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Five-Dimensional Cosmological Model with One Dimensional Cosmic String Coupled with Zero Mass Scalar Field in Lyra Manifold

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Abstract

A five-dimensional Bianchi type-III string cosmological model is studied with a one-dimensional cosmic string in the presence of zero mass scalar field in the context of the Lyra manifold. Exact solutions of Einstein's field equations are obtained by assuming quadratic equation of state (EoS) of the form $p = k\rho^2 + \rho$, where k is a constant and strictly $k > 0$. The physical and geometrical aspects of the investigated model are analyzed in detail.

Keywords: One-dimensional cosmic string; Zero mass scalar field; Higher dimensional Bianchi- III universe.

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1. Introduction

At the time of phase transition in the early universe, the universe's temperature lowered, and the symmetry of the universe was broken spontaneously, which gives topologically stable defects known as vacuum domain walls, strings, and monopoles [1]. As a result, string cosmological models have received significant attention from researchers due to their importance in structure formation in the early stages of the universe's evolution.

By introducing the gauge function into the structure-less Manifold, Lyra [2] proposed a modification of Riemannian geometry, which shows a remarkable similitude to Weyl's geometry [3]. In consequent investigations, Sen [4], Sen, and Dunn [5] expressed a new scalar-tensor theory of gravitation and constructed an analog of Einstein's field equations based on Lyra's geometry. Halford [6] has shown that the scalar-tensor treatment based on Lyra's geometry predicts the same effects as in general relativity. At present, it is fascinating to study string cosmology in five-dimensional space-time in the framework of general relativity in addition to Lyra geometry. Numerous authors have studied different cosmological models in various theories of gravitation along with zero mass scalar field. Ram *et al*. [7] presented anisotropic dark energy with a massive scalar field in the evolution of a spatially homogeneous Bianchi type-VI_0 cosmological model in the

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framework of Lyra geometry. Reddy [8] investigated a spatially homogeneous and anisotropic Bianchi type-V dark energy model in the presence of scalar meson fields in general relativity. Katore *et al.* [9] examined a cylindrically symmetric Einstein-Rosen cosmological model with bulk viscosity and zero-mass scalar field in Lyra geometry. Mete [10] studied the cosmological model in the Lyra manifold. Reddy *et al.* [11] discussed a new dark energy model in a five-dimensional Kaluza-Klein anisotropic space-time in the presence of scalar-meson fields in general relativity. Kiran *et al.* [12] discussed spatially homogeneous and anisotropic Bianchi type-III bulk viscous fluid string cosmological model in $f(R,T)$ gravity. Singh *et al.* [13] considered a five-dimensional spherically symmetric space-time within the framework of Saez-Ballester theory, wherever minimal dark energy-matter interaction occurs. Bhabor *et al.* [14] studied five-dimensional Bianchi type-I string cosmological models with bulk viscous fluid in Lyra geometry. Mete *et al.* [15] studied a five-dimensional plane-symmetric bulk viscous string cosmological model in general relativity. Lambat *et al.* [16] explored Bianchi type VIo inflationary model with scalar field and flat potential in the context of Lyra geometry.

The equation of state in relativity and cosmology, which is however the relationship among temperature, pressure, combined matter, energy, and energy density for any region of space, plays a vital role. Reddy *et al.* [17] have studied the Bianchi type-I cosmological model with the quadratic equation of state in the general theory of relativity. Ananda *et al.* [18] examined cosmological dynamics and dark energy with a quadratic EoS; they have shown that the behavior of the anisotropy at the singularity found in the brane scenario can be recreated in the general relativistic context by considering the general form of quadratic EoS. Several researchers, including Rao *et al.* [19], Adhav *et al.* [20], and Mollah *et al.* [21,22], have investigated various cosmological models in five-dimensional space-time in different aspects. Recently Beesham *et al*. [23] and Baro *et al*. [24] pointed out different cosmological models with a quadratic EoS in general and modified theories of gravitation. Kantowaski-Sachs cosmological model with bulk viscous and cosmic string in the context of $f(T)$ gravity has been investigated by Bhoyar *et al.* [25]. Basumatary *et al*. [26] have explored the Bianchi type VI_0 dark energy model with a particular form of scale factor in Sen-Dunn's theory of gravitation. Brahma *et al*. [27] have reviewed Bianchi type-V dark energy cosmological model with the electromagnetic field in Lyra based on $f(R,T)$ gravity.

Inspired by the above discussions, in this paper, a higher dimensional bulk viscous fluid cosmological model with one-dimensional cosmic string is studied in the presence of interacting zero mass scalar field in Lyra geometry with the aid of Bianchi type-III space-time. Exact solutions ofEinstein's field equations are obtained by assuming quadratic EoS in the form$p = k\rho^2 + \rho$, where k is a constant and strictly $k > 0$. Some physical and geometrical properties are also examined with present-day observations.

2. Metric and Field Equations

Bianchi type- III cosmological models in five-dimension are given by

$$ds^2 = -dt^2 + a^2 dx^2 + b^2(e^{-2x} dy^2 + dz^2) + c^2 dm^2 \tag{1}$$

where $a, b, c$are functions of time t and m is extra dimensions.

The field equations based on Lyra's Manifold as presented by Sen [4] in the normal gauge are written as,

$$R_{ij} - \frac{1}{2}Rg_{ij} + \frac{3}{2}\varphi_i\varphi_j - \frac{3}{4}g_{ij}\varphi^k\varphi_k = -T_{ij} \quad (2)$$

where φ_i is displacement vector field of the Lyra manifold defined as,

$$\varphi_i = (0,0,0,0,\beta(t)) \quad (3)$$

and other symbols have their usual meaning, as in Riemannian geometry.

Let us consider the energy-momentum tensor for a bulk viscous fluid containing a one-dimensional cosmic string corresponding to interacting zero-mass scalar fields as

$$T_{ij} = (\rho + \bar{p})u_i u_j + \bar{p}g_{ij} - \lambda x_i \rightleftarrows x_j + \left(\psi_{,i}\psi_{,j} - \frac{1}{2}g_{ij}\psi_{,\alpha}\psi^{,\alpha}\right) , \quad (4)$$

whereρ is the rest energy density for a cloud of strings loaded with particles which are given by$\rho = \lambda + \rho_p$, ρ_p being particle energy density, λ is tension in the string and

$$\bar{p} = p - \xi\theta, \quad (5)$$

where $\bar{p}$ is the total pressure which consists of the proper pressure p, ξ stands for the coefficient of bulk viscosity, $H$is the Hubble parameter, $\theta = u^i_{;i}$ the scalar expansion factor, u^i the five velocity vectors and x^i is a space-like vector that represents the directions of the strings.

The velocity vector u^i and direction of the string $x^i$are given by

$$u^i = (0,0,0,0,1) \quad (6)$$

and

$$x^i = (0,0,0,\frac{1}{c},0) \quad (7)$$

In the co-moving coordinates system, $u^i$and x^isatisfy the conditions

$$u_i u^i = -x_i x^i = -1 \quad \text{and} \quad u^i x_i = 0 \quad (8)$$

The scalar field ψ satisfies the equation

$$\psi^{i}_{;\,i} = 0 \quad (9)$$

Let us assume an equation of state (EoS) in the general form $p = p(\rho)$ for the matter of distribution and considered it in the quadratic form [17] as

$$p = k\rho^2 + \rho, \quad (10)$$

where k is a constant and strictly$k \neq 0$.

In the commoving coordinate system, equation (4) reduces to

$$T_1^1 = T_2^2 = T_3^3 = \bar{p}; \quad T_4^4 = \bar{p} + \lambda; \quad T_5^5 = -\rho \text{ and } T_j^i = 0 \text{ for } i \neq j$$

If $R(t)$ be the average scale factor then the spatial volume is

$$V = ab^2c = R^4 \quad (11)$$

From equations (2) - (8), line element (1) leads the following system of equations

$$2\frac{\ddot{b}}{b}+\frac{\ddot{c}}{c}+\frac{\dot{b}^2}{b^2}+2\frac{\dot{b}\dot{c}}{bc}+\frac{3}{4}\beta^2=-\bar{p}-\frac{1}{2}\dot{\psi}^2 \tag{12}$$

$$\frac{\ddot{a}}{a}+\frac{\ddot{b}}{b}+\frac{\ddot{c}}{c}+\frac{\dot{a}\dot{b}}{ab}+\frac{\dot{b}\dot{c}}{bc}+\frac{\dot{a}\dot{c}}{ac}+\frac{3}{4}\beta^2=-\bar{p}-\frac{1}{2}\dot{\psi}^2 \tag{13}$$

$$\frac{\ddot{a}}{a}+\frac{\ddot{b}}{b}+\frac{\ddot{c}}{c}+\frac{\dot{a}\dot{b}}{ab}+\frac{\dot{b}\dot{c}}{bc}+\frac{\dot{a}\dot{c}}{ac}-\frac{1}{a^2}+\frac{3}{4}\beta^2=-\bar{p}-\frac{1}{2}\dot{\psi}^2 \tag{14}$$

$$\frac{\ddot{a}}{a}+2\frac{\ddot{b}}{b}+\frac{\dot{b}^2}{b^2}+2\frac{\dot{a}\dot{b}}{ab}-\frac{1}{a^2}+\frac{3}{4}\beta^2=-\bar{p}+\lambda-\frac{1}{2}\dot{\psi}^2 \tag{15}$$

$$\frac{\dot{b}^2}{b^2}+2\frac{\dot{a}\dot{b}}{ab}+2\frac{\dot{b}\dot{c}}{bc}+\frac{\dot{a}\dot{c}}{ac}-\frac{1}{a^2}-\frac{3}{4}\beta^2=\rho+\frac{1}{2}\dot{\psi}^2 \tag{16}$$

$$\frac{\dot{a}}{a}=\frac{\dot{b}}{b} \tag{17}$$

$$\ddot{\psi}+\dot{\psi}\left(\frac{\dot{a}}{a}+2\frac{\dot{b}}{b}+\frac{\dot{c}}{c}\right)=0, \tag{18}$$

where overhead dot (.) denotes ordinary differentiation with respect to time *t*.

3. Solution of Field Equations

Equation (17) gives
$a=lb$, where l is an integrating constant.
Without loss of generality, taking $l=1$, yields

$$a=b \tag{19}$$

Using equation (19) in equations (12) to (18) gives rise to

$$2\frac{\ddot{b}}{b}+\frac{\ddot{c}}{c}+\frac{\dot{b}^2}{b^2}+2\frac{\dot{b}\dot{c}}{bc}+\frac{3}{4}\beta^2=-\bar{p}-\frac{1}{2}\dot{\psi}^2 \tag{20}$$

$$2\frac{\ddot{b}}{b}+\frac{\ddot{c}}{c}+\frac{\dot{b}^2}{b^2}+2\frac{\dot{b}\dot{c}}{bc}-\frac{1}{b^2}+\frac{3}{4}\beta^2=-\bar{p}-\frac{1}{2}\dot{\psi}^2 \tag{21}$$

$$3\frac{\ddot{b}}{b}+3\frac{\dot{b}^2}{b^2}-\frac{1}{b^2}+\frac{3}{4}\beta^2=-\bar{p}+\lambda-\frac{1}{2}\dot{\psi}^2 \tag{22}$$

$$3\frac{\dot{b}\dot{c}}{bc}+3\frac{\dot{b}^2}{b^2}-\frac{1}{b^2}-\frac{3}{4}\beta^2=\rho+\frac{1}{2}\dot{\psi}^2 \tag{23}$$

$$\ddot{\psi}+\dot{\psi}\left(3\frac{\dot{b}}{b}+\frac{\dot{c}}{c}\right)=0. \tag{24}$$

From the above five highly nonlinear independent equations with seven unknowns $b, c, \beta, \lambda, \rho, \bar{p}, \psi$ can be seen. In order to overcome the difficulties due to the nonlinear nature of the field equations using $\xi=\xi_0$ (Constant). Our aim is to derive the exact solutions of the field equations by assuming the following two extra conditions:
i) The special law of variation for the Hubble parameter proposed by Berman [28], which yields a constant deceleration parameter defined by

$$q=-\frac{R\ddot{R}}{\dot{R}^2}=\text{Constant} \tag{25}$$

ii) Assuming that the shear scalar is proportional to scalar expansion ($\sigma \propto \theta$) proposed by Collins [29], which leads to a condition

$$b = c^n \tag{26}$$

where $n \neq 0$ is constant, and it takes care of the anisotropic nature of the model.
Solving equation (25) yields to

$$R = (Lt + M)^{\frac{1}{1+q}},\ q \neq -1 \tag{27}$$

where $L(\neq 0)$ and M are constants of integration.
This equation point towards the condition for the accelerated expansion of the universe is $(1 + q) > 0$.
The scalar field ψ in the model is obtained as

$$\psi = \psi_0 \left(\frac{1+q}{q-3}\right) t^{\frac{q-3}{1+q}} \tag{28}$$

From equations (11), (19), and (26), the solution to the scale factors $a, b$and c are obtained as

$$a = (Lt + M)^{\frac{4n}{(1+q)(3n+1)}}, q \neq 1, b = (Lt + M)^{\frac{4n}{(1+q)(3n+1)}}, q \neq 1 \tag{29}$$

and

$$c = (Lt + M)^{\frac{4}{(1+q)(3n+1)}},\ q \neq 1 \tag{30}$$

with the suitable choice of constants, the scale factors can be written as,

$$a = b = t^{\frac{4n}{(1+q)(3n+1)}}, \quad c = t^{\frac{4}{(1+q)(3n+1)}}, \quad q \neq 1 \tag{31}$$

After a suitable choice of coordinates and constants, the metric (1) can be written as

$$ds^2 = -dt^2 + t^{\frac{8n}{(1+q)(3n+1)}}(dx^2 + e^{-2x}dy^2 + dz^2) + t^{\frac{8}{(1+q)(3n+1)}}dm^2 \tag{32}$$

4. Physical and Kinematical Parameters

In this section, we compute the values of physical and kinematical parameters of the model (32), which plays a significant role in the discussion of the cosmological model of the universe.
The spatial volume for the model is given by

$$V = t^{\frac{4}{(1+q)}} \tag{33}$$

The expansion scalar (θ), the shear scalar (σ^2) and the average Hubble's parameter(H) are obtained as

$$\theta = \frac{4}{(1+q)} t^{-1} \tag{34}$$

$$\sigma^2 = \frac{6(n-1)^2}{(1+q)^2(3n+1)^2 t^2} \tag{35}$$

$$H = \frac{1}{(1+q)} t^{-1}. \tag{36}$$

Now by using quadratic EoS in the form $p = k\rho^2 + \rho$, the energy density ρ, and the string tension density λ are given as

$$\rho = \frac{1}{\sqrt{k}}\left\{-\frac{4(3-q)(2n+1)}{(1+q)^2(3n+1)}t^{-2} + t^{\frac{-8n}{(1+q)(3n+1)}} + \frac{4\xi_0}{(1+q)}t^{-1}\right\}^{\frac{1}{2}} \quad (37)$$

$$\lambda = \frac{4(3-q)(n-1)}{(1+q)^2(3n+1)}t^{-2} - t^{\frac{-8n}{(1+q)(3n+1)}} \quad (38)$$

By using the equations (37) and (38) in the relation $\rho = \lambda + \rho_p$, the energy density of particle ρ_p is obtained as

$$\rho_p = \frac{1}{\sqrt{k}}\left\{-\frac{4(3-q)(2n+1)}{(1+q)^2(3n+1)}t^{-2} + t^{\frac{-8n}{(1+q)(3n+1)}} + \frac{4\xi_0}{(1+q)}t^{-1}\right\}^{\frac{1}{2}} - \frac{4(3-q)(n-1)}{(1+q)^2(3n+1)}t^{-2} + t^{\frac{-8n}{(1+q)(3n+1)}} \quad (39)$$

The displacement vector (β) can be obtained from equation (23) as

$$\beta^2 = \frac{64n(n+1)}{(1+q)^2(3n+1)^2}t^{-2} - \frac{4}{3}t^{\frac{-8n}{(1+q)(3n+1)}} - \frac{4}{3\sqrt{k}}\left\{-\frac{4(3-q)(2n+1)}{(1+q)^2(3n+1)}t^{-2} + t^{\frac{-8n}{(1+q)(3n+1)}} + \frac{4\xi_0}{(1+q)}t^{-1}\right\}^{\frac{1}{2}} - \frac{2}{3}\psi_0{}^2 t^{\frac{-8}{(1+q)}} \quad (40)$$

Also, the total pressure $\bar{p}$ and the proper pressure$p$can be derived from equations (10) and (20) as

$$\bar{p} = -\frac{4(3-q)(2n+1)}{(1+q)^2(3n+1)}t^{-2} + t^{\frac{-8n}{(1+q)(3n+1)}} + \frac{1}{\sqrt{k}}\left\{-\frac{4(3-q)(2n+1)}{(1+q)^2(3n+1)}t^{-2} + t^{\frac{-8n}{(1+q)(3n+1)}} + \frac{4\xi_0}{(1+q)}t^{-1}\right\}^{\frac{1}{2}} \quad (41)$$

$$p = -\frac{4(3-q)(2n+1)}{(1+q)^2(3n+1)}t^{-2} + t^{\frac{-8n}{(1+q)(3n+1)}} + \frac{1}{\sqrt{k}}\left\{-\frac{4(3-q)(2n+1)}{(1+q)^2(3n+1)}t^{-2} + t^{\frac{-8n}{(1+q)(3n+1)}} + \frac{4\xi_0}{(1+q)}t^{-1}\right\}^{\frac{1}{2}} + \frac{4\xi_0}{(1+q)}t^{-1} \quad (42)$$

5. Physical Interpretation of the Solutions

From Fig. 1, it has been observed that the Hubble parameter (H) given by equation (36) is always positive. As time t increases, the Hubble parameter (H) constantly decreases, and after some time, that is, whenever $t \to \infty$ it becomes zero.

The behavior of the expansion scalar (θ) by the use of equation (34) is depicted in Fig. 2. It is observed that the expansion scalar (θ) decreases as time t increases, and after some finite time, it vanishes as $t \to \infty$.

From equation (33) and Fig. 3, it has been noticed that the evolution of spatial volume (V) is finite whenever $t = 0$, and it increases with time , and after some finite time whenever time$t \to \infty$, it reaches to infinity.

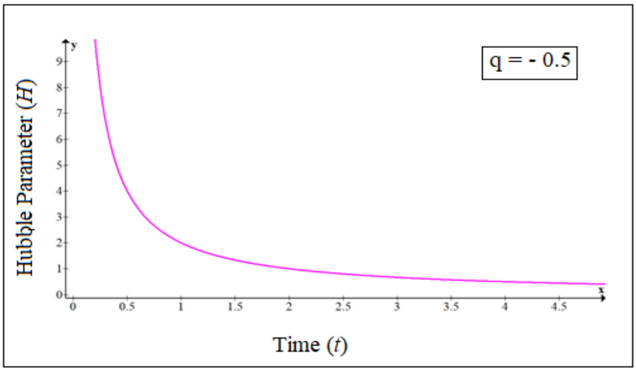

Fig. 1. Variation of Hubble parameter H versus time t.

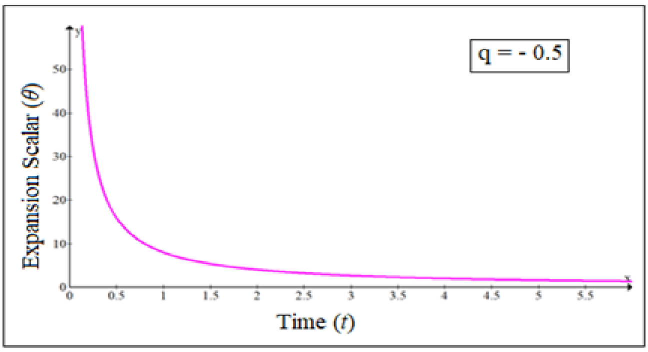

Fig. 2. Variation of expansion scalar θ versus time t.

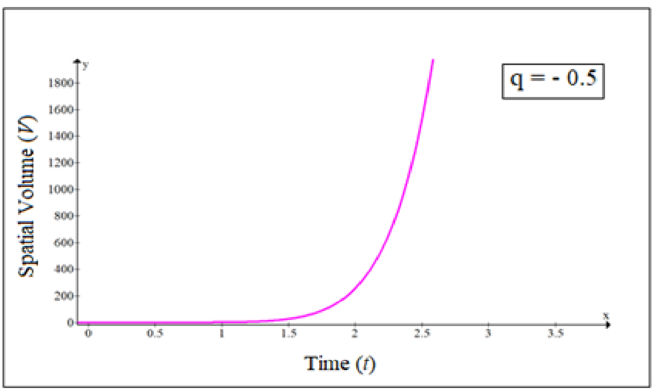

Fig. 3. Variation of spatial volume V versus time t.

6. Conclusion

In this paper, five-dimensional Bianchi type- III metrics in the framework of Lyra geometry with matter source as a bulk viscous fluid with one-dimensional cosmic string has been studied, and exact solutions of Einstein's field equations are obtained by using the quadratic equation of state in the form $p = k\rho^2 + \rho$. It has been observed that the expansion scalar (θ), the Hubble parameter (H), and the displacement vector (β) are always positive and tend to zero whenever t approaches infinity. At an initial epoch $t = 0$, the spatial volume $V = 0$ and increases as time t increases, which exhibits the accelerated expansion of the universe (since $(1 + q) > 0$), and finally, it turns out to be infinity whenever time $t \to \infty$. We also note that $\frac{\sigma}{\theta}$ tends to constant as t tends to infinity for $n \neq 1$ which shows that the anisotropy in the universe is maintained throughout. However, for $n = 1$, the model approaches isotropy.

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