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Anisotropic Bianchi Type VI_0 Cosmological Models in a Modified $f(R,T)$ Gravity

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Abstract

We investigate the spatially homogeneous and anisotropic Bianchi type VI_0 cosmological models with modified Holographic Ricci Dark Energy subjected to $f(R,T)$ gravity. We have obtained the solution of the field equations with the help of the power law relation among the scale factors. Dynamical parameters are calculated by using the connection between pressure and energy density. The kinematical implications of the models are also examined. Graphical presentation of different cosmological parameters is shown for distinct values of the parameter of the model. The models studied provide feasible devices to describe the early and present Universe.

Keywords: $f(R,T)$ gravity; Bianchi type VI_0 metric; Modified holographic Ricci dark energy.

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1. Introduction

The astronomical scrutiny discovered that our Universe is passing through the acceleration phase. Firstly, this was observed by High-redshift supernova Ia [1-7]. Cosmic microwave background radiations [8,9] and large-scale structures [10-15] confirm that the Universe is expanding speedily. To have such acceleration, one needs to establish an element with high negative pressure named Dark Energy (DE). Our Universe added up to 1/3 dark matter and 2/3 dark energy. Concerning the character of DE and dark matter, there are various options of the models such as quintessence scalar field models [16,17], K-essence [18,19], phantom field [20,21], tachyon field [22,23], Chaplygin gas [24,25], quintom [26,27] and so on. Also, some DE models using Alternative theories of gravitation have been investigated by Pawar *et al.* [28], Samanta [29,30], and Pawar and Solanke [31]. Einstein's theory of gravitation was highly favorable for explaining the evolution of the Universe, the origin of the Universe, and also for creating cosmological models. But it fails to describe late-time acceleration. Hence multiple efforts have been taken to modify the theory of gravity. For this modification, Gravitational action is reinstated by an arbitrary function of R (Ricci Scalar) and is named $f(R)$ gravity [32].

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Copeland *et al.* [33] gave a full-scale review on $f(R)$ gravity. Certain views of $f(R)$ gravity models have been investigated by Chiba *et al.* [34], which show late time acceleration and early time inflation. One of the impressive and presumptive forms of modified theory is $f(R,T)$ gravity, which was introduced by Harko *et al.* [35] where $L = f(R,T)$, i.e., gravitational Lagrangian L is an arbitrary function of R(Ricci scalar) and T(trace of stress-energy tensor), by deviating the action of gravitational field equations with regard to the metric tensor, field equations of this theory conceivably determined. Samanta and Myrzakulov [36] have analyzed the cosmological model in $f(R,T)$gravity with imperfect fluid. Sharif [37], Keskin [38], Reddy *et al.* [39,40], Chaubey and Shukla [41] have discussed cosmological models in $f(R,T)$ gravity by using distinct Bianchi-type space-times.

Spatially homogeneous and anisotropic cosmological models constitute a major part of explaining the large-scale behavior of the Universe and are also extensively designed to explore the natural picture of the early stages of the Universe in General Relativity. However, Bianchi type I space-time is the leading component for analyzing the feasible results of anisotropy in the primary stages of the Universe as well as the recent observation. Also, certain models (e.g., B-VI_0) explain an anisotropic space-time and develop interest among physicists [42-45]. Amrhashchi *et al.* [46] studied the variable equation of state for B-VI_0 dark energy models. Adhav *et al.* [47] studied Bianchi type VI_0 cosmological models with anisotropic dark energy. Pradhan and Bali [48] and Bali *et al.* [49] studied B-VI_0 space-time in connection with massive strings. Pradhan *et al.* [50] introduced "Dark energy models in an anisotropic B-VI_0 space-time".

Nowadays, the DE models which are encouraging great cosmologists are the holographic dark energy (HDE) models. Many components of holographic dark energy have been studied by Mishra *et al.* [51], Sahu *et al.* [52]. Also, Gao *et al.* [53] presented an HDE model, dubbed as "Ricci dark energy model" (RDE), structured by restoring the future event horizon with the inverse of the Ricci Scalar curvature, i.e., a Holographic Ricci Dark Energy (HRDE) model, in which length- scale is the inverse of the Ricci Curvature Scalar, i.e., $L \approx |R|^{-\frac{1}{2}}$. A new holographic Ricci dark energy model is suggested by Granda and Oliveros [54]. Cohen *et al.* [55], Chen and Jing [56], and Hsu [57] have made some modification to this model by considering the density of DE, which constitute H (Hubble parameter), first and second-order derivative of H (i.e., $\dot{H}, \ddot{H}$) such as,

$$\rho_\Lambda = (3\eta_1 H^{-1}\ddot{H} + 3\eta_2\dot{H} + 3\eta_3 H^2).$$

where η_1, η_2, η_3 are arbitrary dimensionless parameters. Naidu *et al.* [58] have acquired "Dynamics of axially symmetric anisotropic MHRDE model in Brans-Dicke theory of gravitation". Recently, Pawar *et al.* [59] examined "A Modified Holographic Ricci Dark Energy Model in $f(R,T)$ theory of gravity". Basumatary *et al.* [60] have reviewed Bianchi type VI_0 dark energy model with a particular form of scale factor in Sen-Dunn's theory of gravitation. Brahma *et al.* [61] have explored the Bianchi type V dark energy model with the electromagnetic field in Lyra based on $f(R,T)$ theory of gravity. Ugale *et* *al*. [62] studied the Bianchi type IX cosmological model with perfect fluid in $f(R,T)$ theory of gravity.

Inspired by the above discussion, we have studied Bianchi type VI_0 space-time loaded with modified Holographic Ricci Dark Energy in the context of $f(R,T)$gravity. The physical and geometrical aspects of the models are also studied. The objective of this work is to study MHRDE under $f(R,T)$gravity theory, and the profile of the paper is as follows: In section 2, Description of $f(R,T)$ model is given. In section 3, we have presented the metric and field equations. Section 4 contains the solution of field equations. In the last section (sect 5), we have given conclusions of the present work.

2. Description of $f(R,T)$ Gravity

We precisely describe $f(R,T)$ gravity which is a modification of Einstein's theory of relativity. Hilbert-Einstein action for this theory is stated by

$$S = \int L_m \sqrt{-g} d^4x + \frac{1}{2k}\int f(R,T)\sqrt{-g} d^4x \tag{1}$$

here $f(R,T)$ is an arbitrary function of R (Ricci Scalar) and T (trace of the energy-momentum tensor). By differing the action (1) of the gravitational field regarding the metric tensor g_{ij} we got the field equations for $f(R,T)$ gravity by

$$f_R(R,T)R_{ij} - \frac{1}{2}f(R,T)g_{ij} - \left(\Delta_i\Delta_j - g_{ij}\Box\right)f_R(R,T) = kT_{ij} - f_T(R,T)T_{ij} - f_T(R,T)\Theta_{ij} \tag{2}$$

where ∇_i is the derivative along the tangent vector, $f_R(R,T) = \frac{\partial f_R(R,T)}{\partial R}$, $f_T(R,T) = \frac{\partial f_R(R,T)}{\partial T}$, $\Box = \nabla^i\nabla_i$,

$\theta_{ij} = g^{lk}\frac{\delta T_{lk}}{\delta g^{ij}}$, $k = \frac{8\pi G}{c^4}$ here c is the speed at which light travels in a vacuum, G represents the Newtonian Gravitational constant.

$$T_{ij} = T'_{ij} + \bar{T}_{ij} \tag{3}$$

here T_{ij} represents the standard energy-momentum tensor obtained from matter Lagrangian L_m in which T'_{ij} serves as energy-momentum tensor and $\bar{T}_{ij}$ represents the energy-momentum tensor in order to modify holographic Ricci dark energy.

$$T'_{ij} = \rho_M u_i u_j\ ,\ i,j = 1,2,3,4 \tag{4}$$

$$\bar{T}_{ij} = (\rho_\Lambda + p_\Lambda)u_i u_j + g_{ij}p_\Lambda \tag{5}$$

The four-velocity vector in co-varying coordinates $u^i = (0,0,0,1)$ satisfies the condition $u^i\nabla_j u_i = 0$ and $u^i u_i = 1$. In this current work, we have chosen $f(R,T)$in a particular way as $f(R,T) = R + 2f(T)$, here $f(T)$ is the random function of T. So, the analogs field equations are expressed by

$$G_{ij} = R_{ij} - \frac{1}{2}Rg_{ij} = kT_{ij} + 2f_T T_{ij} + [f(T) + 2p_\Lambda f_T]g_{ij} \tag{6}$$

where f_T is the first derivative of f respecting T. Let $f(T) = \lambda T$, here λ is constant. From Equation (5), we have

$$\bar{T}_j^i = diag[-1, \omega_x, \omega_y, \omega_z]\rho_\Lambda$$
$$\bar{T}_j^i = diag[-1, \omega_\Lambda, (\omega_\Lambda + \delta), (\omega_\Lambda + \gamma)]\rho_\Lambda \quad (7)$$

here $\omega_x, \omega_y, \omega_z$ represents the directional equation of state parameter with x, y, z axes respectively,

The equation of state parameter is taken by

$$\omega_\Lambda = \frac{p_\Lambda}{\rho_\Lambda} \quad (8)$$

here we take $\omega_\Lambda = 1$.

3. Metric and Field Equations

The spatially homogeneous and anisotropic Bianchi type VI_0 space-time is stated by the metric,

$$ds^2 = -dt^2 + A^2(t)dx^2 + B^2(t)e^{-2lx}dy^2 + C^2(t)e^{2lx}dz^2 \quad (9)$$

where A, B, and C are the functions of cosmic time t , l is an arbitrary constant.

By making use of a co-moving coordinate system, equation (6) by using equation (4) and equation (7) for the metric (9), credibly described by

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} + \frac{l^2}{A^2} = p_\Lambda + \lambda(8p_\Lambda + \rho_M) \quad (10)$$

$$\frac{\ddot{A}}{A} + \frac{\dot{A}}{A}\frac{\dot{c}}{c} + \frac{\ddot{C}}{C} - \frac{l^2}{A^2} = p_\Lambda + \lambda(8p_\Lambda + \rho_M) \quad (11)$$

$$\frac{\ddot{A}}{A} + \frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\ddot{B}}{B} - \frac{l^2}{A^2} = p_\Lambda + \lambda(8p_\Lambda + \rho_M) \quad (12)$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{C}\dot{B}}{CB} - \frac{l^2}{A^2} = p_\Lambda + \rho_M + \lambda(6p_\Lambda + 3\rho_M + 2\rho_\Lambda) \quad (13)$$

$$\frac{\dot{B}}{B} - \frac{\dot{C}}{C} = 0 \quad (14)$$

An overhead dot (·) shows the derivative respecting to cosmic time t .

For solving the field equations, let us define the dynamical parameters, which are also helpful for the physical analysis of the solution.

The average scale factor $a(t)$ of the Bianchi type VI_0 space-time is defined as

$$a(t) = (ABC)^{\frac{1}{3}} \quad (15)$$

and the spatial volume (V) of the metric is

$$V = a^3(t) = ABC \quad (16)$$

The directional Hubble parameter is given by

$$H_1 = \frac{\dot{A}}{A}, H_2 = \frac{\dot{B}}{B}, H_3 = \frac{\dot{C}}{C} \quad (17)$$

The average Hubble parameter (H) is expressed by

$$H = \frac{1}{3}(H_1 + H_2 + H_3) = \frac{1}{3}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) \tag{18}$$

The scalar expansion θ and shear scalar $\sigma^2$are as follows

$$\theta = 3H = \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) \tag{19}$$

$$\sigma^2 = \frac{1}{2}\left(\frac{\dot{A}^2}{A^2} + \frac{\dot{B}^2}{B^2} + \frac{\dot{C}^2}{C^2}\right) - \frac{1}{6}\theta^2 \tag{20}$$

The mean anisotropic parameter Δ is

$$\Delta = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2 \tag{21}$$

The deceleration parameter q is given by

$$q = -1 + \frac{d}{dt}\left(\frac{1}{H}\right) \tag{22}$$

The positive sign of q denotes the standard decelerating model, and the negative sign shows inflation.

4. Solution of Field Equations

After integration, equation (14) gives

$$B = C \tag{23}$$

Here the integral constant is absorbed in B or C. Using (23), equations (10)-(13) reduce to

$$\frac{2\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} + \frac{l^2}{A^2} = p_\Lambda + \lambda(8p_\Lambda + \rho_M) \tag{24}$$

$$\frac{\ddot{A}}{A} + \frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\ddot{B}}{B} - \frac{l^2}{A^2} = p_\Lambda + \lambda(8p_\Lambda + \rho_M) \tag{25}$$

$$\frac{2\dot{A}\dot{B}}{AB} + \frac{\dot{B}^2}{B^2} - \frac{l^2}{A^2} = p_\Lambda + \rho_M + \lambda(6p_\Lambda + 3\rho_M + 2\rho_\Lambda) \tag{26}$$

There are three equations having four unknowns A, B, ρ_M, p_Λ. Hence to figure out the above equations, we require one physical condition for that power law relation between scale factors A and B can be used, which leads to

$$A = B^n \tag{27}$$

Where n is a constant. Now to find exact solutions of (24)-(26) combining (16) and (27), we obtain

$$A = V^{\frac{n}{n+1}}, \mathrm{B} = V^{\frac{1}{n+1}}. \tag{28}$$

Subtraction of (25) from (24) gives

$$\frac{\ddot{B}}{B} - \frac{\ddot{A}}{A} - \frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\dot{B}^2}{B^2} + \frac{2l^2}{A^2} = 0 \tag{29}$$

Substituting (28) into (29), we get

$$\ddot{V} = \frac{2l^2(n+2)}{n-1} V^{\frac{-n+2}{n+2}} \tag{30}$$

On integrating the above equation, I got the first integral as

$$\int \frac{dV}{V^{\frac{4}{n+2}} + d} = \frac{l(n+2)t}{(n-1)^{\frac{1}{2}}} \tag{31}$$

where d is an arbitrary constant. Assuredly (31) required some restriction on the option of n such as $n > 1$. To solve the above integration, we have to choose either d or n so that (31) be integrable. For that, we consider the subsequent cases.

4.1. *Case I*

When $d = 0$

For this case, the solution of (31) is

$$V = \left(\frac{ln}{\sqrt{n-1}}\right)^{\frac{n+2}{n}} (t + c_1) \tag{32}$$

where c_1 is an arbitrary constant. From (28) and (32) we get the scale factor as

$$A = \frac{ln}{\sqrt{n-1}} (t + c_1) \tag{33}$$

$$B = \left(\frac{ln}{\sqrt{n-1}}\right)^{\frac{1}{n}} (t + c_1)^{\frac{1}{n}} \tag{34}$$

With the above scale factors, the metric (9) becomes

$$ds^2 = -dt^2 + \left(\frac{ln}{\sqrt{n-1}}\right)^2 (t + c_1)^2 dx^2 + \left(\frac{ln}{\sqrt{n-1}}\right)^{\frac{2}{n}} (t + c_1)^{\frac{2}{n}} (e^{-2lx} dy^2 + e^{2lx} dz^2) \tag{35}$$

We figure out the following cosmological parameters for the model (35), as

The average scale factor $a(t)$ determined to be

$$a(t) = \left(\left(\frac{ln}{\sqrt{n-1}}\right)^{\frac{n+2}{n}} (t + c_1)\right)^{\frac{1}{3}} \tag{36}$$

The directional Hubble parameters are

$$H_1 = nH_2 = nH_3 = \frac{1}{(t+c_1)} \tag{37}$$

The Average Hubble parameter is

$$H = \frac{n+2}{3n(t+c_1)} \tag{38}$$

The Scalar expansion θ is

$$\theta = 3H = \frac{n+2}{n(t+c_1)} \tag{39}$$

The Shear scalar is

$$\sigma^2 = \frac{n-1}{\sqrt{3}n(t+c_1)} \tag{40}$$

The mean anisotropy parameter gives a value

$$\Delta = \frac{4}{3}\left(\frac{n-1}{n+2}\right)^2 \tag{41}$$

The deceleration parameter q is

$$q = -\frac{2}{n+2} \tag{42}$$

Now from Equations (8), (10), (11), (33), and (34), the modified holographic Ricci dark energy density and pressure is obtained as

$$\rho_\Lambda = p_\Lambda = \frac{1}{1+8\lambda}\left[\frac{2-n}{n^2(t+c_1)^2} - \frac{2\lambda}{n(t+c_1)^2(1+2\lambda)}\right],\ \lambda \neq \frac{-1}{8} \tag{43}$$

Also, from Equations (8), (10), (11), and (43), we have the matter-energy density expression as

$$\rho_M = \frac{2}{n(1+2\lambda)(t+c_1)^2},\ \lambda \neq \frac{-1}{2} \tag{44}$$

The above outcomes are productive in studying the behavior of the model (35). We have successive statements.

1) The model (35) describes an expanding, accelerating, shearing, and non-rotating Universe. The above model is free from an initial singularity.
2) A positive value of the Hubble parameter H and a negative value of q indicates that Universe is expanding and accelerating.
3) With increasing cosmic time, the average scale factor increases, as shown in Fig. 1, and Ricci dark energy pressure decreases with increasing cosmic time and becomes zero as $t \rightarrow \infty$ (Fig. 3).
4) As cosmic time tends to infinity spatial volume increases (Fig. 1).
5) The average Hubble parameter and dynamical scalar expansion are decreasing functions of cosmic time (Fig. 2).

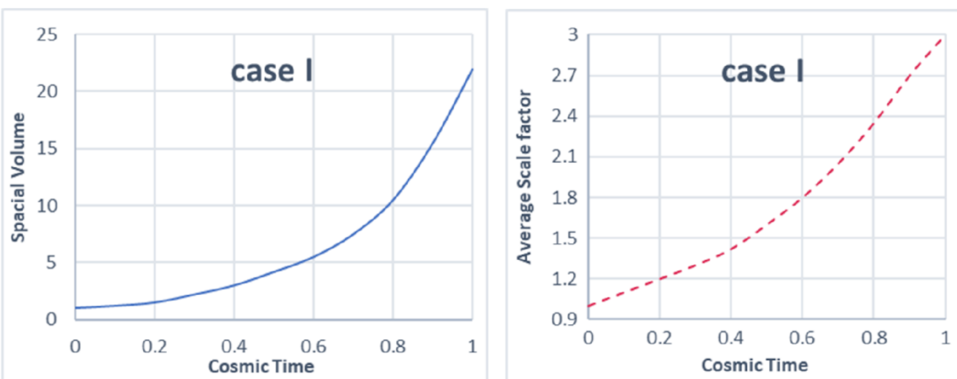

Fig. 1. Variations of spatial volume V (left) and of average scale factor (right) with respect to cosmic time t. All terms have arbitrary units.

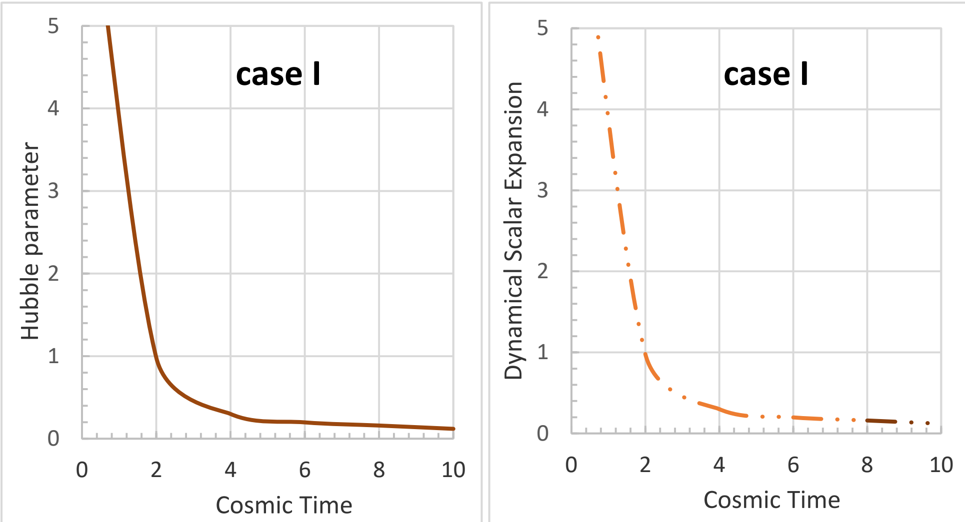

Fig. 2. Graph of Hubble Parameter H (left) and Dynamical scalar expansion θ (right) vs cosmic time t. where $t, a(t)$, H and θ all quantities have arbitrary values.

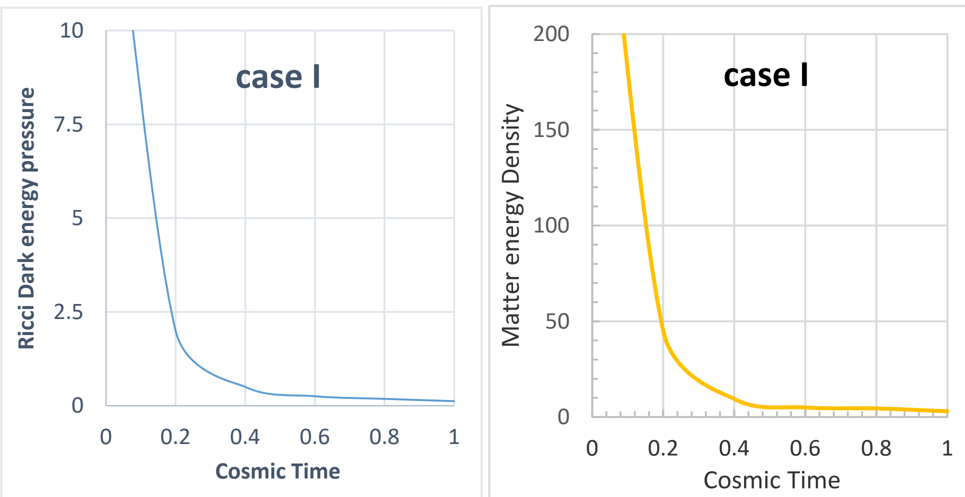

Fig. 3. Variations of Ricci Dark energy pressure p_{Λ}(left) and of Matter energy density ρ_M (right) as a function of cosmic time t. where $t, a(t)$, ρ_M and p_{Λ} all quantities are in arbitrary units.

4.2. *Case II*

When $d \neq 0$

When $d \neq 0$ equation (31) is not integrable for general values of n. Nevertheless, for $n = 2$, it gives

$$\int \frac{dV}{\sqrt{V+d}} = 4lt \tag{45}$$

which, after integration, becomes

$$V = 4l^2t^2 + 2\alpha t + \beta \quad (46)$$

where α and β are arbitrary constants. The constant d is absorbed in β. From (28) and (46), we gained the scale factors as

$$A = \sqrt{4l^2t^2 + 2\alpha t + \beta} \quad (47)$$

$$B = (4l^2t^2 + 2\alpha t + \beta)^{\frac{1}{4}} \quad (48)$$

Hence, the metric (9) can be expressed in the form

$$ds^2 = -dt^2 + (4l^2t^2 + 2\alpha t + \beta)dx^2 + (4l^2t^2 + 2\alpha t + \beta)^{\frac{1}{2}}(e^{-2lx}dy^2 + e^{2lx}dz^2) \quad (49)$$

The expressions for directional Hubble parameters and Average Hubble parameters are as follows

$$H_1 = \frac{4l^2t+\alpha}{4l^2t^2+2\alpha t+\beta} \quad (50)$$

$$H_2 = H_3 = \frac{1}{2}\left(\frac{4l^2t+\alpha}{4l^2t^2+2\alpha t+\beta}\right) \quad (51)$$

$$H = \frac{2}{3}\left(\frac{4l^2t+\alpha}{4l^2t^2+2\alpha t+\beta}\right) \quad (52)$$

The Scalar expansion θ is given by

$$\theta = 2\left(\frac{4l^2t+\alpha}{4l^2t^2+2\alpha t+\beta}\right) \quad (53)$$

The Shear scalar is obtained as

$$\sigma = \frac{1}{\sqrt{3}}\left(\frac{4l^2t+\alpha}{4l^2t^2+2\alpha t+\beta}\right) \quad (54)$$

The deceleration parameter q is given by

$$q = -\frac{2l^2(4l^2t^2+2\alpha t+\beta)}{(4l^2t+\alpha)^2} \quad (55)$$

The mean anisotropy parameter gives value

$$\Delta = \frac{1}{9} \neq 0 \quad (56)$$

Now from Equations (8), (10), (11), (47), and (48), the modified holographic Ricci dark energy density and pressure are as

$$\rho_\Lambda = p_\Lambda = \left[\frac{6\lambda}{(1+8\lambda)(1+2\lambda)} + \frac{5}{(1+8\lambda)}\right]\frac{l^2}{(4l^2t^2+2\alpha t+\beta)} - \frac{5(4l^2t+\alpha)^2}{2(4l^2t^2+2\alpha t+\beta)^2}\left[\frac{1}{2(1+8\lambda)} + \frac{\lambda}{(1+8\lambda)(1+2\lambda)}\right], \ \lambda \neq \frac{-1}{8} \quad (57)$$

Also, from Equations (8), (10), (11), and (56), we acquire the matter-energy density

$$\rho_M = \frac{1}{(1+2\lambda)}\left[\frac{5(4l^2t+\alpha)^2 - 12l^2(4l^2t^2+2\alpha t+\beta)}{2(4l^2t^2+2\alpha t+\beta)^2}\right], \ \lambda \neq \frac{-1}{2} \quad (58)$$

In the model (49),

1) All dynamical parameters decrease the function of time, and for a large time, it eventually tends to zero.
2) As cosmic time goes towards infinity, spatial volume increases (Fig. 4).

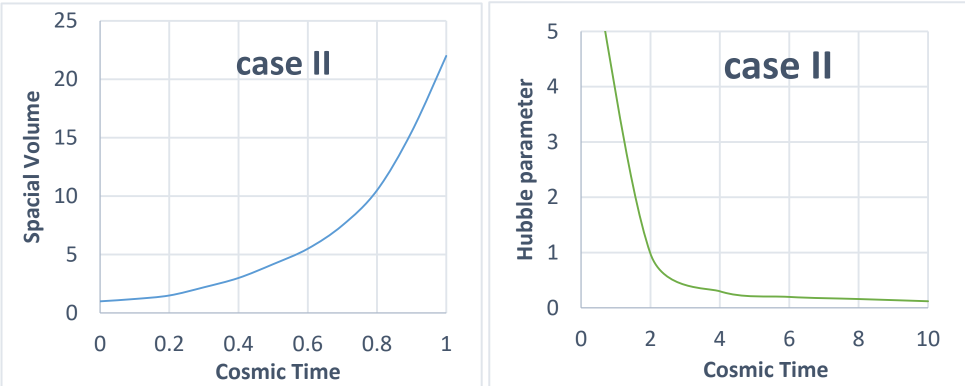

Fig. 4. Variations of spatial volume V (left) and of Hubble parameter (right) (for case II) as a function of cosmic time t . All quantities are in arbitrary units.

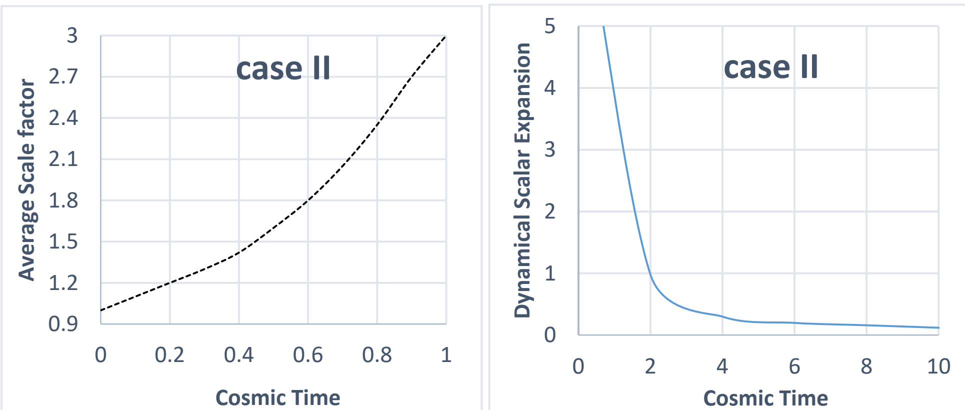

Fig. 5. Graph of Hubble Parameter H (left) and Dynamical scalar expansion θ (right) Vs. cosmic time t where $t, a(t)$ and θ all quantities are in arbitrary units.

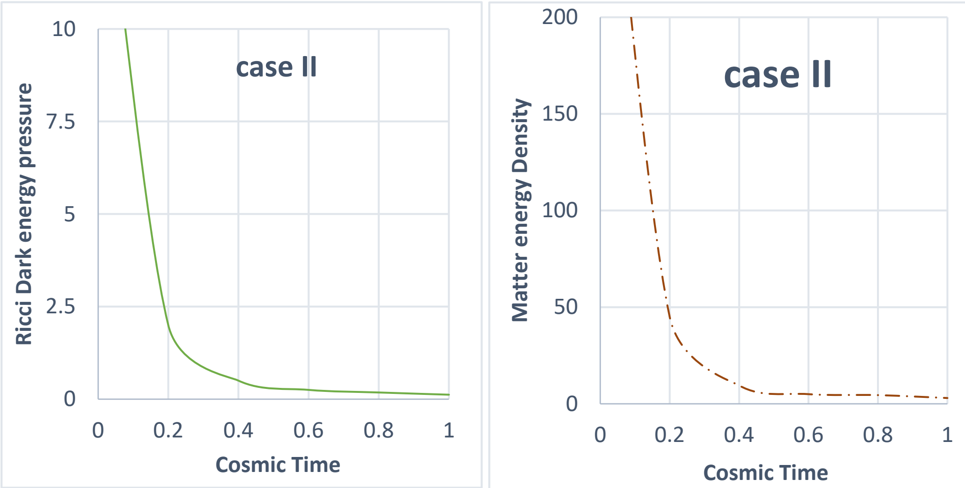

Fig. 6. Variations of Ricci Dark energy pressure p_{Λ}(left) and of Matter energy density ρ_M (right) as a function of cosmic time t. where t, ρ_M and p_{Λ} all quantities are in arbitrary units.

5. Conclusion

In this paper, we have presented the modified Holographic Ricci Dark Energy model in the framework of $f(R,T)$ gravity theory with the help of Bianchi types VI_0 space-time. For the description of the solution of the field equations, we used a power law relation among the scale factors A and B. Here we have acquired exact solutions of field equations for two different cases, which depend on the choice of parameters, i.e., Case I) when $d = 0$ and Case II) when $d \neq 0$. Hence, when $d = 0$, the metric potential $A(t)$ and $B(t)$ of the Universe are constant at $t = 0$, i.e., the obtained model has no initial singularity. This demonstrates that the expansion of the Universe starts with finite volume. A positive value of the Hubble parameter and a negative value of the deceleration parameter shows that Universe is expanding and accelerating. The average scale factor increases as cosmic time increases, and Ricci dark energy pressure decreases. We also observe that the directional Hubble parameter, Average Hubble parameter, and dynamical scalar expansion are decreasing as cosmic time increases. Here, matter energy density is decreasing function of t. It is seen that, $\frac{\sigma^2}{\theta^2}$ gives a finite value, and the anisotropic parameter (Δ) does not vanish. So, this model approaches anisotropy. When $d \neq 0$ at $t = 0$, The metric potential $A(t)$ and $B(t)$ are constant. The deceleration parameter q gives a negative value, and the Hubble parameter gives a positive value. Hence the universe is expanding as well as accelerating. All physical parameters $H, \sigma, \theta, \rho_\Lambda$ and ρ_M are decreasing function of time, and all tend to zero for a long time. As $\frac{\sigma^2}{\theta^2} \neq 0$, in this case, the model does not approach isotropy. Also, the mean anisotropic parameter Δ remains constant all over the evolution of the universe as it is not dependent on cosmic time. Thus, the models studied in this paper provide a feasible device to describe the early and present universe.

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