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Stability of Cosmological Model in Self-Creation Theory of Gravitation

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Abstract

In the present paper, we considered Bianchi type-II space-time in the presence of a macroscopic body in the self-creation theory formulated by Barber. The relation between metric coefficient and state equation has helped present the exact cosmological model in theory. The features, stability, and some physical & kinematical properties of the obtained model are also discussed.

Keywords: Bianchi type-II; Barber self-creation theory; macroscopic body.

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1. Introduction

Einstein's general theory of relativity is one of the most beautiful structures in theoretical physics. It describes the successful theory of gravity in terms of geometry. It also serves as the basis of the universe model. Two "self-creation" theories based on two sets of general relativity field equations, including matter and scalar fields, are proposed by Barber [1]. This theory explained that the gravitational relationship of Einstein's field equations could be a variable scalar in the space-time manifold. The second theory proposed by Barber is a modification of general relativity, including continuous creation and being within the observable range. Doing so changed the general theory of relativity, making it a variable theory. The scalar field is not directly affected by gravity but only shares the matter tensor with the scalar, and the scalar acts as an anti-gravitational constant.

The Barber field equation in the second self-creation theory can be expressed as

$$R_{ij} - \frac{1}{2} R g_{ij} = -8\pi \varphi^{-1} T_{ij} \tag{1}$$

And

$$\Box \varphi = \varphi_{;k}^{\prime k} = \frac{8\pi\lambda}{3} T \tag{2}$$

Where φ is the Barber's scalar, T_{ij} is the energy-momentum tensor, $\Box$ is the invariant D'Alembertian, T is the trace of the energy-momentum tensor T_{ij} and λ is a coupling

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constant. In the limit $\lambda \to 0$, this theory approaches Einstein's general theory of relativity in every respect.

Due to the nature of space-time, Barber's scalar φ is a function of 't.' Venkateswarlu *et al*. [2] presented Bianchi type I cosmological models in the self-creation theory of gravitation for perfect fluid distribution. Shanthi *et al*. [3] investigated Bianchi Type-II and III models in self-creation cosmology. Pawar *et al*. [4] discussed the magnetized Bianchi type IX cosmological model in Barber's second self-creation theory. Kumar *et al.* [5] investigated the field equations for a five-dimensional Kaluza-Klein model in the presence of bulk viscosity within the framework of Barber's second self-creation theory. Vinutha *et al*. [6] studied the Dynamics of FRW type Kaluza-Klen mhrde cosmological model in self-creation theory. Hegazy [7] investigated Bulk Viscous Bianchi Type VI_0 Cosmological Model in the self-creation Theory of Gravitation and in the General Theory of Relativity. Wankhade *et al*. [8] studied Wet Dark Fluid Cosmological Model in Barber Self-Creation Theory of Gravitation. Singh *et al*. [9,10] discussed Bianchi type-II cosmological models in Brans–Dicke theory and in Lyra Geometry. Yang [11] investigated the energy of the Universe in the Bianchi type-II cosmological model.

Also, Katore *et al*. [12,13], Nasr Amhmed *et al*. [14], and Shah *et al.* [15] are among the authors who have analyzed the stability of cosmological models.

The current work aims to get Bianchi type-II cosmological model in the presence of a macroscopic body. The present paper is organized as follows. In section 2, Metric and Field Equations**.** Section 3, Solutions of field Equations, Section 4, is mainly concerned with the physical and Kinematical properties of the model, and Section 5, Stability Solution. The last section contains some conclusions.

2. Metric and Field Equation

We consider the Bianchi type II space-time in the form,

$$ds^2 = dt^2 - A^2(dx - zdy)^2 - B^2dy^2 - C^2dz^2, \tag{3}$$

Where A, B, and C are functions of 't' only.

The energy momentum-tensor for a macroscopic body (Landue and Lifshitz) [16] is given by

$$T^{ik} = (p+\varepsilon)u^i u^k - pg^{ik} \tag{4}$$

Here p is the pressure, ε is the energy density, and u_i is the four-velocity vectors of the distribution, respectively. Where u^i will satisfy $u_i u^i = -1$,

From equation (4) we get

$$T_1^1 = T_2^2 = T_3^3 = -p, and\ T_4^4 = \varepsilon, \tag{5}$$

Using the equations (1), (2), and (4), the field equations of metric (3) are

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4C_4}{BC} - \frac{3A^2}{4B^2C^2} = -8\pi\phi^{-1}p\ , \tag{6}$$

$$\frac{A_{44}}{A}+\frac{C_{44}}{C}+\frac{A_4C_4}{AC}+\frac{A^2}{4B^2C^2}=-8\pi\phi^{-1}p, \tag{7}$$

$$\frac{A_{44}}{A}+\frac{B_{44}}{B}+\frac{A_4B_4}{AB}+\frac{A^2}{4B^2C^2}=-8\pi\phi^{-1}p, \tag{8}$$

$$\frac{A_4B_4}{AB}+\frac{B_4C_4}{BC}+\frac{A_4C_4}{AC}-\frac{A^2}{4B^2C^2}=8\pi\phi^{-1}\varepsilon, \tag{9}$$

$$\phi_{44}+\left(\frac{A_4}{A}+\frac{B_4}{B}+\frac{C_4}{C}\right)\phi_4=\frac{8\pi\lambda}{3}\left(-3p+\varepsilon\right), \tag{10}$$

Where, subscript '4' of the field variable denotes ordinary differentiation with respect to time.

3. Solutions of Field Equations

The equations (6) to (10) are a system of five independent equations with six unknowns$A, B, C, \varphi, p and \varepsilon$. Hence to get a determinate solution, one has to assume the relation between metric coefficients i.e.$A = B^n, C = B^m$ and radiation universe$\varepsilon = 3p$. we get,

$$BB_{44} + k_1 {B_4}^2 = k_2 B^{2(n-m)}, \tag{11}$$

The above equation (11) admits an exact solution given by

$$A = {k_6}^n (k_4 t + k_5)^{\frac{n}{m-n+1}}, \tag{12}$$

$$B = k_6 (k_4 t + k_5)^{\frac{1}{m-n=1}}, \tag{13}$$

$$C = {k_6}^m (k_4 t + k_5)^{\frac{m}{m-n+1}}. \tag{14}$$

Where$k_6 = (m - n + 1)^{\frac{1}{m-n+1}}$

And the scalar field is given by

$$\varphi = \frac{k_9}{(k_4 t + k_5)^{\frac{m+n}{m-n+1}}}. \tag{15}$$

The pressure and energy density are given by

$$p = \frac{{k_4}^2 k_9}{8\pi (k_4 t + k_5)^{\frac{3m-n=2}{m-n+1}}} \left\{\frac{n-m+nm}{(m-n+1)^2} - \frac{3}{4}\frac{{k_6}^{2(n-m-1)}}{{k_4}^2}\right\}, \tag{16}$$

$$\varepsilon = \frac{k_9 {k_4}^2}{8\pi (k_4 t + k_5)^{\frac{3m-n+2}{m-n+1}}} \left\{\frac{n+m+mn}{(m-n+1)^2} - \frac{{k_6}^{2(n-m-1)}}{4{k_4}^2}\right\}. \tag{17}$$

Using equations (12), (13), and (14), the cosmological model in equation (3) takes the form

$$ds^2 = dt^2 - k_6{}^{2n}(k_4t + k_5)^{\frac{2n}{m-n+1}}(dx - zdy)^2 - k_6{}^2(k_4t + k_5)^{\frac{2}{m-n=1}}dy^2 - k_6{}^{2m}(k_4t + k_5)^{\frac{2m}{m-n+1}}dz^2. \quad (18)$$

4. Physical and Kinematical Properties

Spatial volume $V = \sqrt{-g} = k_6{}^{m+n+1}(k_4t + k_5)^{\frac{m+n+1}{m-n+1}}$, (19)

Scalar Expansion $\theta = \frac{1}{3}\left(\frac{m+n+1}{m-n+1}\right)\frac{k_4}{(k_4t+k_5)}$, (20)

Hubble Parameter $H = \left(\frac{m+n+1}{m-n+1}\right)\frac{k_4}{(k_4t+k_5)}$, (21)

Shear Scalar$\sigma^2 = \frac{k_4{}^2}{54(k_4t+k_5)}\left(\frac{m+n+1}{m-n+1}\right)^2$. (22)

Also, the expression for the energy density W, energy flow vector S, and stress tensor $\sigma_{\alpha\beta}$are

$$W = \left(\frac{1+\frac{v^2}{3c^2}}{1-\frac{v^2}{c^2}}\right)\frac{k_9k_4{}^2}{8\pi(k_4t+k_5)^{\frac{3m-n+2}{m-n+1}}}\left\{\frac{n+m+mn}{(m-n+1)^2} - \frac{k_6{}^{2(n-m-1)}}{4k_4{}^2}\right\}, \quad (23)$$

$$S = \left(\frac{4v}{3\left(1-\frac{v^2}{c^2}\right)}\right)\frac{k_9k_4{}^2}{8\pi(k_4t+k_5)^{\frac{3m-n+2}{m-n+1}}}\left\{\frac{n+m+mn}{(m-n+1)^2} - \frac{k_6{}^{2(n-m-1)}}{4k_4{}^2}\right\}. \quad (24)$$

$$\sigma_{\alpha\beta} = \left(\frac{4v_\alpha v_\beta}{c^2\left(1-\frac{v^2}{c^2}\right)} + \delta_{\alpha\beta}\right)\frac{k_9k_4{}^2}{8\pi(k_4t+k_5)^{\frac{3m-n+2}{m-n+1}}}\left\{\frac{n-m+mn}{(m-n+1)^2} - \frac{3k_6{}^{2(n-m-1)}}{4k_4{}^2}\right\}. \quad (25)$$

If the velocity of the macroscopic motion is less in comparison with the velocity of the light, then we have approximately $S = (p + \varepsilon)v$.
Since $\frac{S}{c^2}$ is the momentum density and $\frac{(p+\varepsilon)}{c^2}$ plays the role of the mass density of the body.
From the expression (4), we get

$$T_i^i = \varepsilon - 3p, \quad (26)$$

But, $T_i^i = \sum_a m_a\, c^2\sqrt{1 - \frac{v_a^2}{c^2}}\,\delta(r - r_0)$, (27)

Comparing the relationship (26) with the general formula (27), we saw that it works for the arbitrary system. Since we are considering a macroscopic object, (27) should be averaged over all values per unit volume.
We obtained the result

$$\varepsilon - 3p = \sum_a m_a\, c^2\sqrt{1 - \frac{v_a^2}{c^2}}. \quad (28)$$

Here the summation extends over all particles in unit volume
The right side of this equation tends to zero in the ultra-relativistic limit,
So, in this limit, the equation of state of matter is $p = \frac{\varepsilon}{3}$.

The decomposition of a time-like tidal tensor is

$$u_{a;b} = \frac{k_4}{(m-n+1)(k_4t+k_5)}\left[-nk_6{}^{2n}(k_4t+k_5)^{\frac{2n}{m-n+1}} - k_6{}^2(k_4t+k_5)^{\frac{2}{m-n+1}} - mk_6{}^{2m}(k_4t+k_5)^{\frac{2m}{m-n+1}}\right]. \tag{29}$$

And, Vorticity $\omega_{11} = \omega_{22} = \omega_{33} = \omega_{44} = 0.$ (30)

The vorticity of the model along the *x*, *y*, *z*, and *t*-axes is zero. So, the obtained model is non-rotational. Whereas when vorticity is nonzero, the model is rotating.

Now, let us look at the present model's uniformity with a few observational parameters such as red-shift, look-back time, and luminosity distance. This study focuses on targeting the formation of the Universe, which has astronomical importance.

4.1. *Red-shift*

Red-shift is a very important phenomenon in cosmology and astronomy. Red-shifts are used by astronomers *to* measure how the Universe is expanding. The average scale factor a and red-shift z are related by

$$a = \frac{a_0}{1+z}, \tag{31}$$

Where a_0 is the present value of the scale factor. Hence, we get,

$$a = k_6^{\frac{m+n+1}{3}}(k_4t+k_5)^{\frac{m+n+1}{3(m-n+1)}}, \tag{32}$$

This follows that

$$1+z = \left(\frac{t_0}{t}\right)^{\frac{m+n+1}{3(m-n+1)}}. \tag{33}$$

4.2. *Look –back time*

Look-back time is defined as the difference in the current age of the Universe. i.e. $(z=0)$ and age of the Universe when a particular ray is emitted with a red-shift z. It depends on the dynamics of the Universe.

$$t_L = t_0 - t(z), \tag{34}$$

Where t_0 indicates the current age of the Universe, and z indicates the red-shift of a well-measured amount of light from a distant object such as a galaxy. The expansion of the Universe causes the emission of red shift light. For a given red-shift z, the average universe scale factor is $a_0(t)$ for the current universe scale factor, which is

$$1+z = \frac{a_0(t)}{a(t)}, \tag{35}$$

Solving the above equations, we get the required look-back time

$$t_L = \left(\frac{m+n+1}{m-n+1}\right)\frac{k_4}{H_0}\left(1 - \frac{1}{(1+z)^{\frac{3(m-n+1)}{m+n+1}}}\right). \tag{36}$$

Where $H_0$is the Hubble constant at present, measured in $Km/\,sec/\,M\,pc$ and its value lies between 50 and $100 Km/\,sec/\,M\,pc$.

4.3. *Luminosity-distance red-shift*

The luminosity distance$d_L$of the light source is defined as,

$$d_L = a_0 r_1(z)(1+z). \tag{37}$$

Where the radial coordinate distance $r_1(z)$ of the object at the light emission is

$$r_1(z) = \int_t^{t_0} \frac{dt}{a} = \frac{3(m+n+1)k_4}{2H_0 a_0(m-2n+1)}\left[1-(1+z)^{\frac{-2(m-2n+1)}{m-n+1}}\right], \tag{38}$$

From equations (37) and (38) we get,

$$d_L = \frac{3(m+n+1)(1+z)}{2H_0(m-2n+1)}\left[1-(1+z)^{\frac{-2(m-2n+1)}{m-n+1}}\right]. \tag{39}$$

5. Stability Solutions

Here we discuss the stability of the model by observing the ratio of sound speed given by $\frac{dp}{d\varepsilon} = {c_s}^2$, when the ratio $\frac{dp}{d\varepsilon}$ is positive, i.e., ${c_s}^2 > 0$, we have a stable model. Whereas when the ratio$\frac{dp}{d\varepsilon}$ is negative, i.e.,${c_s}^2 < 0$, we have an unstable model.

In this model,

$$\frac{dp}{d\varepsilon} = \frac{4{k_4}^2(n-m+nm)-3}{4{k_4}^2(n+m+nm)-1}. \tag{40}$$

From equation (40), it is noticed that the ratio of sound speed $\frac{dp}{d\varepsilon}$ is positive for $0 \le m \le 1.9$ *and* $0 \le n \le 1.7$, the model is a stable.

Graphs are plotted for particular values of the physical parameters and other integration constants.

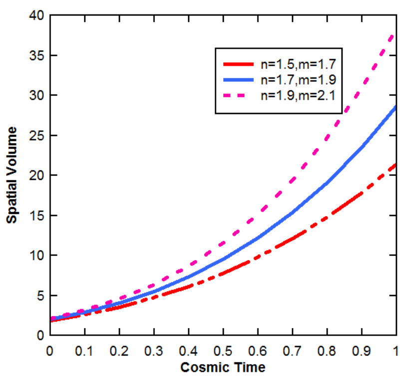

Fig. 1. Plot of spatial volume vs. cosmic time for $k_4 = k_5 = 1$.

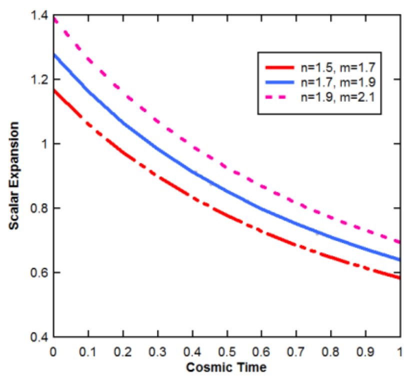

Fig. 2. Plot of expansion scalar vs. cosmic time. for $k_4 = k_5 = 1$.

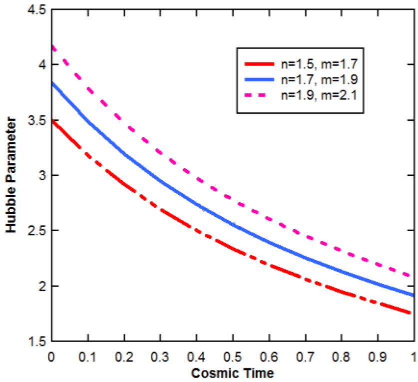

Fig. 3. Plot of Hubble Parameter vs. Cosmic Time for $k_4 = k_5 = 1$.

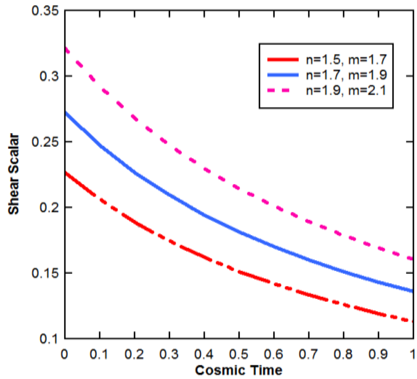

Fig. 4. Plot of Shear Scalar vs. Cosmic Time. for $k_4 = k_5 = 1$.

Conclusion

In this paper, we have considered the Bianchi type-II cosmological model in Barber's second self-creation theory in the presence of the macroscopic body. For solving the field equations, the relation between metric coefficients, i.e., $A = B^n, C = B^m$ and radiation universe is used. Also, it is interesting to note that initially, spatial volume is constant, and as t gradually increases, volume increase (Fig. 1) and the scalar expansion θ (Fig. 2), Hubble parameter H (Fig. 3) decrease, and finally, they vanish when $t \to \infty$. The shear scalar (Fig. 4) was large at the time of the big bang. Therefore, the shape of the Universe was initially different than the present shear scalar and tended to be constant for a large time. The Universe is anisotropic. From equation (40), it is clear that the ratio of sound speed $\frac{dp}{d\varepsilon}$ is positive for $0 \leq m \leq 1.9$ and $0 \leq n \leq 1.7$, the model is stable and the vorticity of the model along *x, y, z,* and *t*-axes is zero. Hence, the model is non-rotational throughout the evolution of the Universe. The Universe is expanding with the increase of cosmic time. From equations (16) and (17) it is observed that the pressure and energy density are decreasing the cosmic time function.

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