1)$$

where Y is an n×1 vector of observations on the dependent variable, X is an n×p matrix of the predictor variables, β is a p×1 vector of unknown regression coefficients, e is an n×1 vector of random error with $e_i \sim N(0,\sigma^2)$.
The Ordinary Least Square (OLS) estimator of β is given as:

$$\hat{\beta}_{OLS} = (X'X)^{-1}\,(X'Y), \quad (2)$$

where $\hat{\beta}_{OLS}$ is a p×1 vector of unknown regression coefficients, $(X'X)$ is a p×p orthogonal matrix, $(X'Y)$ is a p×1 vector and $\hat{\beta} \sim N(\beta, \boldsymbol{\sigma}^2(X'X)^{-1})$. The OLS estimator is unbiased and possesses minimum variance among other estimators. However, one of the notable limitations of this estimator occurs when the predictor variables are highly correlated. This is termed multicollinearity, in the presence of which the OLS becomes unstable and

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gives misleading regression results. Several biased estimators have been proposed in the literature to overcome the problem of multicollinearity.

Hoerl *et al*. [1] proposed the Ridge Regression (RR) estimator.

$$\hat{\beta}_R = (X'X + kI)^{-1}X'Y, \tag{3}$$

where k is the ridge parameter (or biasing constant) and $0 \leq k \leq 1$.

Amin *et al*. [2] proposed the modified ridge regression estimator based on prior information. The estimator is given as:

$$\hat{\beta}_{MRR}(k, b) = (X'X + kI)^{-1}(X'Y + kb), \tag{4}$$

where b is prior information on β. The OLS is a special case of this estimator when $k = 0$. It tends to b as k tends to infinity.

Liu [3] proposed an estimator to overcome the limitations of RR estimator. He combined the benefit of both the estimators given by [1] and [4]. It is given as:

$$\hat{\beta}_{LIU} = (X'X + I)^{-1}(X'X + dI)\hat{\beta}_{OLS} \tag{5}$$

where $0 < d < 1$.

Dorugade [5] and Dorugade [6] introduced a modified two-parameter estimator. The estimator is given as:

$$\hat{\beta}_{MTP}(k, d) = (X'X + kdI)^{-1}(X'X)\hat{\beta}_{OLS}, \tag{6}$$

where $k > 0$, $0 < d < 1$.

This estimator is a general estimator which includes the OLS and RR estimator as special cases, when $k = 0$ or $d = 0$, it gives the OLS estimator and when $d=1$, it gives the RR estimator.

Although these estimators solve the problem of multicollinearity, they are biased estimators. Unbiased estimators have also been proposed by some researchers. The major advantage of unbiased estimators over biased estimators is that they produce unbiased estimates with minimum variance.

Crouse *et al.* [7] proposed an unbiased ridge estimator with prior information *J.* The estimator is defined as:

$$\hat{\beta}(k, J) = (X'X + kI)^{-1}(X'Y + kJ), \tag{7}$$

where J ~ N(β,$(\sigma^2/k)I$) and J is uncorrelated with $\hat{\beta}_{OLS}$.

Amin *et al.* [8] proposed an Almost Unbiased Two-Parameter (AUTP) estimator. AUTP estimator was compared with OLS estimator and Two-Parameter (TP) estimator based on MSE criterion. The estimator is given as:

$$\hat{\beta}(k, d) = \hat{\beta}_{TP} + k(1 - d)(X'X + kI)^{-1}\hat{\beta}_{TP}, \tag{8}$$

where $\hat{\beta}_{TP} = (X'X + kI)^{-1}(X'X + kdI)\hat{\beta}_{OLS}$. (9)

Wu [9] introduced an Unbiased Two-Parameter (UTP) estimator with prior information based on the Two-Parameter (TP) estimator was by. This is defined as follows:

$$\hat{\beta}_{UTP}(F_{kd}, J) = F_{kd}\hat{\beta}_{OLS} + (I - F_{kd})J, \tag{10}$$

where $F_{kd} = (X'X + kI)^{-1}(X'X + kdI)$. (11)

with J being uncorrelated with $\hat{\beta}_{OLS}$ and J~N$(\beta, (\sigma^2/k(1-d))(S + kdI)S - 1)$ and $S = X'X$.

In this study, a new estimator referred to as an "Unbiased Modified Two-Parameter" (UMTP) Estimator to minimize the effect of multicollinearity in a linear regression model is introduced. The article is organized as follows. In section 2, the new estimator is proposed, and its properties are obtained. The proposed estimator is compared with other two-parameter estimators using the MSE criterion in section 3.

2. Materials and Methodology

2.1. *The new estimator and its properties*

Dorugade [5] introduced a Modified Two-Parameter (MTP) estimator, which was earlier defined in equation (7) as $\hat{\beta}_{MTP} = (X'X + kdI)^{-1}(X'X)\hat{\beta}_{OLS}$, where $R_{kd} = (X'X + kdI)^{-1}X'X$ and $0 < k < 1$, $0 < d < 1$.

Considering the convex estimator below:

$$\hat{\beta}(C,J) = C\hat{\beta}_{OLS} + (I - C)J, \tag{12}$$

where C is a p×p matrix, I is a p×p Identity matrix and $\hat{\beta}(C,J)$ is an unbiased estimator of β.

We define the new estimator, the Unbiased Modified Two-Parameter (UMTP) Estimator, based on prior information as follows:

$$\boldsymbol{\hat{\beta}(R_{kd},J) = R_{kd}\hat{\beta}_{OLS} + (I - R_{kd})J}, \tag{13}$$

where $R_{kd} = (X'X + kdI)^{-1}X'X$ and J~N(β, $\sigma^2(kdI)^{-1}$) for $k > 0$, $0 < d < 1$. (14)

2.1.1. *Determination of the variance of J (Var(J))*

Recall from equation (12); $\hat{\beta}(C,J) = C\hat{\beta}_{OLS} + (I - C)J$,

$$\text{MSE}\{\hat{\beta}(C,J)\} = \sigma^2 CS^{-1}C' + (1 - C)Var(J)(1 - C)'. \quad \text{where } S = X'X$$

$$\frac{\partial MSE\{\hat{\beta}(C,J)\}}{\partial C} = 2C\sigma^2 S^{-1} - 2(I - C)Var(J) = 0.$$

$C = Var(J)[\sigma^2 S^{-1} + Var(J)]^{-1}$, similarly $Var(J) = \sigma^2(I - C)^{-1}CS^{-1}$.

Note that R_{kd} in the proposed estimator corresponds to C in the convex estimator. Therefore, for the proposed estimator, $Var(J) = \sigma^2(I - R_{kd})^{-1}R_{kd}S^{-1} = \sigma^2(kdI)^{-1}$.

2.1.2. *Unbiasedness of the proposed estimator*

$$\begin{aligned}
\text{E}[\hat{\beta}(R_{kd},J)] &= \text{E}[R_{kd}\beta + (I - R_{kd})J], \\
&= \text{E}[R_{kd}\beta] + E[(I - R_{kd})J], \\
&= (S + kdI)^{-1}SE[\beta] + (S + kdI)^{-1}(kdI)E[J],
\end{aligned}$$

$= (S + kdI)^{-1}S\beta + (S + kdI)^{-1}(kdI)\beta,$

$= (S + kdI)^{-1}\beta(S + kdI),$

$$\mathrm{E}[\hat{\beta}(R_{kd}, J)] = \beta \qquad (15)$$

2.1.3. The *variance of the proposed estimator*

$Var[\hat{\beta}(R_{kd}, J)] = Var[R_{kd}\hat{\beta} + (I - R_{kd})J],$

$= Var[(S + kdI)^{-1}S\hat{\beta} + Var[((S + kdI)^{-1}kdI)J],$

$= ((S + kdI)^{-1}S)'\, var[\hat{\beta}]((S + kdI)^{-1}S) + (kdI)^2(S + kdI)^{-1}Var[J](S + kdI)^{-1},$

$= S(S + kdI)^{-1}\sigma^2 S^{-1}(S + kdI)^{-1}S + (kdI)^2(S + kdI)^{-1}\sigma^2(kdI)^{-1}(S + kdI)^{-1},$

$= \sigma^2 S(S + kdI)^{-2} + \sigma^2(S + kdI)^{-2}(kdI)^{-1},$

$= \sigma^2(S + kdI)^{-2}(S + kdI),$

$$Var[\hat{\beta}(R_{kd}, J)] = \sigma^2(S + kdI)^{-1}. \qquad (16)$$

Thus $\hat{\beta}(R_{kd}, J) \sim N(\beta, \sigma^2(S + kdI)^{-1})$.

2.1.4. *Mean Squared Error (MSE) of the proposed estimator*

$\mathrm{MSE}\{\hat{\beta}(R_{kd}, J)\} = Var(\hat{\beta}(R_{kd}, J)) + Bias(\hat{\beta}(R_{kd}, J))^2 = Var(\hat{\beta}(R_{kd}, J)),$

where bias $(\hat{\beta}(R_{kd}, J))^2 = 0$.

$$\mathrm{MSE}\{\hat{\beta}(R_{kd}, J)\} = \sigma^2(S + kdI)^{-1}. \qquad (17)$$

2.2. *Comparison of the proposed estimator with other existing estimators based on MSE vriterion*

Lemma 1: Let b_1 and b_2 be two estimators of β. Then b_2 is said to be MSE superior to b_1 if and only if, $\mathrm{MSE}(b_1) - \mathrm{MSE}(b_2) \geq 0$.

Lemma 2: Let M be a positive definite matrix, such that M > 0, and let α be some vector, then $M - \alpha\alpha' \geq 0$if and only if $\alpha' M^{-1}\alpha \leq 1$.

2.2.1. *Comparison between $\hat{\beta}(R_{kd}, J)$ and $\hat{\beta}_{OLS}$using MSE criterion*

$$\mathrm{MSE}(\hat{\beta}_{OLS}) = \sigma^2(X'X)^{-1} = \sigma^2 S^{-1}. \qquad (18)$$

$\mathrm{MSE}(\hat{\beta}_{OLS}) - \mathrm{MSE}(\hat{\beta}(R_{kd}, J)) = \sigma^2 S^{-1} - \sigma^2(S + kdI)^{-1},$

$$= \sigma^2(S^{-1} - (S + kdI)^{-1}). \qquad (19)$$

which is a non-negative definite matrix for *k* > 0 and 0 < *d* < 1. Thus, according to Lemma **1**$\boldsymbol{\hat{\beta}(R_{kd}, J)}$ is superior to $\boldsymbol{\hat{\beta}_{OLS}}$.

2.2.2. *Comparison between $\hat{\beta}(R_{kd}, J)$ and $\hat{\beta}_{TP}$ using MSE criterion*

Theorem 1: $\widehat{\boldsymbol{\beta}}(\boldsymbol{R_{kd}}, \boldsymbol{J})$ is superior to $\widehat{\boldsymbol{\beta}}_{\boldsymbol{TP}}$ if and only if $\frac{\sigma^2}{k^2(1-d)^2}(C_1C_2^{-1}C_1 - C_2S^{-1}C_2) < \beta\beta'$. **Proof:** Bias$(\hat{\beta}_{TP}) = k(1-d)(S+kI)^{-1}\beta,$ (20)

Bias$(\hat{\beta}_{TP}) = k(1-d)C_1^{-1}\beta,$ where $C_1 = S + kI$ and $C_2 = S + kdI$

Var$(\hat{\beta}_{TP}) = \sigma^2(S+kI)^{-1}(S+kdI)S^{-1}(S+kdI)(S+kI)^{-1}$

Var$(\hat{\beta}_{TP}) = \sigma^2C_1^{-1}C_2S^{-1}C_2C_1^{-1}.$ (21)

MSE$(\hat{\beta}_{TP}) = \sigma^2C_1^{-1}C_2S^{-1}C_2C_1^{-1} + k^2(1-d)^2C_1^{-1}\beta\beta'C_1^{-1}.$ (22)

MSE$(\hat{\beta}(R_{kd}, J)) <$ MSE$(\hat{\beta}_{TP}) = \sigma^2C_2^{-1} < \sigma^2C_1^{-1}C_2S^{-1}C_2C_1^{-1} + k^2(1-d)^2C_1^{-1}\beta\beta'C_1^{-1},$

$= \sigma^2C_2^{-1} - \sigma^2C_1^{-1}C_2S^{-1}C_2C_1^{-1} < k^2(1-d)^2C_1^{-1}\beta\beta'C_1^{-1},$

$= \frac{\sigma^2}{k^2(1-d)^2}(C_1C_2^{-1}C_1 - C_2S^{-1}C_2) < \beta\beta'.$ (23)

2.2.3. *Comparison between $\hat{\beta}(R_{kd}, J)$ and $\hat{\beta}_{UTP}$ using MSE criterion*

Var$(\hat{\beta}_{UTP}) = \sigma^2(S+kI)^{-1}S^{-1}(S+kdI).$

Var$(\hat{\beta}_{UTP}) = \sigma^2C_1^{-1}S^{-1}C_2.$ (24)

MSE$(\hat{\beta}_{UTP}) = \sigma^2C_1^{-1}S^{-1}C_2.$ (25)

MSE$(\hat{\beta}_{UTP}) -$ MSE$(\hat{\beta}(R_{kd}, J)) > 0 = \sigma^2C_1^{-1}S^{-1}C_2 - \sigma^2C_2^{-1} > 0,$

$= \sigma^2(C_1^{-1}S^{-1}C_2 - C_2^{-1}) > 0.$ (26)

which is a non-negative definite matrix for $k > 0$ and $0 < d < 1$. Thus, according to Lemma 1$\widehat{\boldsymbol{\beta}}(\boldsymbol{R_{kd}}, \boldsymbol{J})$ is superior to$\widehat{\boldsymbol{\beta}}_{\boldsymbol{UTP}}$.

2.2.4. *Comparison between $\hat{\beta}(R_{kd}, J)$ and $\hat{\beta}_{MTP}(k, d)$ using MSE criterion*

Theorem 2: The estimator $\widehat{\boldsymbol{\beta}}(\boldsymbol{R_{kd}}, \boldsymbol{J})$ is superior to $\widehat{\boldsymbol{\beta}}_{\boldsymbol{MTP}}(\boldsymbol{k}, \boldsymbol{d})$ if and only if $\beta\beta' > \frac{\sigma^2}{kd}$ with $k>0$ and $0 < d < 1$.

Proof: Bias$(\hat{\beta}_{MTP}(k, d)) = (R_{kd} - I)\beta.$ (27)

Var$(\hat{\beta}_{MTP}(k, d)) = \sigma^2R_{kd}S^{-1}R_{kd}.$ (28)

MSE$(\hat{\beta}_{MTP}(k, d)) = \sigma^2R_{kd}S^{-1}R_{kd} + (R_{kd} - I)\beta\beta'(R_{kd} - I)'.$ (29)

MSE$(\hat{\beta}(R_{kd}, J)) -$ MSE$(\hat{\beta}_{MTP}(k, d)) = \sigma^2(S+kdI)^{-1} - \sigma^2(S+kdI)^{-1}S(S+kdI)^{-1} - (S(S+kdI)^{-1} - I)\beta\beta'(S(S+kdI)^{-1} - I),$

$= \sigma^2(S+kdI)^{-1} - \sigma^2(S+kdI)^{-1}S(S+kdI)^{-1} - (S+kdI)^{-1}kdI\beta\beta'(S+kdI)^{-1}kdI,$

$= \sigma^2(S+kdI)^{-1} - \sigma^2(S+kdI)^{-1}S(S+kdI)^{-1} - k^2d^2I(S+kdI)^{-1}\beta\beta'(S+kdI)^{-1},$

$= (S+kdI)^{-1}[\sigma^2(S+kdI) - \sigma^2S - k^2d^2\beta\beta'](S+kdI)^{-1},$

$= (S+kdI)^{-1}[\sigma^2kdI - k^2d^2I\beta\beta'](S+kdI)^{-1},$

$$= kd(S + kdI)^{-1}[\sigma^2 - kdI\beta\beta'](S + kdI)^{-1}. \quad (30)$$

Since $k > 0$ and $0 < d < 1$, by lemma 2, we obtain that $\text{MSE}(\hat{\beta}(R_{kd},J)) - \text{MSE}(\hat{\beta}_{MTP}(k,d))$ is a non-negative definite matrix if and only if $\beta\beta' \leq \frac{\sigma^2}{kd}$. Thus, we conclude that $\hat{\boldsymbol{\beta}}(\boldsymbol{R_{kd}},\boldsymbol{J})$ is superior to $\hat{\boldsymbol{\beta}}_{\boldsymbol{MTP}}(\boldsymbol{k},\boldsymbol{d})$ if and only if $\beta\beta' > \frac{\sigma^2}{kd}$.

2.2.5. *Selection of bias parameters k and d*

The bias parameters *k* and *d* are used in estimating two-parameter estimators. They are very important and crucial as they play a vital role in controlling the regression bias towards the mean of the response variable [10].

To examine the performance of the new proposed estimator over other existing estimators, the bias parameters *k* and *d* are chosen. In this study, we choose

$$k = \frac{p\hat{\sigma}^2}{\sum_i^p \hat{\alpha}_i^2}. \quad [11] \quad (31)$$

and

$$d = \sum_i^p \left[\frac{(\lambda_i + k)(\hat{\sigma}^2 + \lambda_i\hat{\alpha}_i^2) - \lambda_i}{k\alpha_i^2}\right]. \quad [5] \quad (32)$$

where parameters k and d are ridge parameters and constant

3. Results and Discussion

3.1. *Numerical example and stimulation study*

3.1.2. *Numerical example*

The proposed estimator will be illustrated using Portland cement data that exhibit multicollinearity, where multicollinearity is a statistical concept with several independent variables in a model correlated. Multicollinearity occurs when two or more independent variables are highly correlated with one another in a regression model. This means that an independent variable can be predicted from another independent variable in a regression model. Multicollinearity generally occurs when there are high correlations between two or more predictor variables. In other words, one predictor variable can be used to predict the others. This creates redundant information, skewing the results in a regression model; for example, correlated predictor variables are also called multicollinear predictors. Portland cement data set that was initially used by [12]. We computed the regression coefficient and the MSE of the proposed unbiased modified two-parameter estimator $\hat{\beta}(R_{kd},J)$ and also that of the following estimators $\hat{\beta}_{OLS}$, $\hat{\beta}_{MRR}$, $\hat{\beta}_{MLIU}$, $\hat{\beta}_{MTP}$, $\hat{\beta}_{TP}$, and $\hat{\beta}_{UTP}$.

Note that $\hat{\beta}_{MRR}$ is the MSE of Modified Ridge (MRR) estimator, $\hat{\beta}_{MLIU}$ is the MSE of Modified Liu (MLIU) estimator, $\hat{\beta}_{MTP}$ is the MSE of the Modified Two-Parameter (MTP) estimator, $\hat{\beta}_{TP}$ is the MSE of Two- Parameter (TP) estimator and $\hat{\beta}_{UTP}$ is the MSE of Unbiased Two-Parameter (UTP) estimator. The result is presented in Table 1.

Table 1. Estimated regression parameter and MSE Value.

	$\hat{\beta}_{OLS}$	$\hat{\beta}_{MRR}$	$\hat{\beta}_{MLIU}$	$\hat{\beta}_{MTP}$	$\hat{\beta}_{TP}$	$\hat{\beta}_{UTP}$	$\hat{\beta}(R_{kd},J)$
$\hat{\beta}_0$	62.4054	0.1531	0.1611	0.1411	53.0636	0.1629	0.1270
$\hat{\beta}_1$	1.5511	2.2305	2.4105	2.1805	1.6452	2.0822	2.1806
$\hat{\beta}_2$	0.5102	1.4544	1.5644	1.1544	0.6069	1.0939	1.1546
$\hat{\beta}_3$	0.1019	0.8490	0.8990	0.7490	0.1988	0.7192	0.7492
$\hat{\beta}_4$	-0.1441	0.4786	0.4907	0.4866	-0.0494	0.4603	0.4867
MSE	4912.09	3582.51	3612.71	3912.74	3712.21	4361.11	101.26

From Table 1, we observed that the estimated MSE value of the new proposed estimator ($\hat{\beta}(R_{kd},J)$) is smaller than the MSE of OLS ($\hat{\beta}_{OLS}$), Modified Ridge estimator ($\hat{\beta}_{MRR}$), Modified Liu estimator($\hat{\beta}_{MLIU}$), Modified Two-Parameter estimator($\hat{\beta}_{MTP}$), Two-Parameter estimator($\hat{\beta}_{TP}$) and Unbiased Two Parameter estimator ($\hat{\beta}_{UTP}$). This implies that the new proposed estimator performs better in the presence of multicollinearity.

3.2. Simulation study

In order to investigate the performance of the proposed estimator, the MSE of the proposed estimator is compared with those of some existing estimators. The simulation process uses a linear regression model with fixed independent variables such that there exist different levels of multicollinearity among the independent variables. Considering the regression model:

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \beta_3 X_{3i} + \ldots\ldots + \beta_p X_{pi} + \varepsilon_i \quad (33)$$

where i=1, 2,…, n and p=3, 6.

The independent variables were generated by the simulation process used by [2,13,14] and [15-18] as follows:

$$X_{ij} = (1-\rho^2)^{1/2} v_{ij} + \rho v_{ip} \quad i=1, 2,\ldots, n \text{ and } j = 1, 2,\ldots, p \quad (34)$$

where X_{ij} are the generated independent variables, ρ is the correlation between any two independent variables, v_{ij} are random numbers from standard normal distribution and p is the number of independent variables. In this study, we take p = 3, 6 and ρ = 0.8, 0.9, 0.95, and 0.99. X_{ij} are standardized such that $X'X$ are used to generate the dependent variables at specified value of n, p, σ and ε with β = (0.8, 0.1, 0.6) when p =3 and β = (0.8, 0.1, 0.6, 0.15, 0.19, 0.05) when p =6.

The parameter values were chosen such that $\beta'\beta=1$, which is a common restriction in simulation studies of this type [19-25]. The data set are simulated with sample sizes n = 20, 50, 100 and σ = 1, 5, 10. The process is replicated 2000 times. We obtained the estimated MSE values of the following estimators OLS, MRR, MLIU, MTP, TP, UTP, and the proposed UMTP, respectively. Their respective MSE is obtained by the following computation.

$$MSE(\hat{\beta}) = \frac{1}{2000}\sum_{j=1}^{2000} \sum_{i-1}^{p} (\hat{\beta}_{ij} - \beta_i)'(\hat{\beta}_{ij} - \beta_i) \quad (35)$$

where $\hat{\beta}_{ij}$ is the estimate of the i[th] parameter in the j[th] replication and β_i is the true parameter value. The estimated MSE values for different combinations of the n, p, and ρ are presented in Tables 2 and 3.

Table 2. Estimated MSE values of the new estimator and various two-parameter estimators when p =3, β = (0.8, 0.1, 0.6).

				$\rho = 0.8$					
N	20			50			100		
σ	1	5	10	1	5	10	1	5	10
$\hat{\beta}_{OLS}$	0.883	17.219	68.270	0.420	5.709	22.230	0.304	2.765	10.457
$\hat{\beta}_{MRR}$	0.759	14.119	55.868	0.416	5.569	21.670	0.3031	2.749	10.393
$\hat{\beta}_{MLIU}$	0.880	17.141	67.959	0.421	5.700	22.194	0.304	2.763	10.449
$\hat{\beta}_{MTP}$	0.580	9.581	37.711	0.384	4.753	18.405	0.295	2.543	9.566
$\hat{\beta}_{TP}$	0.824	15.753	62.407	0.415	5.535	21.537	0.302	2.726	10.298
$\hat{\beta}_{UTP}$	0.669	11.805	46.606	0.396	5.004	19.403	0.296	2.506	9.406
$\hat{\beta}(R_{kd},J)$	0.571	9.367	36.854	0.383	4.738	18.347	0.295	2.541	9.559
				$\rho = 0.9$					
N	20			50			100		
σ	1	5	10	1	5	10	1	5	10
$\boldsymbol{\hat{\beta}_{OLS}}$	1.780	39.660	158.033	0.615	10.551	41.600	0.398	5.115	19.857
$\boldsymbol{\hat{\beta}_{MRR}}$	1.061	21.625	85.881	0.583	9.734	38.332	0.394	5.004	19.411
$\boldsymbol{\hat{\beta}_{MLIU}}$	1.768	39.357	156.824	0.614	10.520	41.477	0.398	5.108	19.826
$\boldsymbol{\hat{\beta}_{MTP}}$	0.696	12.284	48.497	0.492	7.422	29.079	0.366	4.305	16.611
$\boldsymbol{\hat{\beta}_{TP}}$	1.553	33.978	135.304	0.592	9.970	39.277	0.392	4.969	19.270
$\boldsymbol{\hat{\beta}_{UTP}}$	0.861	16.480	65.292	0.537	8.512	33.432	0.377	4.507	17.411
$\boldsymbol{\hat{\beta}(R_{kd},J)}$	0.669	11.584	45.695	0.489	7.354	28.794	0.366	4.293	16.564
				$\rho = 0.95$					
N	20			50			100		
σ	1	5	10	1	5	10	1	5	10
$\hat{\beta}_{OLS}$	3.683	87.229	348.31	0.992	20.002	79.405	0.584	9.759	38.433
$\hat{\beta}_{MRR}$	1.209	25.134	99.893	0.835	16.034	63.532	0.556	9.075	35.694
$\hat{\beta}_{MLIU}$	3.647	86.330	344.714	0.99	19.906	79.021	0.583	9.732	38.325
$\hat{\beta}_{MTP}$	0.740	12.960	51.148	0.623	10.629	41.896	0.475	6.992	27.354
$\hat{\beta}_{TP}$	3.011	70.407	281.02	0.921	18.194	72.172	0.563	9.247	36.384
$\hat{\beta}_{UTP}$	0.934	18.032	71.465	0.730	13.324	52.680	0.515	7.966	31.245
$\hat{\beta}(R_{kd},J)$	0.697	I1.835	46.642	0.613	10.366	40.866	0.473	6.931	27.11
				$\rho = 0.99$					
N	20			50			100		
σ	1	5	10	1	5	10	1	5	10
$\hat{\beta}_{OLS}$	19.703	487.731	1952.318	3.935	93.531	373.520	2.047	46.351	194.798
$\hat{\beta}_{MRR}$	1.135	21.770	86.276	1.373	29.237	116.315	1.205	25.190	100.141
$\hat{\beta}_{MLIU}$	19.436	481.062	1923.644	3.897	92.584	369.733	2.033	45.992	183.363
$\hat{\beta}_{MTP}$	0.691	9.528	37.122	0.828	14.362	56.719	0.772	13.904	54.928
$\hat{\beta}_{TP}$	14.742	363.633	1453.916	3.227	75.809	302.628	1.799	39.636	157.854
$\hat{\beta}_{UTP}$	0.855	14.148	55.710	1.047	20.748	82.313	0.973	19.175	76.045
$\hat{\beta}(R_{kd},J)$	0.644	8.156	31.653	0.758	13.025	51.360	0.740	13.086	51.648

Table 3. Estimated MSE values of the new estimator and various two-parameter estimators when p =6, β = (0.8, 0.1, 0.6, 0.15, 0.19, 0.05).

				$\rho = 0.8$					
N	20			50			100		
σ	1	5	10	1	5	10	1	5	10
$\hat{\beta}_{OLS}$	1.517	33.072	131.680	0.725	13.297	52.583	0.393	4.99	19.356
$\hat{\beta}_{MRR}$	1.279	27.093	107.779	0.703	12.748	50.389	0.392	4.962	19.244
$\hat{\beta}_{MLIU}$	1.511	32.923	131.085	0.724	13.269	52.472	0.493	4.986	19.341
$\hat{\beta}_{MTP}$	0.938	20.459	73203	0.601	10.454	41.129	0.377	4.591	17.759
$\hat{\beta}_{TP}$	1.405	30.264	120.447	0.714	12.773	50.489	0.390	4.929	19.070
$\hat{\beta}_{UTP}$	1.105	22.659	90.006	0.650	11.334	44.722	0.377	4.593	17.396
$\hat{\beta}(R_{kd},J)$	0.922	18.052	71.573	0.629	10.381	40.927	0.375	4.588	17.746
				$\rho = 0.9$					
N	20			50			100		
σ	1	5	10	1	5	10	1	5	10
$\hat{\beta}_{OLS}$	2.874	66.991	267.358	1.175	24.548	97.588	0.570	9.421	37.079
$\hat{\beta}_{MRR}$	1.759	39.02	155.443	1.063	21.728	86.308	0.562	9.221	36.278
$\hat{\beta}_{MLIU}$	2.854	66.508	265.425	1.172	24.461	97.239	0.570	9.407	37.022
$\hat{\beta}_{MTP}$	1.127	22.961	91.181	0.826	15.772	62.477	0.520	7.945	31.173
$\hat{\beta}_{TP}$	2.511	57.911	231.031	1.109	22.898	90.986	0.560	9.154	36.011
$\hat{\beta}_{UTP}$	1.416	30.288	120.501	0.943	18.669	74.060	0.528	8.287	32.530
$\hat{\beta}(R_{kd},J)$	1.083	21.831	86.657	0.817	15.550	61.587	0.511	7.924	31.089
				$\rho = 0.95$					
N	20			50			100		
σ	1	5	10	1	5	10	1	5	10
$\hat{\beta}_{OLS}$	5.660	136.664	546.048	2.043	46.482	184.324	0.925	18.291	72.558
$\hat{\beta}_{MRR}$	2.075	46.692	186.105	1.561	34.185	136.132	0.875	17.032	67.566
$\hat{\beta}_{MLIU}$	5.607	148.33	540.712	2.032	45.973	184.072	0.923	18.241	72.358
$\hat{\beta}_{MTP}$	1.228	25.046	99.470	1.064	21.646	85.964	0.722	13.164	52.048
$\hat{\beta}_{TP}$	4.663	111.694	446.159	1.850	41.399	164.991	0.887	17.342	68.765
$\hat{\beta}_{UTP}$	1.589	34.329	136.628	1.309	27.774	110.477	0.795	14.942	59.148
$\hat{\beta}(R_{kd},J)$	1.155	23.187	92.028	1.037	20.951	83.184	0.718	13.052	51.596
				$\rho = 0.99$					
N	20			50			100		
σ	1	5	10	1	5	10	1	5	10
$\hat{\beta}_{OLS}$	28.462	706.699	2826.192	8.789	212.657	850.024	3.757	89.087	355.745
$\hat{\beta}_{MRR}$	2.157	47.236	188.116	2.448	56.091	223.723	2,152	48.845	194.760
$\hat{\beta}_{MLIU}$	28.085	697.267	2788.460	8.699	251.391	842.960	4.730	88.494	353.009
$\hat{\beta}_{MTP}$	1.178	21.537	85.167	1.354	9.0224	108.160	1.305	27.159	108.940
$\hat{\beta}_{TP}$	21.442	531.082	2123.695	7.006	170.277	680.499	3.244	76.359	334.352
$\hat{\beta}_{UTP}$	1.561	32.676	127.777	1.796	39.944	157.111	1.699	37.286	150.485
$\hat{\beta}(R_{kd},J)$	1.078	19.805	76.422	1.221	26.555	97.477	1.244	25.601	103.400

It is observed in Tables 2 and 3 that the new proposed two-parameter estimator $\hat{\beta}(R_{kd},J)$ is superior to $\hat{\beta}_{OLS}$ and other two-parameter estimators such as $\hat{\beta}_{MRR}, \widehat{\beta}_{MLIU}, \widehat{\beta}_{MTP}, \widehat{\beta}_{TP}, \hat{\beta}_{UTP}$ because it possesses minimum MSE when compared to others. The new proposed estimator performs better for a different number of independent variables and various levels of correlation among independent variables ($\rho =$ 0.8,0.9,0.95,0.99). It also performs better when n is small (n = 20) and for various combinations of variance (σ^2) of the error term. The new estimator is an unbiased estimator that overcomes the problem of multicollinearity and can be used in the place of other estimators considered in this study.

4. Conclusion

A new estimator is proposed, called the Unbiased Modified Two-Parameter (UMTP) estimator, based on prior information to minimize the effect of multicollinearity for the linear regression model. A Monte Carlo simulation study across different combinations of d, k, n, p, ρ and σ are carried out, and the MSE criterion was used to examine the performance of the new estimator over the OLS and other existing two-parameter estimators reviewed in this st udy. Real-life data with multicollinearity problems were also used to evaluate the performance of the new estimator. It was observed that the newly proposed Unbiased Modified Two-Parameter (UMTP) estimator performs better than the existing estimators in the presence of multicollinearity.

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