Publications

Available Online

J. Sci. Res. **15** (3), 571-581 (2023)

JOURNAL OF SCIENTIFIC RESEARCH

www.banglajol.info/index.php/JSR

Bianchi Type-I Cosmological Model with Perfect Fluid in Modified $f(T)$ Gravity

V. G. Mete [1*], P. S. Dudhe[2], V. M. Ingle[3]

[1]Department of Mathematics, R. D. I. K. & K. D. College, Bandera-Amravati (M. S.), India

[2]P. R. M. I. T & R, Badnera-Amravati (M. S.), India

[3]R. D. I. K. & K. D. College, Bandera-Amravati (M. S.), India

Received 14 October 2022, accepted in final revised form 11 June 2023

Abstract

In this paper, the Bianchi type I cosmological models with perfect fluid is investigated in the framework of $f(T)$ theory of gravitation. The functional form of the function $f(T)$ such as $f(T) = T + \beta T^2$ is used for investigation. The physical and kinematical properties of the models are obtained and analyzed. We obtained an accelerating and expanding universe.

Keywords: Bianchi type-I space-time; Theory of gravity; Perfect fluid.

doi: http://doi.org/10.3329/jsr.v15i3.62228 J. Sci. Res. **15** (3), 571-581 (2023)

1. Introduction

Recent cosmological observations indicate that the present observable universe is undergoing an accelerated expansion. It is generally accepted that dark energy, whose origin is still a mystery in modern cosmology, is the reason for this cosmic acceleration. The discovery of the universe's swift expansion shows that it is almost spatially flat and contains roughly 75 % DE, which causes cosmic acceleration. The universe is fairly equally divided throughout with this mysterious energy, which is physically identical to vacuum energy. Many theories have been developed to explain why the cosmic acceleration begins. The most prominent gravitational modification theory is $f(T)$ gravity. The construction of viable modified teleparallel gravity models has developed as an alternative to general relativity, and in recent years, the cosmological applications of this theory have attracted considerable attention in the literature. We still don't fully comprehend the nature of the origin of dark energy. Einstein-Hilbert action is modified by substituting the function of the Ricci scalar for R. These theories come under the purview of $f(R)$ theories of gravitation. Recently proposed modified theories of gravity include $f(G), f(R,G)$and $f(R,T)$. Metric $f(R)$ formalism was used to study the dark energy scenario by Hatkar *et al.* [1]. In the modified $f(G)$ theory of gravitation, FRW domain walls were studied by Katore *et al.* [2]. Another significantly modified theory of

* *Corresponding author*: vmete5622@gmail.com

gravitation is called $f(T)$, where T is a torsion scalar. It is based on Weitzenbock's geometry. The expression of the torsion scalar T in the cosmological background does not include the time derivative of the Hubble parameter H, in contrast to the curvature scalar R of standard general relativity. We note that tetrad components are derived with the Weitzenbock connection in teleparallelism, and metric components are derived with the Levi-Civita connection in the framework of general relativity. This feature offers a significant advantage in the reconstruction procedure of $f(T)$ gravity compared $f(R)$ gravity.

As a result, general relativity is replaced by the teleparallel gravity scenario employing the transformation of the tetrad components to the metric components. To put it differently, the curvature term R from general relativity is transformed into a torsion term T in the teleparallel scenario, and its modified form is transformed from T to $f(T)$ by an arbitrary function in the associated action, known as the $f(T)$ cosmology theory. In this theory, gravitation is attributed to the torsion of a zero-curvature space-time, which acts as a force. Bianchi type-I spatially homogenous models, whose spatial sections are flat, are the most straightforward anisotropic models typically employed to describe the anisotropic effect. An advantage of adopting anisotropic models is their important role in the description of the early stage of the universe. Chirde *et al.* [3] explored the accelerating universe, dark energy, and exponential $f(T)$ gravity. Bianchi type-I homogeneous and anisotropic space-time has been taken into consideration by several researchers. Katore *et al.* [4] studied a higher dimensional Bianchi type-I inflationary universe in general relativity. The study of dark energy in $f(T)$ theories were done by Bamba *et al.* [5], Bianchi type-I metric with massive string was presented by Pradhan *et al.* [6] in general relativity. The accelerating Bianchi-type dark energy cosmological model with cosmic string in $f(T)$ gravity has been studied by Chirde *et al.* [7]. Pawar *et al.* [8] constructed perfect fluid and heat flow in the $f(R,T)$ theory. The Bianchi type-III charged fluid universe in the Brans-Dicke theory of gravitation was examined by Mete *et al.* [9]. Recently Chirde *et al.* [10,11] explored various types of cosmological models in the context of $f(T)$ gravity. Bhoyar *et al.* [12] studied Kantowaski-Sachs cosmological model with bulk viscous and cosmic string in the context of $f(T)$ gravity. Lambat *et al.* [13] investigated Bianchi type VI_0 inflationary model with scalar field and flat potential in the context of Lyra geometry. Brahma *et al.* [14] explored Bianchi type-V dark energy cosmological model with the electromagnetic field in Lyra based on $f(R,T)$ gravity. Mete *et al.* [15] constructed a five-dimensional cosmological model with a one-dimensional cosmic string coupled with zero mass scalar field in the context of the Lyra manifold.

Motivated by the above discussion, we present a cosmological power law solution for the universe's acceleration based on the teleparallel equivalent of general relativity modified as previously mentioned. The functional form of the function $f(T)$ such as $f(T) = T + \beta T^2$ is used and investigated this theory using the cosmic power law scale factor solution. Because this type of solution offers a framework for determining the behavior of more general cosmological solutions in various eras of our universe, such as radiation-dominant, matter-dominant, or dark energy-dominant eras, we are aware that power law solutions play a significant role in standard cosmology.

In this paper, we examine Bianchi type-I space-time with perfect fluid within the framework of $f(T)$ gravity. Preliminary definitions of $f(T)$ gravity is introduced in Section 2. In Section 3, we explore the field equations together with solutions and some physical and kinematic parameters. The conclusions are given in Section 4.

2. Preliminary Definitions and Equation of Motion of $f(T)$ Gravity

In this section, a concise explanation of $f(T)$ gravity, and a thorough derivation of its field equations is given. Let us define the Greek and Latin notations of the Latin subscript as those connected to the space-time coordinates and the tetrad field, respectively. We can define the line element for a general space-time metric as

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu, \tag{1}$$

where $g_{\mu\nu}$ are the components of the metric which is symmetric and possesses ten degrees of freedom. The theory can be expressed either in space-time or in tangent space, allowing us to rewrite the line element that can be transformed into the tetrad described by Minkowski (which represents the dynamic fields of the theory) as follows.

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = \eta_{ij} \theta^i \theta^j \tag{2}$$

$$dx^\mu = e_i^\mu \theta^i, dx^\nu = e_i^\nu \theta^j, \tag{3}$$

where $\eta_{ij} = diag[1, -1, -1, -1]$ is Minkowski metric, $e_i^\mu e_\nu^i = \delta_\nu^\mu$ or $e_i^\mu e_\mu^j = \delta_i^j$. The square root of the metric determinant is given by $\sqrt{-g} = det[\, e_\mu^i] = e$ and the tetrads e_μ^α represent the dynamic fields of the theory. The Weitzenbocks connection components for a manifold, where the Riemann tensor part without the torsion terms is null (contribution of the Levi-Civita connection) and only the nonzero torsion terms exist, are defined as follows

$$\Gamma_{\mu\nu}^\alpha = e_i^\alpha \partial_\nu e_\mu^i = -e_\mu^i \partial_\nu e_i^\alpha \tag{4}$$

which has a zero curvature but nonzero torsion. The main geometrical objects of space-time are constructed from this connection. Through the connection, the components of the tensor torsion are defined by the anti-symmetric part of this connection as

$$T_{\mu\nu}^\alpha = \Gamma_{\mu\nu}^\alpha - \Gamma_{\nu\mu}^\alpha = e_i^\alpha (\partial_\mu e_\nu^i - \partial_\nu e_\mu^i). \tag{5}$$

Also, define the components of the so-called con-torsion tensor as

$$K_\alpha^{\mu\nu} = (-\tfrac{1}{2})(T_\alpha^{\mu\nu} - T_\alpha^{\nu\mu} - T_\alpha^{\mu\nu}). \tag{6}$$

To make clearer the definition of the scalar equivalent to the curvature scalar of Riemannian geometry, we first define a new tensor $S_\alpha^{\mu\nu}$, constructed from the components of the torsion and contortion tensors as

$$S_{\alpha}^{\mu\nu} = \frac{1}{2}(K_{\alpha}^{\mu\nu} - \delta_{\alpha}^{\mu} T_{\beta}^{\beta\nu} - \delta_{\varepsilon}^{\nu} T_{\beta}^{\beta\mu}) \tag{7}$$

Now, we can be able to construct a contraction that is equivalent to the scalar curvature in general relativity. We define then the torsion scalar as

$$T = T_{\mu\nu}^{\alpha} S_{\alpha}^{\mu\nu} \tag{8}$$

Now, we define the action by generalizing the teleparallel gravity, i.e., $f(T)$ theory as

$$S = \int [T + f(T) + L_{Matter}] e d^4 x. \tag{9}$$

Here, $f(T)$ denotes an algebraic function of the torsion scalar *T*. Making the functional variation of the action (9) with respect to the tetrads, we get the following equations of motion

$$S_{\mu}^{\nu\rho} \partial_{\rho} T f_{TT} + [e^{-1} e_{\mu}^{i} \partial_{\rho}(e e_{i}^{\alpha} S_{\mu}^{\nu\rho}) + T_{\lambda\mu}^{\alpha} S_{\alpha}^{\nu\lambda}] f_T + \frac{1}{4} \delta_{\mu}^{\nu} f = 4\pi T_{\mu}^{\nu} \tag{10}$$

The field equation (10) is written in terms of the tetrad and partial derivatives and appears very different from Einstein's equation,

where T_{μ}^{ν} is the energy-momentum tensor, $f_T = {df(T)}/{dT}, f_{TT} = {d^2 f(T)}/{dT^2}$ and by setting $f(T) = a_0 =$ constant, the equations of motion (10) are the same as that of the teleparallel gravity with a cosmological constant, and this is dynamically equivalent to general relativity. These equations clearly depend on the choice made for the set of tetrads.

3. Field Equations for the Bianchi Type-I Model

The line element of homogeneous anisotropic Bianchi type-I is given by

$$ds^2 = dt^2 - A^2(t)dx^2 - B^2(t)(dy^2 + dz^2), \tag{11}$$

where the metric potentials *A* and *B* be functions of cosmic time *t* only.
Let us choose the following set of diagonal tetrads related to the metric (11)

$$[e_{\mu}^{\nu}] = diag[1, \rightleftarrows A, B, B] \tag{12}$$

The determinant of the matrix (11) is

$$e = AB^2 \tag{13}$$

The components of the tensor torsion (5) for the tetrads (11) are given by

$$T_{01}^{1} = \frac{\dot{A}}{A}, T_{02}^{2} = \frac{\dot{B}}{B}, T_{03}^{3} = \frac{\dot{B}}{B} \tag{14}$$

The components of the tensor $S_{\alpha}^{\mu\nu}$ in (7), are given by

$$S_{1}^{10} = \frac{\dot{B}}{B}, S_{2}^{20} = \frac{1}{2}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right), S_{3}^{30} = \frac{1}{2}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right) \tag{15}$$

The corresponding torsion scalar (8) is given by

$$T = -2\left(2\frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\dot{B}^2}{B^2}\right) \tag{16}$$

Here we take a more general perfect fluid stress-energy tensor in the following form.

$$T_{ij} = (p+\rho)u^{\nu}u_{\mu} - p\delta_{\mu}^{\nu}, \tag{17}$$

where u^{ν} is the four-velocity vector, while ρ and p are the energy density and pressure of the fluid, respectively.

Now, the field equations for space-time (11), in the framework of $f(T)$ gravity, is obtained as

$$f + 4f_T\left(2\frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\dot{B}^2}{B^2}\right) = 16\pi\rho \tag{18}$$

$$f + 4f_T\left(\frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\dot{B}^2}{B^2} + \frac{\ddot{B}}{B}\right) + 4\left(\frac{\dot{B}}{B}\right)\dot{T}f_{TT} = -16\pi p \tag{19}$$

$$f + 2f_T\left(3\frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\dot{B}^2}{B^2} + \frac{\ddot{A}}{A} + \frac{\ddot{B}}{B}\right) + 2\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right)\dot{T}f_{TT} = -16\pi p, \tag{20}$$

where the dot (.) denotes the derivative with respect to time *t*.

Finally, here we have three differential equations with five unknowns, namely, A, B, f, ρ, p. The solution of these equations is discussed in the next section. In the following, we define some important physical quantities of space-time.

We assume the analytic relation between the metric coefficients as

$$A = B^n \tag{21}$$

some kinematical space-time quantities, average scale factor (*a*), and volume (*V*), respectively are defined as

$$a = \sqrt[3]{AB^2}, V = a^3 \tag{22}$$

The generalized mean Hubble parameter (*H*), which describes the volumetric expansion rate of the universe, is

$$H = \frac{1}{3}(H_1 + H_2 + H_3), \tag{23}$$

where H_1 , H_2 , H_3 are the directional Hubble parameters.

Eqns. (22) and (23), reduced to

$$H = \frac{1}{3}\frac{\dot{V}}{V} = \frac{1}{3}(H_1 + H_2 + H_3) = \frac{\dot{a}}{a} \tag{24}$$

We discuss the mean anisotropy parameter (A_m) of the form to analyze whether the model approaches isotropy or not.

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i}{H} - 1\right)^2 \tag{25}$$

The expansion scalar (*θ*) and the shear scalar (σ^2) are defined as

$$\theta = u^{\mu}_{;\mu} = 3H \tag{26}$$

$$\sigma^2 = \frac{3}{2}A_m H^2 \tag{27}$$

3.1. *Physical and kinematical parameters*

Model-I: We consider the value of the average scale factor corresponding to the model of the universe as

$$a = t^{\frac{b}{3}} \tag{28}$$

The value of the deceleration parameter is given for the mean scale factor in Eqn. (28) as

$$q = -1 + \frac{3}{b} \tag{29}$$

It is crucial to remember that to determine the values of metric functions, the average scale factor *a(t)* must be known. Since they define how the cosmos works, the deceleration and Hubble parameters are essential for developing cosmological theories. According to recent discoveries, the universe was previously decelerating and is currently accelerating. The value of the deceleration parameter is, therefore, commonly taken to have both a constant and a time-dependent form. Many researchers have found various average scale factor forms and recommended various time-dependent deceleration parameter forms for the model.

For this model, the associated metric coefficients *A* and *B* become

$$A = t^{\frac{bn}{n+2}} \tag{30}$$

$$B = t^{\frac{b}{n+2}} \tag{31}$$

Using Eqns. (30) and (31), we get

$$ds^2 = dt^2 - t^{\frac{bn}{n+2}} dx^2 - t^{\frac{b}{n+2}} (dy^2 + dz^2) \tag{32}$$

The Torsion scalar *(T)* becomes

$$T = \frac{4(2n+1)b^2}{(n+2)^2 t^3} \tag{33}$$

The spatial volume $(V)$is given as

$$V = t^b \tag{34}$$

The mean Hubble parameter (*H*) and the expansion scalar (θ) turn out to be

$$H = \frac{b}{3t} \tag{35}$$

$$\theta = \frac{b}{t} \tag{36}$$

The average scale factor and spatial volume disappear with time $t \to 0$. As time $t \to 0$, the model begins to expand with a zero volume; when time increases expansion scalar decreases, and as the time $t \to 0$, the mean Hubble parameter is initially large and zero at time $t \to 0$,

The expansion scalar $\theta \to 0$ as time $t \to \infty$, which indicates that the universe is expanding with an increase with time *t.*

The mean anisotropy parameter (A_m) and shear scalar (σ^2) are given by

$$A_m = \frac{2(n-1)^2}{(n+2)^2} \tag{37}$$

$$\sigma^2 = \frac{(n-1)^2 b^2}{3(n+2)^2 t^2} \tag{38}$$

It has been found that the spatial volume vanishes at starting time $t = 0$, expands with time, and becomes infinitely massive at $t = \infty$. Compared to the shear scalar, which is time-dependent and decreases with time as the universe expands, the mean anisotropy parameter is independent of time *t* and remains constant throughout the universe's evolution from early to infinite expansion. This indicates how the universe is expanding with the flow of time while slowing its growth rate to a constant value, showing how the universe began to expand at an infinite rate.

The exact general solution for a viable $f(T)$ a model with a quadratic correction term $f(T) = T + \beta T^2$, will be derived in this section. In particular, considering the basic and usual ansatz $f(T) = T + \beta T^2$ is a good approximation in all realistic cases, and we can use data from planetary motions to constrain β. Houndjo and Momeni [16] investigated cylindrical solutions in modified $f(T)$ gravity with the given function.

$$f(T) = T + \beta T^2, f_T = 1 + 2\beta T, f_{TT} = 2\beta$$

The value of energy density and pressure become,

$$\rho = \frac{1}{\chi^2}\left\{\frac{2(2n+1)b^2}{(n+2)^2 t^2}\left[1 - \frac{6\beta(2n+1)b^2}{(n+2)^2 t^2}\right]\right\} \tag{39}$$

$$p = -\frac{1}{\chi^2}\left\{\begin{aligned} &\frac{4(2n+1)b^2\left[(n+2)^2 t^3 + 4\beta(2n+1)b^2\right]}{(n+2)^4 t^6} + \frac{3\left[(n+2)^2 t^3 + 8\beta(2n+1)b^2\right](b^2-b)(n+2)^2 t}{(n+2)^4 t^6} \\ &- \frac{24\beta(2n+1)(3+n)b^3(n+2)t}{(n+2)^4 t^6} \end{aligned}\right\} \tag{40}$$

In the power law expansion of the universe, the energy density (39) is always positive and decreases as cosmic time *t* grows. The energy density of the cosmos is infinitely massive at first, but it diminishes with expansion and disappears entirely at very great expansion, as seen in Fig. 1. It is evident that pressure (40) takes on a negative value during the period of the cosmic time development displayed in Fig. 2. Negative pressure is necessary to produce an antigravity effect and to propel the acceleration, as is clear from observational evidence.

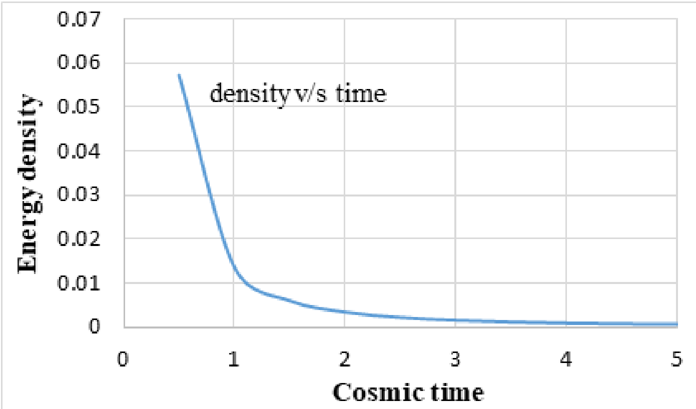

Fig. 1. Energy density versus cosmic time.

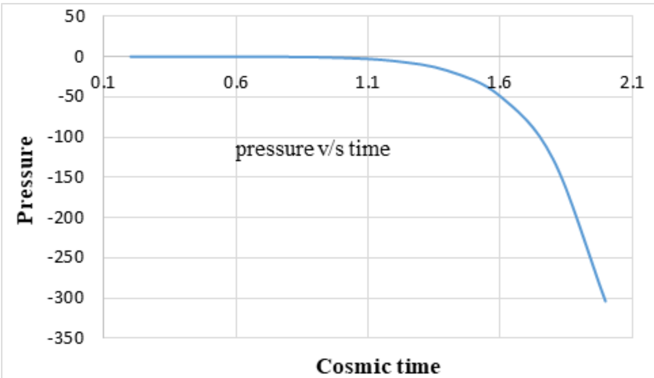

Fig. 2. Pressure versus cosmic time.

Model II: We consider the value of the average scale factor corresponding to the model of the universe as

$$a = (e^{bt} - 1) \tag{41}$$

The value of the deceleration parameter is given for the mean scale factor in Eq. (41) as

$$q = -1 + \frac{1}{e^{bt}} \tag{42}$$

The associated metric coefficients *A* and *B* for this model become

$$A = (e^{bt} - 1)^{\frac{3n}{n+2}} \tag{43}$$

$$B = (e^{bt} - 1)^{\frac{3}{n+2}} \tag{44}$$

Using Eqns. (43)and (44), we get

$$ds^2 = dt^2 - (e^{bt} - 1)^{\frac{3n}{n+2}} dx^2 - (e^{bt} - 1)^{\frac{3}{n+2}} (dy^2 + dz^2) \tag{45}$$

The Torsion scalar *(T)*becomes

$$T = \frac{-18(2n+1)b^2}{(n+2)^2(1-e^{-bt})^2} \tag{46}$$

The spatial volume (V)becomes

$$V = a^3 = (e^{bt} - 1)^3 \tag{47}$$

The mean Hubble parameter (H) and the expansion scalar (θ) turn out to be

$$H = \frac{b}{(1-e^{-bt})} \tag{48}$$

$$\theta = 3H = \frac{3b}{(1-e^{-bt})} \tag{49}$$

The spatial volume disappear with time $t \to 0$. At time $t \to 0$, the model begins to expand with a zero volume. The mean Hubble parameter is initially large and zero as time $t \to \infty$. The expansion scalar $\theta \to 0$ as time$t \to \infty$, which indicates that the universe is expanding with increases with time.

The mean anisotropy parameter (A_m) and shear scalar (σ^2) are given by

$$A_m = \frac{2(n-1)^2}{(n+2)^2} \tag{50}$$

$$\sigma^2 = 3\frac{(n-1)^2 b^2}{(n+2)^2(1-e^{-bt})^2} \tag{51}$$

It has been found that the spatial volume vanishes at starting time $= 0$, expands with time, and becomes infinitely massive at $t = \infty$. Compared to the shear scalar, which is time-dependent and decreases with time as the universe expands, the mean anisotropy parameter is independent of time t. It remains constant throughout the universe's evolution from early to infinite expansion. This indicates how the universe is expanding with the flow of time while slowing its growth rate to a constant value, showing how the universe began to expand at an infinite rate.

The energy density and pressure become,

$$\rho = \frac{1}{\chi^2}\left\{\frac{18(2n+1)b^2}{(n+2)^2(1-e^{-bt})^2}\left[1 - \frac{54\beta(2n+1)b^2}{(n+2)^2(1-e^{-bt})^2}\right]\right\} \tag{52}$$

$$p = -\frac{1}{\chi^2}\left\{\begin{array}{l}\frac{-18(2n+1)b^2\left[(n+2)^2(1-e^{-bt})^2-18\beta(2n+1)^2b^2\right]}{(n+2)^4(1-e^{-bt})^4} \\ +\frac{12b^2\rightleftarrows\left[(n+2)^2(1-e^{-bt})^2-36\beta(2n+1)b^2\right]\left[3n+3+(3-(n+2)e^{-bt})\right]}{(n+2)^4(1-e^{-bt})^4} - \frac{864\beta(2n+1)b^4e^{-bt}}{(n+2)^4(1-e^{-bt})^4}\end{array}\right\} \tag{53}$$

The energy density (52) actions for a suitable selection of constants are shown in Fig. 3. The energy density is a function of time *t* and decreases. According to Fig. 4, it is evident that pressure (53) takes on a negative value as cosmic time progresses. As observational evidence shows, negative pressure is necessary to produce an antigravity effect and drive the acceleration.

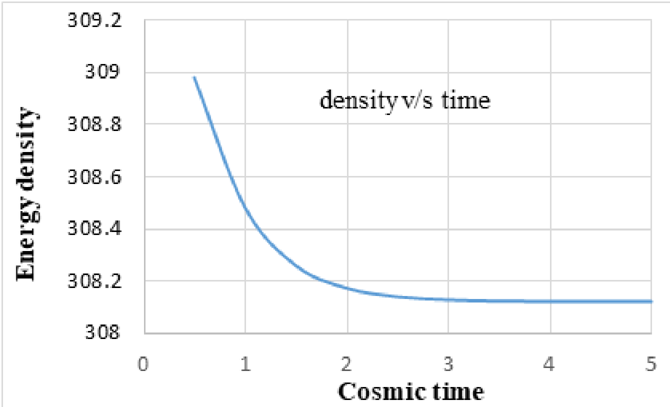

Fig. 3. Energy density versus cosmic time.

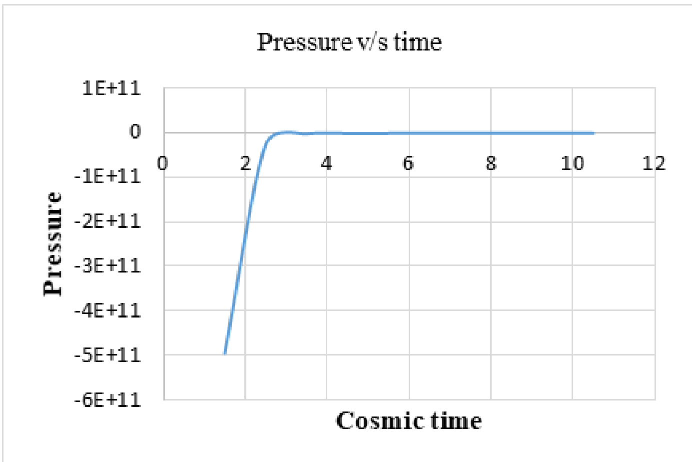

Fig. 4. Pressure versus cosmic time.

4. **Conclusion**

In this paper, we have presented the solution of spatially homogeneous and anisotropic Bianchi type-I cosmological model in the presence of perfect fluid in $f(T)$ theory of gravity. We have derived the exact general solution for a viable $f(T)$ a model with a quadratic correction term $f(T) = T + \beta T^2$. For this purpose, we use scale factor $a = t^{\frac{b}{3}}$,$a = (e^{bt} - 1)$ and $f(T) = T + \beta T^2$. We have evaluated some physical parameters for this solution, such as H, θ, A_m, σ^2. The Hubble parameter is decreasing function of time, i.e., the expansion rate decreases as time increases. It is important to note that in both cases, q is negative. Therefore, the universe is accelerating. For these physical parameters, from both the models, we find that energy density is very large initially, and at a later time, it decreases gradually; for the universe to expand, density must decrease. Pressure assumes a negative value throughout the evolution of cosmic time. As evident from observational data, negative pressure is required to provide an antigravity effect and drive the acceleration.

References

1. S. P. Hatkar, P. S. Dudhe, and S. D. Katore, Found. Phys. **49**, 1067 (2019). https://doi.org/10.1007/s10701-019-00302-x
2. S. D. Katore, S. P. Hatkar, and P. S. Dudhe, Grav. Astrophys. **64**, 103 (2021). https://doi.org/10.1007/s10511-021-09672-5
3. V. R. Chirde and S. H. Sheikh, The Afr. Rev. Phys. **10,** 145 (2015).
4. S. D. Katore, K. S. Adhav, V. G. Mete, and A. Y. Shaikh, Pramana **78**, 101 (2012). https://doi.org/10.1007/s12043-011-0208-y
5. K. Bamba, C. Q. Geng, C. C. Lee, and L. W. Luo, J. Cosmo. Astroparticle Phys. **2011**, ID 021 (2011). https://doi.org/10.1088/1475-7516/2011/01/021
6. A. Pradhan and D. S. Chouhan, Astrophys. Space Sci. **331**, 697 (2011). https://doi.org/10.1007/s10509-010-0478-8
7. S. H. Sheikh, V. R. Chirde, Astrophys. Space Sci. **365**, 1 (2020). https://doi.org/10.1007/s10509-020-03772-y
8. D. D. Pawar, R. V. Mapari, and J. L. Pawade, Pramana **95**, 1 (2021). https://doi.org/10.1007/s12043-020-02058-w
9. V. G. Mete, K. R. Mule, and V. M. Ingle, Int. J. Sci. Res. Phys. Appl. Sci. **6**, 57 (2018). https://doi.org/10.26438/ijsrpas/v6i5.5761
10. V. R. Chirde, S. P. Hatkar, and S. D. Katore, Int. J. Modern Phys. D **29**, ID 2050054 (2020). https://doi.org/10.1142/S0218271820500546
11. V. R. Chirde and S. H. Sheikh, Afr. Rev. Phys. **10,** 475 (2016).
12. S. R. Bhoyar, V. R. Chirde, and S. H. Shekh, J. Sci. Res. **11**, 249 (2019). http://dx.doi.org/10.3329/jsr.v11i3.39220
13. P. M. Lambat and A. M. Pund, J. Sci. Res. **14**, 435 (2022). http://dx.doi.org/10.3329/jsr.v14i2.55557
14. B. P. Brahma and M. Dewri, J. Sci. Res. **14**, 721 (2022). http://dx.doi.org/10.3329/jsr.v14i3.56416
15. V. G. Mete and V. S. Deshmukh, J. Sci. Res. **15**, 351 (2023). http://dx.doi.org/10.3329/jsr.v15i2.61442
16. M. J. S.Houndjo, D. Momeni, and R. Myrzakulov, Int. J. Mod. Phys. D **21**, ID 1250093 (2012). https://doi.org/10.1142/S0218271812500939