Laminar Forced Convection MHD Couette-Poiseuille Flow with Viscous and Joule Dissipations

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Abstract

The laminar forced convection MHD Couette-Poiseuille flow of a viscous incompressible fluid with the viscous and Joule dissipations has been studied. Two different orientations of the wall thermal boundary-conditions have been considered, namely: the constant heat-flux at the upper moving plate with the adiabatic stationary lower plate and the constant heat flux at the stationary lower plate with an adiabatic moving upper plate. The governing equations are solved analytically. It is observed that the fluid velocity increases near the stationary plate and it decreases near the moving plate with an increase in magnetic parameter. The temperature field is significantly affected by the modified Brinkman number. The fluid temperature increases when the lower plate is adiabatic and the upper plate is at positive constant heat flux while it decreases in case the lower plate is at negative constant heat flux and the upper plate is adiabatic with an increase in modified Brinkman number for the combined effects of viscous and Joule dissipations. Further, the fluid temperature decreases for positive heat flux case while it increases for negative heat flux case with an increase in either magnetic parameter or velocity parameter when the combined effects of viscous and Joule dissipations are taken into account.

Keywords: MHD; Couette-Poiseuille flow; Viscous dissipation; Joule dissipation and modified Brinkman number.

1. Introduction

Couette-Poiseuille flow in the presence of transverse magnetic field between two plane parallel plates is an interesting phenomenon. For many decades engineers have studied such flows for both steady and unsteady cases. This flow is important in many material-processing applications such as extrusion, metal forming, continuous casting as well as wire and glass fiber drawing. The viscous dissipation changes the temperature distribution by playing a roll like an internal heat generation source in the energy transfer. This heat source is caused by the shearing of fluid layers. The merit of the effect of viscous dissipation depends on whether the plate is being cooled or heated. Apart from the viscous dissipation in MHD flows, the Joule dissipation also acts as a volumetric heat source. Singer [1] has assessed the unsteady free convection heat transfer with

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magnetohydrodynamic effects in a channel regime. The effect of Joule heating on MHD combined heat and mass transfer flow of an electrically conducting viscous incompressible fluid past an infinite plate has been considered by Hossain [2]. Sacheti *et al.* [3] have discussed an exact solution for unsteady magnetohydrodynamic free convection flow with constant heat flux. El-Hakiem *et al.* [4] have studied Joule heating effects on magnetohydrodynamic free convection flow of a micro-polar fluid. Davaa et al. [5] have numerically studied fully-developed laminar heat transfer to non-Newtonian fluids flowing between parallel plates with the axial movement of one of the plates with an emphasis on the viscous dissipation effect. Hashemabadi *et al.* [6] have obtained an analytical solution to predict the fully-developed, steady and laminar heat transfer of viscoelastic fluids between parallel plates. Aydin [7,8] has investigated the effects of viscous dissipation on the heat transfer in a forced pipe-flow of different types. Aydin and Avci [9] have studied the viscous dissipation effects on the heat transfer in a Poiseuille flow. Duwairi [10] has presented the effects of Joule heating and viscous dissipation on the forced convection flow in the presence of thermal radiation. Aydin and Avci [11] have analytically examined laminar forced convection in a Couette-Poiseuille flow of a Newtonian fluid with constant properties by taking the viscous dissipation into account. Analysis of laminar heat transfer in micro-Poiseuille flow has been investigated by Aydin and Avci [12]. Umavathi *et al*. [13] have studied the magnetohydrodynamics Couette-Poiseuille flow and heat transfer in an inclined channel. Tso *et al.* [14] have obtained viscous dissipation effects of power- law fluid within parallel plates with constant heat fluxes. Combined effects of Hall currents and radiation on MHD free convective Couette flow in a rotating system have been studied by Sarkar *et al.* [15]. Sheela-Francisca et al. [16] have investigated the heat transfer with viscous dissipation in Couette-Poiseuille flow under asymmetric wall heat fluxes. Mecili and Mezaache [17] have examined slug flow heat transfer in parallel plate microchannel including slip effects and axial conduction. Jamalabadi and Park [18] have investigated thermal radiation, Joule heating, and viscous dissipation effects on MHD forced convection flow with uniform surface temperature. Omowaye and Koriko [19] have investigated Steady Arrhenius Laminar free convective MHD flow and heat transfer past a vertical stretching sheet with viscous dissipation. Effect of Hall Current on Unsteady MHD Couette Flow and Heat Transfer of Nanofluids in a Rotating System have been studied by Ali *et al.* [20]. Developing the laminar MHD forced convection flow of water/FMWNT carbon nanotubes in a microchannel imposed the uniform heat flux has been obtained by Karimipour *et al*. [21]. Kuiry and Bahadur [22] have studied an unsteady MHD flow of a Dusty Visco-elastic fluid between parallel plates with exponentially decaying pressure gradient in an inclined magnetic field. Reddy *et al*. [23] have discussed an MHD free convection heat transfer couette flow in rotating system. Ramesh [24] has studied effects of viscous dissipation and Joule heating on the Couette and Poiseuille flows of a Jeffrey fluid with slip boundary conditions. Mathematical Model and Solution for an Unsteady MHD Fourth Grade Fluid Flow over a Vertical Plate in a Porous Medium with Magnetic Field and Suction/Injection Effects have been investigated by Fenuga *et al*. [25]. Uwaezuoke [26] has studied Laminar forced

convection heat transfer in Couette-Poiseuille flow with viscous dissipation effects on asymmetric wall heat fluxes. Recently, Uwaezuoke and Ihekuna [27] have discussed Laminar forced convection with viscous dissipation in Couette-Poiseuille flow of Pseudo-Plastic fluids.

Our present paper is devoted to study the laminar MHD Couette-Poiseuille flow of a viscous incompressible fluid with a simultaneous pressure gradient and the axial movement of the upper plate with the viscous and Joule dissipations. Three different conditions of the upper plate are considered: (i) stationary, (ii) moving in the positive z-direction and (iii) moving in the negative z-direction. The effects of the magnetic parameter, velocity parameter and the modified Brinkman number on the fluid velocity and temperature distributions are discussed for two different thermal boundary conditions.

2. Formulation of the Problem and Its Solutions

Consider a steady hydromagnetic and thermally developed laminar flow of a viscous incompressible fluid between two infinite parallel plates separated by a distance h in the presence of a transverse magnetic field on taking viscous and joule dissipation into account. The upper plate is assumed to move at a uniform velocity u_0 in the direction of the flow while the lower one is stationary. Choose a cartesian co-ordinates system with z-axis along the lower plate in the direction of flow, the y-axis is normal to the plates. A uniform magnetic field of strength B_0 is applied perpendicular to the plates. Since the plates are infinitely long, all physical variables, except pressure, depend on y only. The axial heat-conduction in the fluid and in the plates is neglected.
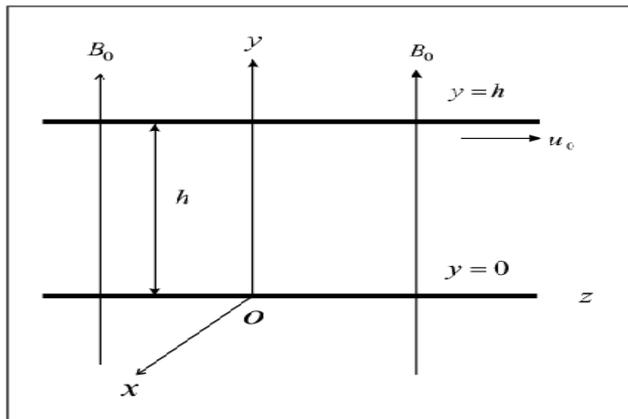

Fig. 1. Geometry of the problem.

The momentum equation in the z-direction can be written as

$$\frac{1}{\rho}\frac{\partial p}{\partial z} = \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho} u,$$

(1)

The equation of energy including viscous and Joule dissipations is given by

$$u \frac{\partial T}{\partial z} = \frac{k}{\rho c_p} \frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho c_p} \left(\frac{\partial u}{\partial y} \right)^2 + \frac{\sigma B_0^2}{\rho c_p} u^2, \tag{2}$$

where u is the fluid velocity in the z-direction, T the fluid temperature, ν the kinematic coefficient of viscosity, ρ the fluid density, k the thermal conductivity, c_p the specific heat at constant pressure and μ the coefficient of viscosity.

The boundary conditions for velocity field are

$$u = 0 \quad \text{at} \quad y = 0; \quad u = u_0 \quad \text{at} \quad y = h. \tag{3}$$

Introducing the non-dimensional variables

$$u_1 = \frac{u}{u_m}, \quad \eta = \frac{y}{h}, \tag{4}$$

equation (1) becomes

$$\frac{d^2 u_1}{d\eta^2} - M^2 u_1 = -C, \tag{5}$$

where $M^2 = \frac{\sigma B_0^2 h^2}{\rho \nu}$ is the magnetic parameter, $C = -\frac{h^2}{\rho \nu u_m} \left(\frac{\partial p}{\partial z} \right)$ a constant and u_m the mean velocity.

The boundary conditions for u_1 are

$$u_1 = 0 \quad \text{at} \quad \eta = 0; \quad u_1 = \lambda \quad \text{at} \quad \eta = 1, \tag{6}$$

where $\lambda = \frac{u_0}{u_m}$ is the velocity parameter.

The constant heat-flux at the plate is assumed as

$$k \frac{\partial T}{\partial \eta} \bigg|_{\eta = h} = q_h, \tag{7}$$

where q_h is positive when its direction is along the fluid in case of the hot plate, otherwise it is negative in case of the cold plate.

For the uniform plate heat-flux case

$$\frac{\partial T}{\partial z} = \frac{dT_h}{dz}. \tag{8}$$

Introducing the non-dimensional temperature

$$\theta = \frac{T - T_h}{\frac{q_h h}{k}}, \tag{9}$$

equation (2) becomes

$$\frac{d^2 \theta}{d\eta^2} = au_1 - 2Br \left[\left(\frac{du_1}{d\eta} \right)^2 + \varepsilon M^2 u_1^2 \right], \tag{10}$$

where $a = \frac{u_m kPrh}{\nu q_h} \frac{\partial T_h}{\partial z}$, $Pr = \frac{\rho \nu c_p}{k}$ the prandtl number, $Br = \frac{\mu u_m^2}{2hq_h}$ the modified Brinkman number and ε is a constant, $\varepsilon = 0$ or 1 according as Joule dissipation is neglected or taken into account.

The solution of the equations (5) subject to the boundary conditions (6) is

$$u_1 = \left[\frac{M \sinh M + \lambda(1 - \cosh M)}{M \sinh M + 2(1 - \cosh M)} \right] \left[1 - \frac{\sinh M\eta}{\sinh M} - \frac{\sinh M(1-\eta)}{\sinh M} \right] + \lambda \frac{\sinh M\eta}{\sinh M}. \tag{11}$$

If $M^2 = 0$, then the equation (11) is identical with the equation (5) of Aydin and Avci [11].

Two different forms of thermal boundary conditions are applied. These two different cases are separately discussed as follows:

2.1. Case-A: Consider a constant positive heat-flux at the upper moving plate with an adiabatic lower stationary plate.

In this case, the dimensionless thermal boundary conditions are

$$\theta = 0 \text{ and } \begin{cases} \left. \dfrac{\partial \theta}{\partial \eta} \right|_{\eta=1} = 1 \quad at \ \eta = 1 \\ \\ \left. \dfrac{\partial \theta}{\partial \eta} \right|_{\eta=0} = 0 \quad at \ \eta = 0. \end{cases} \tag{12}$$

On the use of equation (11), the solution of equation (10) subject to the thermal boundary conditions (12) is

$$\theta(\eta) = \left[C_1 \left(\frac{\eta^2}{2} - \frac{\sinh M\eta}{M^2 \sinh M} - \frac{\sinh M(1-\eta)}{M^2 \sinh M} \right) + \lambda \frac{\sinh M\eta}{M^2 \sinh M} \right] a$$

$$-2Br \left[\frac{M^2}{2\sinh^2 M} \left\{ \left(C_1^2 + 2(\lambda C_1 - C_1^2) \cosh M + (C_1 - \lambda)^2 \right) \frac{\eta^2}{2} \right. \right.$$

$$+ \frac{C_1^2}{4M^2} \cosh 2M(1-\eta) + \frac{(\lambda C_1 - C_1^2)}{2M^2} \cosh M(1-2\eta) + \frac{(C_1 - \lambda)^2}{4M^2} \cosh 2M\eta \bigg\}$$

$$+ \frac{\varepsilon M^2}{\sinh^2 M} \left\{ \left(C_1^2 \sinh^2 M - (\lambda C_1 - C_1^2) \cosh M - \frac{1}{2} C_1^2 - \frac{1}{2} (C_1 - \lambda)^2 \right) \frac{\eta^2}{2} \right.$$

$$+ \frac{2(\lambda C_1 - C_1^2)}{M^2} \sinh M \sinh M\eta - \frac{2C_1^2}{M^2} \sinh M \sinh M(1-\eta)$$

$$+ \frac{(\lambda C_1 - C_1^2)}{4M^2} \cosh M(1-2\eta) + \frac{C_1^2}{8M^2} \cosh 2M(1-\eta) + \frac{(C_1 - \lambda)^2}{8M^2} \cosh 2M\eta \bigg\} \bigg]$$

$$+ C_2\eta + C_3, \tag{13}$$

where

$$C_1 = \frac{M \sinh M + \lambda(1 - \cosh M)}{M \sinh M + 2(1 - \cosh M)},$$

$$A_1 = C_1^2 \sinh 2M + 2(\lambda C_1 - C_1^2) \sinh M,$$

$$A_2 = 6(\lambda C_1 - C_1^2)\sinh M + 3C_1^2 \sinh 2M,$$

$$A_3 = 2M[C_1^2 + 2(\lambda C_1 - C_1^2)\cosh M + (C_1 - \lambda)^2],$$

$$A_4 = 2(\lambda C_1 - C_1^2)\sinh M + (C_1 - \lambda)^2 \sinh 2M,$$

$$A_5 = 2M[2C_1^2 \sinh^2 M - 2(\lambda C_1 - C_1^2)\cosh M - C_1^2 - (C_1 - \lambda)^2],$$

$$A_6 = 4(\lambda C_1 - C_1^2)\sinh 2M + 8C_1^2 \sinh M + 2(\lambda C_1 - C_1^2)\sinh M + (C_1 - \lambda)^2 \sinh 2M,$$

$$a = \frac{M[4\sinh^2 M + 2BrM((A_1 + A_3 + A_4) + \varepsilon(A_5 + A_6 - A_2))]}{4\sinh M[C_1(M\sinh M + 2(1 - \cosh M)) - \lambda(1 - \cosh M)]}, \tag{14}$$

$$A_7 = 2(\lambda C_1 - C_1^2)\cosh M + (C_1 - \lambda)^2 \cosh 2M + C_1^2,$$

$$A_8 = 16(\lambda C_1 - C_1^2)\sinh^2 M + 2(\lambda C_1 - C_1^2)\cosh M + C_1^2 + (C_1 - \lambda)^2 \cosh 2M,$$

$$C_2 = \frac{a}{M\sinh M}[C_1(1 - \cosh M) - \lambda] - \frac{BrM}{2\sinh^2 M}(A_1 - \varepsilon A_2),$$

$$C_3 = -\left[C_1\left(\frac{1}{2} - \frac{1}{M^2}\right) + \frac{\lambda}{M^2}\right]a + \frac{Br}{4\sinh^2 M}[(MA_3 + A_7) + \varepsilon(MA_5 + A_8)] - C_2.$$

Without magnetic field ($M^2 = 0$) and without joule dissipation ($\varepsilon = 0$) into account the equation (13) is same as the equation (12) of Aydin and Avci [11].

2.2. Case-B: Consider a constant negative heat-flux at the lower stationary plate with an adiabatic upper moving plate.

In this case, the dimensionless thermal boundary conditions are

$$\theta = 0 \text{ and } \begin{cases} \left.\dfrac{\partial\theta}{\partial\eta}\right|_{\eta=0} = -1 \quad \text{at } \eta = 0 \\[3mm] \left.\dfrac{\partial\theta}{\partial\eta}\right|_{\eta=1} = 0 \quad \text{at } \eta = 1. \end{cases} \tag{15}$$

The solution of equation (10), on using (11), subject to the boundary conditions (15) is

$$\theta(\eta) = \left[C_1\left(\frac{\eta^2}{2} - \frac{\sinh M\eta}{M^2 \sinh M} - \frac{\sinh M(1-\eta)}{M^2 \sinh M}\right) + \lambda\frac{\sinh M\eta}{M^2 \sinh M}\right]a$$

$$-2Br\left[\frac{M^2}{2\sinh^2 M}\left\{\left(C_1^2 + 2(\lambda C_1 - C_1^2)\cosh M + (C_1 - \lambda)^2\right)\frac{\eta^2}{2}\right.\right.$$

$$+\frac{C_1^2}{4M^2}\cosh 2M(1-\eta) + \frac{(\lambda C_1 - C_1^2)}{2M^2}\cosh M(1-2\eta) + \frac{(C_1 - \lambda)^2}{4M^2}\cosh 2M\eta\Big\}$$

$$+\frac{\varepsilon M^2}{\sinh^2 M}\left\{\left(C_1^2 \sinh^2 M - (\lambda C_1 - C_1^2)\cosh M - \frac{1}{2}C_1^2 - \frac{1}{2}(C_1 - \lambda)^2\right)\frac{\eta^2}{2}\right.$$

$$+ \frac{2(\lambda C_1 - C_1^2)}{M^2} \sinh M \sinh M\eta - \frac{2C_1^2}{M^2} \sinh M \sinh M(1-\eta)$$

$$+ \frac{(\lambda C_1 - C_1^2)}{4M^2} \cosh M(1-2\eta) + \frac{C_1^2}{8M^2} \cosh 2M(1-\eta) + \frac{(C_1 - \lambda)^2}{8M^2} \cosh 2M\eta \Bigg\} \Bigg]$$

$$+ C_2^* \eta + C_3^*, \tag{16}$$

where

$$A_7^* = C_1^2 \cosh 2M + 2(\lambda C_1 - C_1^2) \cosh M + (C_1 - \lambda)^2,$$

$$A_8^* = C_1^2 \cosh 2M + (C_1 - \lambda)^2 - 16C_1^2 \sinh^2 M + 2(\lambda C_1 - C_1^2) \cosh M,$$

$$C_2^* = -1 + \frac{a}{M \sinh M} [C_1(1 - \cosh M) - \lambda] - \frac{BrM}{2\sinh^2 M}(A_1 - \varepsilon A_2), \tag{17}$$

$$C_3^* = \frac{aC_1}{M^2} + \frac{Br}{4\sinh^2 M}(A_7^* + \varepsilon A_8^*).$$

and a, A_1, A_2 and C_1 are given by (14).

Without magnetic field ($M^2 = 0$) and without joule dissipation ($\varepsilon = 0$) into account the equation (16) is identical with the equation (14) of Aydin and Avci [11].

3. Results and Discussion

We have presented the non-dimensional velocity and temperature distributions for several values of the magnetic parameter M^2, the velocity parameter λ and the modified Brinkman number Br in Figs. 2-18. Figs. 2 and 3 represent the fluid velocity $u_1(\eta)$ against η for several values of the magnetic parameter M^2 and the velocity parameter λ. Interestingly one can make a note from Fig. 2 that the fluid velocity $u_1(\eta)$ increases in the region $0 < \eta \le 0.41$ and then changes its behaviour and decreases for $\eta > 0.41$ with an increase in magnetic parameter M^2. That means magnetic field acts as an acting body force in the region $0 < \eta \le 0.41$ but in general a magnetic field normal to the flow direction has the tendency to slow down the movement of the fluids due to opposite direction of Lorentz force which is shown for $\eta > 0.41$. Magnetic field can not regulate the flow field at $\eta = 0.41$. The practice of magnetic fields has effectively been applied to monitoring the fluid flow in many engineering applications. It is seen from Fig.3 that the fluid velocity $u_1(\eta)$ decreases in the region $0 < \eta \le 0.68$ and it has an increasing trend for $\eta > 0.68$ with an increase in velocity parameter λ. It is also noticed from Fig. 3 that if the upper plate is moving in the opposite direction to the flow field then the reverse flow occurs near the upper plate.
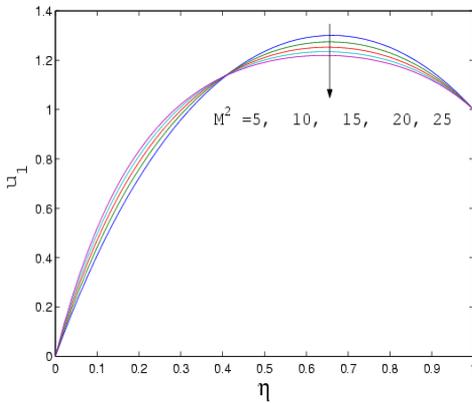
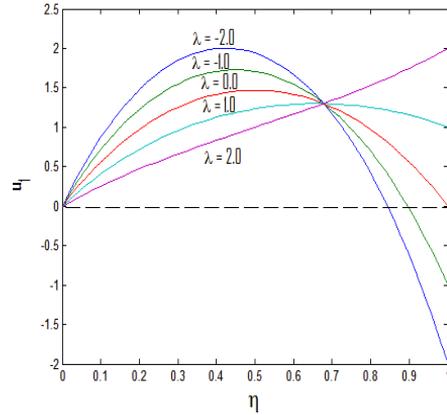Fig. 2. Velocity profiles for M^2 when $\lambda = 1$

Fig. 3. Velocity profiles for λ when $M^2 = 5$

Figs. 4-18 represent the fluid temperature $\theta(\eta)$ against η for several values of the magnetic parameter M^2, the velocity parameter λ and the modified Brinkman number Br for the case-A (positive heat-flux) and case-B (negative heat-flux). The bulk fluid is heated for positive heat-flux and it is cooled for negative heat-flux. The comparision of the effects of physical parameters on the temperature fields without joule dissipation ($\varepsilon = 0$) and with joule dissipation ($\varepsilon = 1$) respectively has been shown with the helps of graphs. Fig.4 reveals that the fluid temperature $\theta(\eta)$ increases for the case-A while it decreases for the case-B with different magnitude of upper plate velocity (indicating by velocity parameter λ) in the absence of viscous dissipation ($Br = 0$). Positive values of velocity parameter λ represents the velocity movement of the upper plate is in the positive z-direction, while the negative values indicates in the opposite direction, $\lambda = 0$ means the upper plate is stationary. It is known that the viscous dissipation behaves like an energy source due to viscosity of the fluid and Joule dissipation behaves like an energy source due to magnetic field. Both of these dissipation enhance the temperature of the bulk fluid. For the positive heat flux case at the plate, the increasing viscous dissipation will result in decreasing temperature-differences between the plate and the bulk fluid whereas for the negative heat-flux case at the plate, it will increase temperature differences between the plate and the bulk fluid which is the main driving mechanism for heat transfer from plate to fluid. It is seen from Figs. 5, 7 and 9 that for case-A the fluid temperature $\theta(\eta)$ increases in the vicinity of the lower plate and it decreases away from the stationary plate for enhance of the viscous dissipation only (increase of Br) whereas it increases for the combined effects of viscous and Joule dissipations with an increase in modified Brinkman number Br. In case-B for negative heat-flux Figs. 6, 8 and 10 demonstrate that the fluid temperature $\theta(\eta)$ decreases with an increase in modified Brinkman number Br either for only viscous dissipation or both viscous and Joule dissipations taken into account. It is interesting to see that the fluid temperature distribution with joule dissipation effect ($\varepsilon = 1$) is in general greater than the fluid

temperature distribution without joule dissipation ($\varepsilon = 0$). This is a good agreement of joule dissipation effect in fluid temperature distribution. It is seen from Fig.11 that for positive heat-flux, the fluid temperature $\theta(\eta)$ increases for only viscous dissipation effect whereas $\theta(\eta)$ decreases for the combined effects of viscous and Joule dissipations with an increase in velocity parameter λ. It is observed from Fig12 that for negative heat-flux, the fluid temperature $\theta(\eta)$ decreases for only viscous dissipation effect whereas $\theta(\eta)$ increases for the combined effects of viscous and Joule dissipations with an increase in velocity parameter λ. It means that joule dissipation effect has a influence to reverse the effects of upper plate velocity on the fluid temperature distribution. It is observed from Figs. 13, 15 and 17 that for positive heat-flux, the fluid temperature $\theta(\eta)$ increases with an increase in magnetic parameter M^2 for only viscous dissipation and the reversed effects are shown for the combined effects of viscous and Joule dissipations. It is also observed from Figs. 14, 16 and 18 that for negative heat-flux, the fluid temperature $\theta(\eta)$ decreases with an increase in magnetic parameter M^2 for viscous dissipation only and the reversed effects are shown for combined effects of viscous and Joule dissipations.
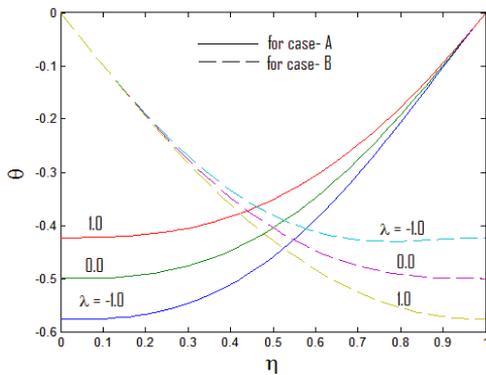
Fig. 4. Temperature profiles for λ when $M^2 = 5$, $Br = 0$
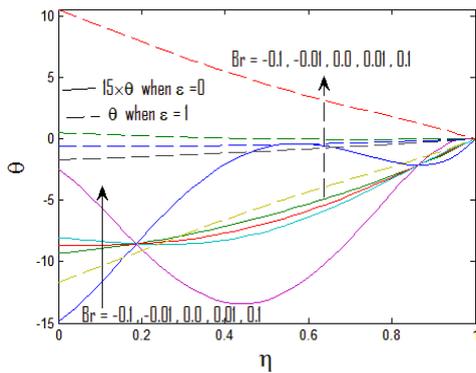
Fig. 5. Temperature profiles for Br when $M^2 = 5$, $\lambda = -1$ for case-A.
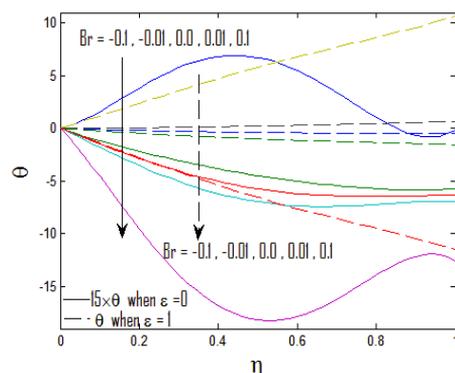
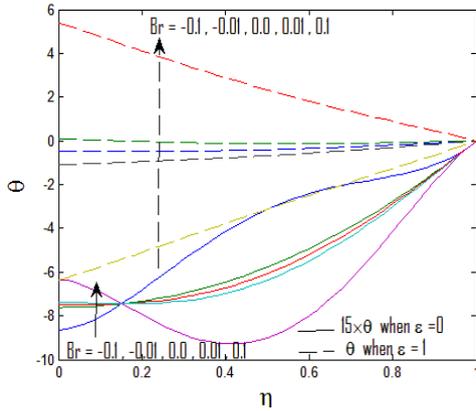Fig. 6. Temperature profiles for Br when $M^2 = 5$, $\lambda = -1$ for case-B.

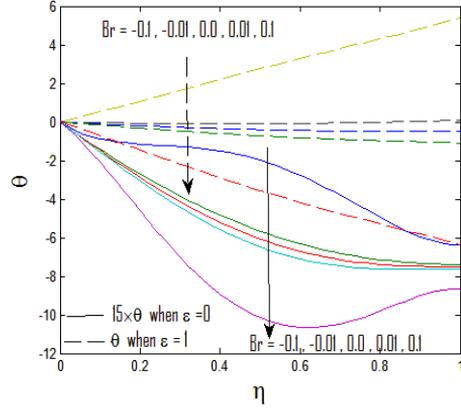Fig. 7. Temperature profiles for Br when $M^2 = 5$, $\lambda = 0$ for case-A.

Fig. 8. Temperature profiles for Br when $M^2 = 5$, $\lambda = 0$ for case-B.
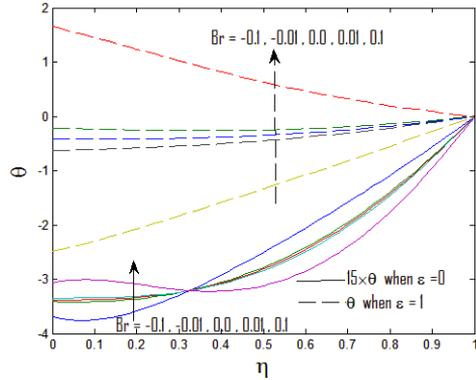

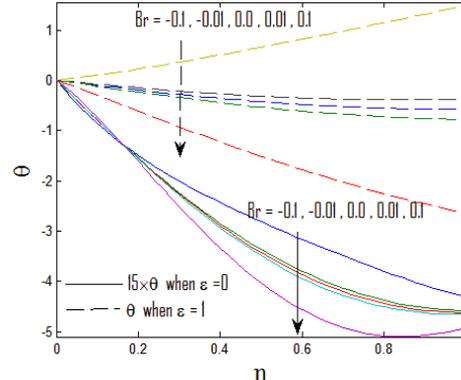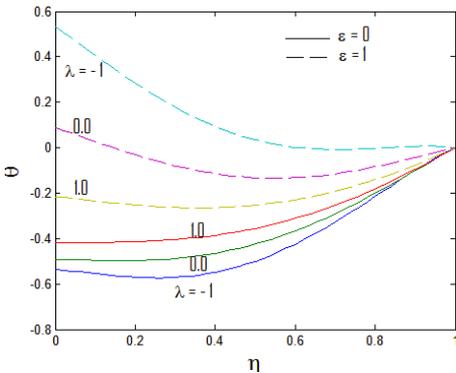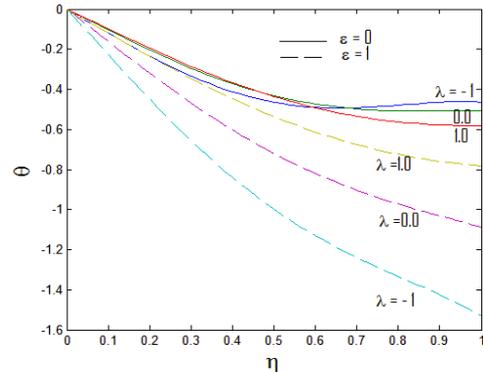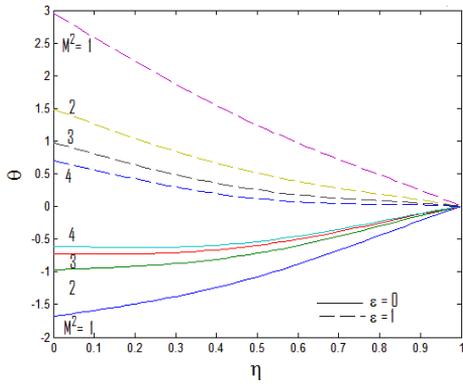Fig. 9. Temperature profiles for Br when $M^2 = 5$, $\lambda = 1$ for case-A.

Fig. 10. Temperature profiles for Br when $M^2 = 5$, $\lambda = 1$ for case-B.

Fig. 11. Temperature profiles for λ when $M^2 = 5$, $Br = 0.01$ for case-A.

Fig. 12. Temperature profiles for λ when $M^2 = 5$, $Br = 0.01$ for case-A.

Fig. 13. Temperature profiles for M^2 when $\lambda = -1$, $Br = 0.01$ for case-A.
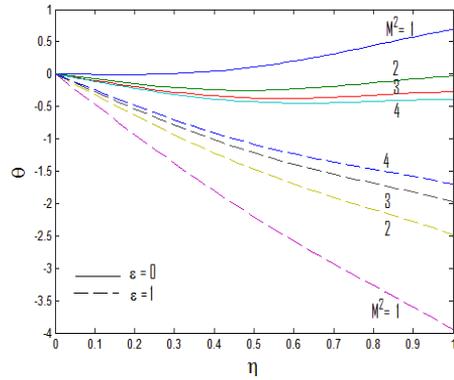

Fig. 14. Temperature profiles for M^2 when $\lambda = -1$, $Br = 0.01$ for case –B.
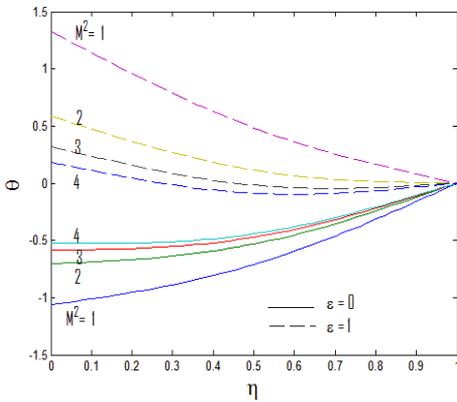

Fig. 15. Temperature profiles for M^2 when $\lambda = 0$, $Br = 0.01$ for case-A.
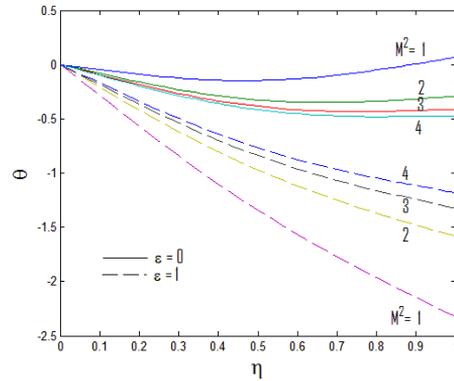

Fig. 16. Temperature profiles for M^2 when $\lambda = 0$, $Br = 0.01$ for case-B.
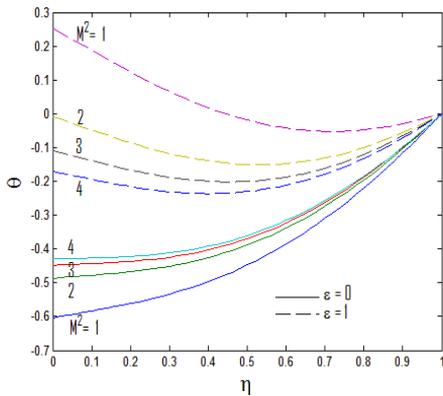

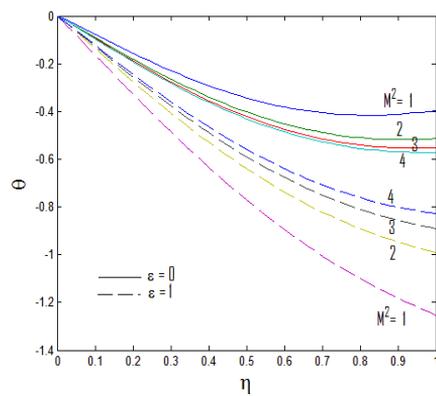Fig. 17. Temperature profiles for M^2 when $\lambda = 1$, $Br = 0.01$ for case-A.

Fig. 18. Temperature profiles for M^2 when $\lambda = 1$, $Br = 0.01$ for case-B.

4. Conclusion

The laminar forced convection MHD Couette-Poiseuille flow of a viscous incompressible fluid between plane parallel plates with a simultaneous pressure-gradient and the movement of the upper plate with viscous and Joule dissipations into account has been investigated. Two different thermal boundary-conditions have been considered in Case A and Case B. In Case A, a constant heat-flux at the upper plate with an adiabatic lower plate and in Case B, a constant heat-flux at the lower plate with an adiabatic upper plate have been considered respectively. It is found that the magnetic field accelerates the fluid velocity in the vicinity of the stationary plate and it retards the fluid velocity near the moving plate. The fluid temperature field is significantly affected by the modified Brinkman number. The fluid temperature increases in case the lower plate is adiabatic and the upper plate is at positive constant heat flux while it decreases when the lower plate is at negative constant heat flux and the upper plate is adiabatic with an increase in modified Brinkman number for combined effects of viscous and Joule dissipations. Further, the magnetic field tends to reduce the temperature field for positive heat flux case while it has a tendency to enhance the temperature field for negative heat flux case when combined effects of viscous and Joule dissipations are taken. The velocity of the upper plate in z-direction or opposite to the z-direction involves significant effects on temperature distribution. It is interesting to note that viscous and Joule dissipations behave like an energy source which increases the temperature of the bulk fluid.

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References

1. R. M. Singer, Zeitschrift für angewandte Mathematik und Physik ZAMP **16**, 483 (1965). https://doi.org/10.1007/BF01593924
2. M. A. Hossain, Int. Center for Theoret. Phys. (ICTP), (1990) Italy, Internal Print no IC/90/265.
3. N. C. Sacheti, P. Chandran and A. K. Singh, Int. Commun. Heat Mass Transfer **21**, 1 (1994). https://doi.org/10.1016/0735-1933(94)90090-6
4. M. A. El-Hakiem, A. A. Mohammadein, S. M. M. EL-Kabier, and R. S. R. Gorla, Int. Commun. Heat and Mass Transfer **26**, 2 (1999). https://doi.org/10.1016/S0735-1933(99)00008-1
5. G. Davaa, T. Shigechi, and S. Momoki, Int. Commun. Heat Mass Transfer **31**, 5 (2004). https://doi.org/10.1016/S0735-1933(04)00053-3
6. S. H. Hashemabadi, S.G. Etemad, and J. Thibault, Int. J. Heat Mass Transfer **47**, 3985 (2004). https://doi.org/10.1016/j.ijheatmasstransfer.2004.03.026
7. O. Aydin, Energy Convers Manag. **46**, 757 (2005). https://doi.org/10.1016/j.enconman.2004.05.004

8. O. Aydin, Energy Convers Manag. **46**, 3091 (2005). https://doi.org/10.1016/j.enconman.2005.03.011
9. O. Aydin and M. Avci, Appl. Energy **83**, 5 (2006). https://doi.org/10.1016/j.apenergy.2005.03.003
10. H. M. Duwairi, Int. J. Nume. Meth. Heat Fluid Flow **15**, 5 (2005). https://doi.org/10.1108/09615530510593620
11. O. Aydin and M. Avci, Appl. Energy **83**, 856 (2006). https://doi.org/10.1016/j.apenergy.2005.08.005
12. O. Aydin and M. Avci, Int. J. Thermal Sci. **46**, 30 (2007). https://doi.org/10.1016/j.ijthermalsci.2006.04.003
13. J. C. Umavathi, I. -C. Liu, and J. P. Kumar, J. Mechanics **26**, 525 (2010). https://doi.org/10.1017/S172771910000472X
14. C. P. Tso, J. S. Francisca, and Y. M. Hung, Non-Newton. Fluid Mech. **165**, 625 (2010). https://doi.org/10.1016/j.jnnfm.2010.02.023
15. B. C. Sarkar, S. Das, and R. N. Jana, Pelagia Res. Library **3**, 6 (2012).
16. J. Sheela-Francisca, C. P. Tso, and D. Rilling, Open J. Fluid Dyn. **2**, 111 (2012). http://dx.doi.org/10.4236/ojfd.2012.24011
17. M. Mecili and E. H. Mezaache, Energy Procedia **36**, 268 (2013). http://dx.doi.org/10.1016/j.egypro.2013.07.031
18. M. Y. A. Jamalabadi and J. H. Park, Open J. Fluid Dyn. **4**, 125 (2014). http://dx.doi.org/10.4236/ojfd.2014.42011
19. A. J. Omowaye and O. K. Koriko, Nigerian Math. Soc. **33**, 259 (2014).
20. A. O. Ali, O. D. Makinde, and Y. N. Gyekye, Appl. Comput. Math. **4**, 232 (2015). http://dx.doi.org/10.11648/j.acm.20150404.12
21. A. Karimipour, A. Taghipour, and A. Malvandi, J. Magn. Magn. Mater. **419**, 420 (2016). https://doi.org/10.1016/j.jmmm.2016.06.063
22. D. R. Kuiry and S. Bahadur, J. Sci. Res. **8**, 149 (2016). https://doi.org/10.3329/jsr.v8i2.25745
23. G. J. Reddy, P. Manideep, and R. S. Raju, AIP Conf. Proc. **1953**, ID 140143 (2018). https://doi.org/https://doi.org/10.1063/1.5033318
24. K. Ramesh, Propulsion and Power Res. **7**, 329 (2018). https://doi.org/10.1016/j.jppr.2018.11.008
25. O. J. Fenuga, S. J. Aroloye, and S. O. Salawu, J. Sci. Res. **12**, 485 (2020). https://doi.org/10.3329/jsr.v12i4.45551
26. M. U. Uwaezuoke, Int. J. Eng. Sci. Invention **10**, 8 (2021). https://doi.org/10.35629/6734-1008010817
27. M. U. Uwaezuoke and S. O. Ihekuna, Int. J. Eng. Sci. Invention **10**, 43 (2022). https://doi.org/10.35629/6734-1010014359