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Amplification of Electrostatic Wave at the Cost of Kinetic Alfvén Wave Turbulence Energy in Auroral Zone

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Abstract

The low-frequency electromagnetic kinetic Alfvén (KAW) wave is among the most significant waves in the Earth's magnetospheric plasma environment. This study investigates the generation process of high-frequency electrostatic and non-resonant upper-hybrid (UH) waves at the expense of kinetic Alfvén waves in the Earth's auroral zone. The plasma particles that sustain long-term phase alignment with the kinetic Alfvén wave turbulent field are accelerated. The accelerated plasma particles may transfer energy and momentum to the unstable high-frequency wave via a modulation field. Considering a Maxwell-Boltzmann distribution function and using the Vlasov-Poisson system of equations, fluctuating parts of distribution functions resulting from the resonant KAW, modulated wave, and nonlinear upper-hybrid wave are estimated. Data available from various space probes in Earth's magnetosphere are used to calculate the growth rates of the upper-hybrid wave from nonlinear dispersion relations. The calculations demonstrate that the amplification of upper-hybrid wave is possible at the expense of kinetic Alfvén wave turbulent energy.

Keywords: Magnetospheric plasma; Kinetic Alfvén wave; Upper-hybrid wave; Resonant wave, Growth rate.

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1. Introduction

Alfvén wave is one of the most common electromagnetic phenomena in Earth's magnetospheric plasma environment. In a recent study, it has been demonstrated in a laboratory experiment by direct measurement using LAPD (Large Plasma Device) the mechanism of energization of electrons by Alfvén wave under conditions relevant to the auroral zone [1]. The team has claimed that the work has provided a direct and definitive experimental confirmation of the causal relationship between Alfvén waves and accelerated electrons, mainly responsible for the aurora formation. Here, a study on the nonlinear interaction of electrostatic upper-hybrid (UH) wave with kinetic Alfvén wave (KAW) turbulence present in magnetized plasmas has been carried out using the plasma-maser mechanism. The KAW accelerates the thermal particles traveling along the wave

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through resonant interactions. The energized particles transfer their energy and momentum to UH waves through a modulated field.

Originally, Hasegawa [2] introduced the KAW in the study of space plasma physics. Information from satellites/spacecraft like FAST, Freja, Cluster, and Polar observations establish that KAWs exist in magnetospheric plasma environments and play a vital role in the acceleration of particles in the auroral zone [3-6]. In a conventional plasma, the velocity of KAW can easily be controlled as it is less than the velocity of light (c) [7]. Also, the KAW can propagate along and across the magnetic field. As a result, collision-less damping occurs because of its coupling with electrostatic waves. From their works on the plasma maser effect, Saikia *et al*. [8] and Deka [9] have suggested that the up-conversion process is very effective in the presence of electromagnetic low-frequency KAW turbulence compared to electrostatic turbulences. Also, KAWs play significant roles in plasma particles' heating, acceleration, and transport processes in magnetized plasmas [10].

On the other hand, high-frequency electrostatic UH wave fluctuation is a common feature in the inner region of the Earth's magnetosphere environment and the Van Allen radiation belt [11,12]. EXOS-D satellite observations confirm the presence of upper hybrid waves at the height of over 1000 km in the auroral zone and that these waves are continuously related to electromagnetic radiation in the auroral ovals [13]. Space observations have revealed the existence of the Alfvén wave and UH wave instabilities associated with auroral emissions in the Io plasma torus of Jupiter and many other planetary magnetospheres [14,15].

The auroral zone is full of intense plasma activities. The strong magnetization ensures that this zone is low beta plasma. Here electron plasma frequency can be much less than the electron gyrofrequency, i.e., $\omega_{pe} << \Omega_e$. Parallel electric fields, low-density plasmas, strong gradients in density, temperature, and magnetic fields are some of the characteristics of this region enough to attract the attention of researchers [16]. In a recent study using the plasma-maser mechanism, Deka and Gogoi [17] found that energy up-conversion of plasma wave is possible in the mid-altitude ionospheric plasma region.

The plasma-maser is a mode-mode interaction process identified by many authors as an effective tool for studying plasma wave instabilities in space [18-21]. In this energy up-conversion process, energy from the resonant (low-frequency) wave is transferred to the non-resonant (high-frequency) wave even if the frequency difference is high. This process requires only a small anisotropy of resonant waves or particles to work. Both resonant and non-resonant waves must be contained in the system for the plasma-maser effect to occur. In plasma physics, a wave is a resonant wave if the Cherenkov resonance condition $\Omega - \vec{k}.\vec{v} = 0$ is met, and a wave is a non-resonant wave if the scattering and Cherenkov requirements are not met. i.e., $\Omega - \omega - (\vec{K} - \vec{k}).\vec{v} \neq 0$ and $\Omega - \vec{K}.\vec{v} \neq 0$, Where ω and $\vec{k}$ stand for the resonant wave's frequency and wave number, respectively, Ω and $\vec{K}$ stand for the non-resonant wave's equivalent.

In the magnetosphere, most wave energy is found in low-frequency turbulent fields like MHD waves and drift waves [22,23]. So, we can expect numerous radio phenomena in space to be described based on the plasma-maser effect. Plasma-maser effect predicts the possibility of generating high-frequency electrostatic waves from low-frequency waves.

Due to its long parallel wavelength, KAW turbulences may play a more effective role in anomalous transport phenomena for the excitation of non-resonant high-frequency waves in a space plasma environment. For some of its special properties, KAWs can interact with other waves more effectively than non-dispersive MHD Alfvén waves [24]. In this study, it is observed by applying observational data that excitation of UH wave is possible while KAW can undergo damping.

2. Formulation of the Problem

We consider magnetized plasmas and derive the dispersion relation of the upper hybrid mode wave in the presence of kinetic Alfvén wave turbulence. Let the propagating vector of KAW be $\vec{k} = (k_{\perp}, 0, k_{\parallel})$, the wave field $E_l = (E_{\perp}, 0, E_{\parallel})$, and magnetic field $B_l = (0, B_{ly}, 0)$ The Maxwellian particle distribution function in the magnetized plasma is given as

$$f_{0e}(\vec{v}) = \left(\frac{m}{2\pi T_e}\right)^{\frac{3}{2}} exp\left[-\frac{m}{2T_e}\left(v_{\perp}^2 + v_{\parallel}^2\right)\right] \tag{1}$$

Where e refers to electron, f_{0e} is the space and time average part of the distribution, ∥ and ⊥ mean parallel and perpendicular to the external magnetic field, respectively, T_e stands for electron temperature.

The interaction of low-frequency KAW and high-frequency UH wave turbulence is governed by the Vlasov equation:

$$\left[\frac{\partial}{\partial t} + \vec{v}\cdot\frac{\partial}{\partial r} - \frac{e}{m_e}\left(\vec{E} + \frac{\vec{v}\times\vec{B}}{c}\right)\cdot\frac{\partial}{\partial\vec{v}}\right]\cdot F_{0\mathrm{e}}(\vec{r},\vec{v},t) = 0 \tag{2}$$

$\vec{E}$ is self-consistently determined by the Poisson equation:

$$\vec{\nabla}.\ \vec{E}(\vec{r},t) = -4\pi e \int f(\vec{r},\vec{v},t)\, d\vec{v} \tag{3}$$

Where $\vec{v}$ stands for three-dimensional volume element in velocity space.

The unperturbed particle distribution function for electrons, the unperturbed electric and magnetic fields are taken as

$$\vec{F}_{oe} = f_{0e} + \epsilon f_{1e} + \epsilon^2 f_{2e}\,,$$

$$\vec{E}_{0l} = \epsilon\vec{E}_l + \epsilon^2\vec{E}_2\,,$$

$$\text{and } \vec{B}_{0l} = \epsilon\vec{B}_0 + \epsilon^2\vec{B}_l\,. \tag{4}$$

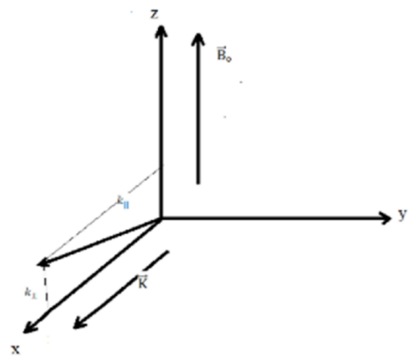

Fig. 1. Geometry of model.

In Fig, $\vec{K} = (K_\perp, 0,0)$ is the propagation vector of the upper hybrid wave, $\vec{k} = (k_\perp, 0, k_\parallel)$ is the propagation vector of KAW.
Here, f_{1e} and f_{2e} are the fluctuating parts and is ϵ a small parameter of the turbulence.
Using these in equation (2) and linearizing to the order of ϵ we get

$$\left[\frac{\partial}{\partial t} + \vec{v}.\frac{\partial}{\partial r} - \frac{e}{m}\left(\vec{E} + \frac{\vec{v}\times\vec{B}}{c}\right).\frac{\partial}{\partial \vec{v}}\right].f_{1e}(\vec{k}) = \frac{e}{m}\left(\vec{E}_l + \frac{\vec{v}\times\vec{B}_l}{c}\right).\frac{\partial}{\partial \vec{v}} \quad f_{0e}. \tag{5}$$

We use the Fourier transforms

$$F(\vec{r}, \vec{v}, t) = \sum_{\vec{k},\, \omega} F(\vec{k}, \omega)\, exp\left[i(\vec{k}.\vec{r} - \omega t)\right], \tag{6}$$

to find the fluctuating parts of the low-frequency turbulent field f_{1e}, as

$$f_{1e}(\vec{k}, \omega) = -\frac{ie}{m}\left[\frac{a\Omega_e}{k_\perp v_\perp} E_{l\perp}(\vec{k}, \omega)\frac{\partial f_{0e}}{\partial v_\perp} + E_{l\parallel}(\vec{k}, \omega)\frac{\partial f_{0e}(\vec{v})}{\partial v_\parallel}\right] X_{a\acute{a}}, \tag{7}$$

Where, $X_{aa'} = \sum_{a,a'} \frac{J_a(\alpha'')J_{a'}(\alpha'')\, exp\{i(a' - a)\theta\}}{a\Omega_e + k_\parallel v_\parallel - \omega + i0^+}$, $\alpha'' = \frac{k_\perp v_\perp}{\Omega_e}$, $i0^+$ is the small imaginary part.

Now we perturb the quasi-steady state by the test non-resonant U.H. wave field $\mu\delta\vec{E}_h$ with propagating vector $\vec{K} = (K_\perp, 0,0)$ electric field $\delta\vec{E}_h = (\delta E_h, 0, 0)$ and a frequency Ω. Thus, the total perturbed electric field, magnetic field, and particle distribution function due to this perturbation are

$$\delta\vec{E} = \mu\delta\vec{E}_h + \mu\epsilon\delta\vec{E}_{lh} + \mu\epsilon^2\Delta\vec{E}, \tag{8}$$

$$\delta\vec{B} = \mu\epsilon\delta\vec{B}_{lh}, \tag{9}$$

$$\delta f = \mu\delta\vec{f}_h + \mu\epsilon\delta f_{lh} + \mu\epsilon^2\Delta f, \tag{10}$$

Using equation (8) and equation (10) in a total electrostatic field $\vec{E} + \delta\vec{E}$, magnetic field $\vec{B} + \delta\vec{B}$, and distribution $f + \delta f$ in (2) and linearizing to the order of $\mu, \mu\epsilon$, and $\mu\epsilon^2$ we get

$$P\delta f_{lh} = \frac{e}{m}\left(\delta\vec{E}_{lh} + \frac{\vec{v}\times\delta\vec{B}_{lh}}{c}\right)\cdot\frac{\partial}{\partial\vec{v}}f_{0e} + \frac{e}{m}\left(\delta\vec{E}_{l} + \frac{\vec{v}\times\delta\vec{B}_{l}}{c}\right)\cdot\frac{\partial}{\partial\vec{v}}\delta f_{h} + \frac{e}{m}\delta\vec{E}_{h}\cdot\frac{\partial}{\partial\vec{v}}f_{1e} \quad (11)$$

$$P\delta f_{h} = \frac{e}{m}\delta\vec{E}_{h}\cdot\frac{\partial}{\partial\vec{v}}f_{0e} \quad (12)$$

$$P\Delta f = \frac{e}{m}\left(\vec{E}_{l} + \frac{\vec{v}\times\vec{B}_{l}}{c}\right)\cdot\frac{\partial}{\partial\vec{v}}\delta f_{lh} + \frac{e}{m}\left(\delta\vec{E}_{lh} + \frac{\vec{v}\times\delta\vec{B}_{lh}}{c}\right)\cdot\frac{\partial}{\partial\vec{v}}\delta f_{1e} \quad (13)$$

Here, $P \equiv \frac{\partial}{\partial t} + \vec{v}\cdot\frac{\partial}{\partial\vec{r}} + \frac{e}{m}\left(\vec{E} + \frac{\vec{v}\times\vec{B}}{c}\right)\cdot\frac{\partial}{\partial\vec{v}}$ and second-order quantities are ignored, considering the random phase approximation principles.

We use Fourier transforms and integrate along the unperturbed orbit to find the fluctuating parts δf_h, over the particle trajectories [25]. Here

$$\delta f_h\left(\vec{K},\Omega\right) = \frac{e}{m}\int_{-\infty}^{0}\delta E_h\frac{\partial f_{0e}(\vec{v})}{\partial v_\perp}[i\{K_\perp(x'-x)-\Omega\tau\}]\mathrm{d}\tau$$

$$= -\frac{ie}{m}\delta E_h\frac{t\Omega_e}{K_\perp v_\perp}\frac{\partial f_{0e}(\vec{v})}{\partial v_\perp}Y_{tt'}. \quad (14)$$

$$Y_{tt'} = \sum_{t,t'}\frac{J_t(\alpha)J_{t'}(\alpha)\,exp\{i(t'-t)\theta\}}{t\Omega_e-\Omega},\ \alpha = \frac{K_\perp v_\perp}{\Omega_e}$$

In a similar way, we calculate δf_{lh} and Δf and use Ampere's equations

$$\nabla\times\delta\vec{B}_{lh} = -\frac{1}{c}\frac{\partial}{\partial t}\delta\vec{E}_{lh} + \frac{4}{c}\vec{J},\qquad \vec{J} = -en\int\vec{v}\sum f_{lhj}\left(\vec{K}-\vec{k}\right)d\vec{v}.$$

to get the Fourier component of the mixed mode as follows:

$$\delta E_{lh}\left(\vec{K}-\vec{k}\right) = -\frac{i4\pi en(\Omega-\omega)}{c^2k_\perp{}^2-(\Omega-\omega)^2}\int v_\parallel\delta f_{lh}\left(\vec{K}-\vec{k}\right)d\vec{v}$$

Which gives

$$\delta E_{lh}\left(\vec{K}-\vec{k}\right) = -\frac{\omega_{pe}^2(\Omega-\omega)}{M\{c^2k_\perp{}^2-(\Omega-\omega)^2\}}\int v_\parallel\left[\left[\frac{p\Omega_e}{K'_\perp v_\perp}\frac{\partial}{\partial v_\perp}E_{l\perp}\left(\vec{k}\right)\left\{\frac{\partial}{\partial v_\perp} - \frac{k_\parallel}{\omega}\left(v_\parallel\frac{\partial}{\partial v_\perp} - v_\perp\frac{\partial}{\partial v_\parallel}\right)\right\}\right.\right.$$

$$+E_{l\parallel}\left(\vec{k}\right)\left\{\frac{\partial}{\partial v_\parallel} + \frac{p\Omega_e}{K'_\perp v_\perp}\frac{k_\perp}{\omega}\left(v_\parallel\frac{\partial}{\partial v_\perp} - v_\perp\frac{\partial}{\partial v_\parallel}\right)\right\}\delta f_h\left(\vec{K}-\vec{k}\right)$$

$$\left.+\frac{p\Omega_e}{K'_\perp v_\perp}\delta E_h\frac{\partial}{\partial v_\perp}f_{1e}\left(\vec{K}-\vec{k}\right)\right]Z_{pp'}\,d\vec{v} \quad (15)$$

Where, $M = 1 + \frac{\omega_{pe}^2(\Omega-\omega)}{c^2k_\perp{}^2-(\Omega-\omega)^2}\int v_\parallel Z_{pp'}\left[\left(\frac{k_\parallel}{|\vec{K}-\vec{k}|}\frac{\partial}{\partial v_\parallel} - \frac{t\Omega_e}{|\vec{K}-\vec{k}|\,v_\perp}\frac{\partial}{\partial v_\perp}\right)\right.$

$$\left.+\frac{|\vec{K}-\vec{k}|}{\Omega-\omega}\left(v_\parallel\frac{\partial}{\partial v_\perp} - v_\perp\frac{\partial}{\partial v_\parallel}\right)f_{0e}(\vec{v})\right]Z_{pp'}d\vec{v} \quad (16)$$

$$Z_{pp'} = \frac{J_p(\alpha')J_{p'}(\alpha')\exp\{i(p'-p)\}}{\Omega-\omega+k_\parallel v_\parallel-p\Omega_e},\ \alpha' = \frac{K'_\perp v_\perp}{\Omega_e}\ and\ K'_\perp = K_\perp - k_\perp$$

3. Nonlinear Dispersion Relation

We use the Poisson equation to get Fourier components of the nonlinear dielectric constant of the UH wave turbulence as

$$\delta E_h\left(\vec{K},\Omega\right) = \frac{i4\pi e}{K}\int\left[\delta f_h\left(\vec{K},\Omega\right) + \Delta f\left(\vec{K},\Omega\right)\right]d\vec{v}\ .$$

From this, we derive the dispersion relation of the electrostatic upper hybrid wave as

$$\epsilon_h\left(\vec{K},\Omega\right) = \epsilon_0\left(\vec{K},\Omega\right) + \epsilon_d\left(\vec{K},\Omega\right) + \epsilon_p\left(\vec{K},\Omega\right). \tag{17}$$

Where ϵ_0, ϵ_d and ϵ_p are the linear part, direct coupling part, and polarization coupling part, respectively.

ϵ_0 is given by

$$\epsilon_0\left(\vec{K},\Omega\right) = 1 - \frac{\omega_{pe}^2}{K_\perp}\int \frac{t\Omega_e}{K_\perp v_\perp}\frac{\partial}{\partial v_\perp} f_{0e}(\vec{v})\, Y_{tt'} d\vec{v} \tag{18}$$

Also,

$$\epsilon_d\left(\vec{K},\Omega\right) = \frac{\omega_{pe}^2}{K_\perp}\left(\frac{e}{m}\right)^2 \int Y_{tt'}\left[E_{l\perp}\frac{t\Omega_e}{K_\perp v_\perp}\left\{\frac{\partial}{\partial v_\perp} - \frac{k_\perp}{\omega}\left(v_\parallel\frac{\partial}{\partial v_\perp} - v_\perp\frac{\partial}{\partial v_\parallel}\right)\right\}\right.$$
$$\left. + E_{l\parallel}\left(\vec{k}\right)\left\{\frac{\partial}{\partial v_\parallel} - \frac{k_\perp}{\omega}\left(v_\parallel\frac{\partial}{\partial v_\perp} - v_\perp\frac{\partial}{\partial v_\parallel}\right)\right\}\right] Z_{pp'}$$
$$\times\left[\left[\frac{p\Omega_e}{K_\perp' v_\perp}\frac{\partial}{\partial v_\perp} E_{l\perp}\left\{\frac{\partial}{\partial v_\perp} - \frac{k_\parallel}{\omega}\left(v_\parallel\frac{\partial}{\partial v_\perp} - v_\perp\frac{\partial}{\partial v_\parallel}\right)\right\}\right.\right.$$
$$\left. + E_{l\parallel}\left(\vec{k}\right)\left\{\frac{\partial}{\partial v_\parallel} + \frac{t\Omega_e}{K_\perp v_\perp}\frac{k_\perp}{\omega}\left(v_\parallel\frac{\partial}{\partial v_\perp} - v_\perp\frac{\partial}{\partial v_\parallel}\right)\right\}\right] Y_{tt'}\frac{t\Omega_e}{K_\perp v_\perp}\frac{\partial}{\partial v_\perp} f_{0e}(\vec{v})$$
$$\left. + \frac{p\Omega_e}{K_\perp' v_\perp}\frac{\partial}{\partial v_\perp} X_{aa'}\left(\frac{a\Omega_e}{k_\perp v_\perp}E_{l\perp}\left(\vec{k},\omega\right)\frac{\partial}{\partial v_\perp} + E_{l\parallel}\left(\vec{k},\omega\right)\frac{\partial}{\partial v_\parallel}\right) f_{0e}(\vec{v})\right] d\vec{v} \tag{19}$$

And

$$\epsilon_p = -\frac{\omega_{pe}^2}{K_\perp^2}\left(\frac{e}{m}\right)^2 \sum_k \frac{\omega_{pe}^2(\Omega-\omega)}{M\left(\vec{K}-\vec{k}\right)\left[c^2 {k_\perp}^2 - (\Omega-\omega)^2\right]} \times Im\{\,(A\times D) + (B\times C)\,\} \tag{20}$$

Where, terms A, B, C, D are:

$$\mathrm{A} = \int Y_{tt'}\left[E_{l\perp}\left(\vec{k}\right)\frac{t\Omega_e}{K_\perp v_\perp}\left\{\frac{\partial}{\partial v_\perp} - \frac{k_\perp}{\omega}\left(v_\parallel\frac{\partial}{\partial v_\perp} - v_\perp\frac{\partial}{\partial v_\parallel}\right)\right\}\right.$$
$$\left. + E_{l\parallel}\left(\vec{k}\right)\left\{\frac{\partial}{\partial v_\parallel} - \frac{k_\perp}{\omega}\left(v_\parallel\frac{\partial}{\partial v_\perp} - v_\perp\frac{\partial}{\partial v_\parallel}\right)\right\}\right] Z_{pp'}$$
$$\times\left[\left(\frac{k_\parallel}{\left|\vec{K}-\vec{k}\right|}\frac{\partial}{\partial v_\parallel} - \frac{t\Omega_e}{\left|\vec{K}-\vec{k}\right| v_\perp}\frac{\partial}{\partial v_\perp}\right) + \frac{\left|\vec{K}-\vec{k}\right|}{\Omega-\omega}\right]\left(v_\parallel\frac{\partial}{\partial v_\perp} - v_\perp\frac{\partial}{\partial v_\parallel}\right)\Big] f_{0e}(\vec{v}) d\vec{v}$$

$$\mathrm{B} = \int Y_{tt'}\left[\frac{t\Omega_e}{K_\perp v_\perp}\frac{\vec{K}-\vec{k}}{\left|\vec{K}-\vec{k}\right|}\frac{\partial}{\partial v_\perp} - \frac{k_\parallel}{\left|\vec{K}-\vec{k}\right|}\frac{\partial}{\partial v_\parallel} - \frac{\left|\vec{K}-\vec{k}\right|}{\Omega-\omega}\left(v_\parallel\frac{\partial}{\partial v_\perp} - v_\perp\frac{\partial}{\partial v_\parallel}\right)\right] X_{a\acute{a}}$$
$$\times\left(\frac{a\Omega_e}{k_\perp v_\perp}E_{l\perp}\left(\vec{k},\omega\right)\frac{\partial}{\partial v_\perp} + E_{l\parallel}\left(\vec{k},\omega\right)\frac{\partial}{\partial v_\parallel}\right) f_{0e}(\vec{v})\Big] d\vec{v}$$

$$\mathrm{C} = \int Z_{pp'}\left[\frac{p\Omega_e}{K_\perp' v_\perp}\frac{\partial}{\partial v_\perp} E_{l\perp}\left(\vec{k}\right)\left\{\frac{\partial}{\partial v_\perp} - \frac{k_\parallel}{\omega}\left(v_\parallel\frac{\partial}{\partial v_\perp} - v_\perp\frac{\partial}{\partial v_\parallel}\right)\right\}\right.$$
$$\left. + E_{l\parallel}\left(\vec{k}\right)\left\{\frac{\partial}{\partial v_\parallel} + \frac{p\Omega_e}{K_\perp' v_\perp}\frac{k_\perp}{\omega}\left(v_\parallel\frac{\partial}{\partial v_\perp} - v_\perp\frac{\partial}{\partial v_\parallel}\right)\right\}\right] Y_{tt'}\frac{t\Omega_e}{K_\perp v_\perp}\frac{\partial}{\partial v_\perp} f_{0e}(\vec{v}) d\vec{v}$$

$$\mathrm{D} = \int Z_{pp'} \left[\frac{p\Omega_e}{K'_\perp v_\perp} \frac{\partial}{\partial v_\perp} \left\{X_{a\acute{a}} \left(\frac{a\Omega_e}{k_\perp v_\perp}\ E_{l\perp}(\vec{k},\omega) \frac{\partial}{\partial v_\perp}\ +\ E_{l\parallel}(\vec{k},\omega)\ \frac{\partial}{\partial v_\parallel}\right) f_{0e}(\vec{v})\right] d\vec{v}$$

In evaluating the above relations, we have considered the fact ω << **Ω** so that the terms containing higher orders of ε can be ignored.

4. The Plasma-maser Interaction

We estimate the growth rate of the U.H. waves with the help of the following formula:

$$\frac{\gamma\,(\vec{K},\Omega)}{\Omega} = -\left[\frac{Im\epsilon_d\,(\vec{K},\Omega) + Im\epsilon_p\,(\vec{K},\Omega) + \frac{1\partial^2\epsilon_0}{2\partial\Omega\partial t}}{\Omega\,\frac{\partial\epsilon_0}{\partial\Omega}\,(\vec{K},\Omega)}\right]_{\Omega_r} \tag{21}$$

The third part of the formula given by equation (21) is due to the reverse absorption effect. In an open system, the electron distribution function is controlled externally by particles from outside, and the contribution from the reverse absorption effect is zero.

Considering the condition $\omega = k_\parallel v_\parallel$ for plasma-maser and assuming $Kv_\parallel > \Omega$, we first calculate ϵ_0 which is the linear part of the dielectric constant of the U H wave. We consider the fact that for UH wave turbulence, the most dominant contribution to Bessel's function is achieved from the terms $a, a' = 0,\ t, t', p, p' = 1$. From equation (18), we have,

$$\epsilon_0\left(\vec{K},\Omega\right) = 1 - \frac{2\omega_{pe}^2}{\Omega^2 - \Omega_e^2}\frac{\lambda_1}{\beta_e}. \tag{22}$$

Where $\lambda_1 = \int_0^\infty J_1(\alpha)^2\ f_{0e}(v_\perp) 2\pi v_\perp dv_\perp$, or $\lambda_1 = I_0(\beta_e) exp\ (-\beta_e)$, $\beta_e = \frac{K_\perp^2 T_e}{m\Omega_e^2}$,

$I_1(\beta_e)$ is modified Bessel function.

Thus, $\quad \frac{\partial\epsilon_0}{\partial\Omega}\left(\vec{K},\Omega\right) = \frac{\partial}{\partial\Omega}\left(1 - 2\frac{\omega_{pe}^2}{\Omega^2 - \Omega_e^2}\frac{\lambda_1}{\beta_e}\right) = \frac{4\Omega\omega_{pe}^2}{(\Omega^2 - \Omega_e^2)^2}\frac{\lambda_1}{\beta_e}$.

The dispersion relation of the non-resonant wave can be deduced from equation (22) as

$$\Omega^2 = \Omega_e^2 + 2\omega_{pe}^2\frac{\lambda_1}{\beta_e}.$$

where $\Omega_e = \frac{eB_0}{m}$ is the cyclotron (angular) frequency.

Now we calculate the growth rate due to the polarization coupling term. Here, in this problem, we have considered E⊥ >> E□. Thus, we get

$$Im\epsilon_p\left(\vec{K},\,\Omega\right) = \frac{\omega_{pe}^2}{K_\perp^2}\left(\frac{e}{m}\right)^2 \sum_k \frac{\omega_{pe}^2(\Omega-\omega)|E_{l\perp}|^2}{M(\vec{K}-\vec{k})[(\Omega-\omega)^2 - c^2{k_\perp}^2]}\{\,(A\times ImD) + (C\times ImB)\,\} \tag{23}$$

First, we calculate M from equation (16) for the small arguments ω and $\vec{k}$, to the lowest order as

$$\frac{-1}{M(\vec{K}-\vec{k})\{c^2{k_\perp}^2 - (\Omega-\omega)^2\}} \cong \frac{1}{c^2 k_\perp^2} \tag{24}$$

Integrating A, B, C, and D in equation (20) by parts and with the help of small argument expansion of
Bessel function, we have

$$A = \frac{E_{l\perp}(\vec{k},\omega)}{(\Omega^2-\Omega_e^2)^2}\frac{K_\perp(K_\perp-k_\perp)^2}{|K_\perp-k_\perp|},\ C = \frac{K_\perp(K_\perp-k_\perp)}{(\Omega^2-\Omega_e^2)^2}E_{l\perp}(\vec{k},\omega),$$

$$ImB = \frac{2\sqrt{\pi}E_{l\parallel}(\vec{k})K_\perp(K_\perp-k_\perp)}{(\Omega^2-\Omega_e^2)v_e^2|k_\parallel|\,|K_\perp-k_\perp|}\left(\frac{\omega}{v_e k_\parallel}\right) exp\left\{\left(-\frac{\omega^2}{{k_\parallel}^2 v_e^2}\right)\right\},$$

and $ImD = \frac{E_{l\parallel}(\vec{k})K_\perp}{\Omega^2-\Omega_e^2}\frac{2\sqrt{\pi}}{v_e^2|k_\parallel|}\left(\frac{\omega}{v_e k_\parallel}\right) exp\left\{\left(-\frac{\omega^2}{k_\parallel^2 v_e^2}\right)\right\}.$

From equation (23), using the dispersion relation $\Omega^2 - \Omega_e^2 = 2\omega_{pe}^2\frac{\lambda_1}{\beta_e}$ and considering dominant terms only, $Im\epsilon_p$ due to polarization coupling term is obtained as,

$$Im\epsilon_p(\overrightarrow{K},\ \Omega) = 4\sqrt{\pi}\left(\frac{e}{m}\right)^2\frac{\omega_{pe}^4(\Omega-\omega)}{(\Omega^2-\Omega_e^2)^3}\frac{E_{l\parallel}^2(\vec{k},\omega)}{c^2k_\perp^2}\ \frac{(K_\perp-k_\perp)^2}{|k_\parallel||\vec{K}-\vec{k}|}\left(\frac{1}{v_e^3}\right)\left(\frac{\omega}{k_\parallel}\right)\left(\frac{k_\perp}{k_\parallel}\right) exp\left\{\left(-\frac{v_A^2}{v_e^2}\right)\right\} \quad (25)$$

The normalized turbulence energy of KAW (W_T) is given by

$$W_T = \sum_k\frac{|B_{ly}|^2}{16\pi nT_e} = \sum_k\frac{E_{l\parallel}^2(\vec{k},\omega)}{16\pi nT_e}\left(\frac{k_\perp}{k_\parallel}\right)^2\left(\frac{c}{v_A}\right)^2 (Q+1)^2 .$$

Where Q is related to the amplitude of the electric field components of KAW and

$$\frac{E_{l\parallel}(\vec{k})}{E_{l\perp}(\vec{k})} = -\frac{k_\parallel}{k_\perp}\frac{T_e}{T_i}\left[1 - I_0(\beta_i)exp(-\beta_i)\right] \sim \frac{k_\parallel}{k_\perp}Q^{-1} .$$

Here, T_i and I_o are ion temperature and modified Bessel function, respectively.
From equation (21) and equation (25), the growth rate $\frac{\gamma_p}{\Omega}$ is given by

$$\frac{\gamma_p}{\Omega} \sim \sqrt{\pi}\left(\frac{\beta_e}{\lambda_1}\right)^2\frac{\omega_{pe}^2}{\Omega_e}\left(\frac{\Omega-\omega}{\Omega}\right)\frac{1}{{v_e}^2k_\perp^3}\frac{(K_\perp-k_\perp)^2}{K_\perp|K_\perp-k_\perp|}\frac{k_\parallel}{|k_\parallel|}\ W_T\left(\frac{v_A}{c}\right)^2\left(\frac{v_e}{c}\right)^2\left(\frac{v_A}{v_e}\right)\frac{Q}{(Q+1)^2}exp\left\{\left(-\frac{v_A^2}{v_e^2}\right)\right\} \quad (26)$$

Where v_A represents the velocity of KAW.
In a similar manner, considering contributions from dominant terms only, the imaginary part $Im\epsilon_d$ from equation (19) is obtained as

$$Im\epsilon_d(\vec{K},\Omega) = 4\sqrt{\pi}\frac{K_\perp'}{k_\perp}\frac{\omega_{pe}^4}{(\Omega^2-\Omega_e^2)^2}\frac{k_\parallel}{|k_\parallel|}\ W_T\frac{Q}{(Q+1)^2}\left(\frac{v_A}{c}\right)^2\left(\frac{v_A}{v_e}\right)exp\left\{\left(-\frac{v_A^2}{v_e^2}\right)\right\} . \quad (27)$$

From equation (21) and equation (27) the growth rate $\frac{\gamma_d}{\Omega}$ is given by

$$\frac{\gamma_d}{\Omega} \sim \frac{\beta_e}{\lambda_1}\sqrt{\pi}\frac{K_\perp'}{k_\perp}\frac{\omega_{pe}^2}{\Omega_e^2}\ W_T\frac{Q}{(Q+1)^2}\frac{k_\parallel}{|k_\parallel|}\left(\frac{v_A}{c}\right)^2\left(\frac{v_A}{c}\right)^2\left(\frac{v_A}{v_e}\right) exp\left\{\left(-\frac{v_A^2}{v_e^2}\right)\right\} \quad (28)$$

5. Results and Discussion

It is observed that the growth of the upper hybrid wave is possible through the plasma maser mechanism at the cost of KAW turbulence energy. Both the direct coupling term and polarization coupling term contribute to the growth. In the present case, the polarization term comes out to be mainly responsible for the destabilization effect in plasma-maser interaction. The dominant role of polarization coupling terms over direct coupling terms was also observed in previous studies, elucidating unstable electrostatic wave generations through theoretical methods [9,20].

It is well established in the study of space plasma that Alfvén wave fluctuations accelerate energetic electrons, either as field-line resonances in the Earth's dipolar magnetic field or as waves propagating towards the auroral ionosphere [2, 26]. They are instrumental in the aurora formation process. In most studies involving the plasma-maser effect, electrostatic turbulences are considered for investigating interactions in magnetized and unmagnetized plasma. But magnetohydrodynamic (MHD) waves have long parallel wavelengths. Hence, they are the dominant source of turbulence energy in space plasma environments [8]. In this paper, KAWs have been considered, which can interact with other waves even more efficiently than MHD Alfvén waves [24]. For instance, whistler wave, electron wave, and ion-acoustic wave turbulence are all amplified in the presence of kinetic Alfvén wave turbulence. [9,27]. Given its significance, Hasegawa and Mima [28] suggested that the presence of KAW in plasma might be regarded as a universal property of large-scale plasmas.

Also, the observational data from the EXOS-D satellite confirmed that upper hybrid waves exist at the height of over 1000 km in the auroral zone [13]. The UH wave is directly connected to electromagnetic radiation while propagating through the plasma environment in an inhomogeneous region. It is to be noticed that auroral plasma is a mixture of cold ionospheric plasma (T.1eV) and hot magnetospheric plasma (T100 eV.). The orbit of the satellite Freza covered the lower part of the auroral acceleration region (600 -1750 km). Thus, information from Freza can be applied to study the interactions of magnetospheric and ionospheric (50 - 1000 km) plasma and the resulting energization.

Data are available from the Freza satellite at an altitude near 1700 km in the high latitude magnetosphere in the northern hemisphere auroral oval are considered for this empirical study. The typical plasma parameters of the Earth's aurora region and that of the topside polar ionosphere are applied to estimate growth rates [29,30].

(a) Parameters for auroral altitude (of 1700 km): $v_e \sim 4.2 \times 10^7 cm/s,\ \ v_A \sim 9 \cdot 25 \times 10^8 m/s,\ \ k_\parallel \sim 1.24 \times 10^{-7}/cm,\ \ \mathrm{K}_\perp \sim 2\pi 10^{-6}\ \mathrm{cm}^{-1},\ \ k_\perp \sim 3 \cdot 1 \times 10^{-6}/cm,$ (b) Plasma parameters of the topside polar ionosphere (from Freja): $\omega_{pe} \sim (1-4) \times 10^6 \mathrm{s}^{-1}, \Omega_\mathrm{e} \sim 5 \times 10^6 \mathrm{s}^{-1}$. Also, (c) we have reasonably assumed $K_\perp - k_\perp \sim 3 \times 10^{-8}$ as $K_\perp >> K_\perp - k_\perp$. and $\Omega_e \sim \Omega$

Using these data and equation (26), the growth rate of the polarization coupling term is obtained as

$$\frac{\gamma_p}{\Omega} \simeq 10^{-3} W_T. \tag{29}$$

And from equation (28), the growth rate of direct coupling term is obtained as

$$\frac{\gamma_d}{\Omega_e} \simeq 10^{-4} W_T. \tag{30}$$

6. Conclusion

The results show that both direct coupling and polarization coupling contribute to the amplification process of UH waves at the cost of KAWs. Further, the contribution of polarization coupling is prominent over direct coupling, and the growth rate is also sufficiently high for amplifying the high-frequency non-resonant wave. Thus, this study implies that the plasma maser effect may be one of the possible mechanisms for predicting the instability of high-frequency electrostatic and electromagnetic waves in Earth's magnetospheric plasma.

References

1. J. W. R Schroeder, G. G. Howes, F. Skiff, T. A. Carter, S. Vincena, S. Dorfman, and C. A. Kletzing, Nat. Commun. **11**, 1668 (2021).
2. A. Hasegawa, J. Geophys, **81**, 5083 (1976). https://doi.org/10.1029/JA081i028p05083
3. C. C. Chaston, C. W. Carlson, R. E. Ergun, and J. P. McFadden, Phys. Scr. T **84**, 64 (2000). https://doi.org/10.1238/Physica.Topical.084a00064
4. C. C. Chaston, J. W. Bonnell, C. W. Carlson, J. P. McFadden, R. E. Ergun, and R. J. Strangeway, J. Geophys. Res. **108**, 8003 (2003). https://doi.org/10.1029/2001JA007537
5. Y. Koyaintsev, N. Ivchenko, K. Stasiewicz, and M. Berthomier, Phys. Scr. T**84**, 151 (2000). https://doi.org/10.1238/Physica.Topical.084a00151
6. J. R. Wygant, A. Keiling, C. A. Cattell, M. Johnson, *et al*., J. Geophys. Res. **105**, 18675 (2000). https://doi.org/10.1029/1999JA900500
7. A. Hasegawa and C. Uberoi, The Kinetic Alfven Wave (Technical Information Center, U.S. Dept. of Energy, 1982).
8. B. J. Saikia, P. N. Deka, and S. Bujarbarua, Contrib. Plasma Phys. **35**, 263 (1995). https://doi.org/10.1002/ctpp.2150350308
9. P. N. Deka, Ph. D. Thesis (Gauhati University, Guwahati. India, 1997).
10. L. Chen, F. Zonca and Y. Lin, Reviews of Modern Plasma Physics **5**, 1(2021) https://doi.org/10.1007/s41614-020-00049-3
11. W. S. Kurth, S. D. Pascuale, J. B. Faden, C. A. Kletzing, et al., J. Geophys. Res. **120**, 904, (2015). https://doi.org/10.1002/2014JA020857
12. H. Y. Peter, S. Kim, J. Hwang, and D. K. Shin, J. Geophys. Res.: Space Phys. **122**, 5365 (2016) https://doi.org/10.1002/2016JA023321**.**
13. H. Oya, A. Morioka, M. Lizima, K. Kobyashi, and T. Ono, J. Geomag. Geoelectr., **42**, 411 (1990). https://doi.org/10.5636/jgg.42.411
14. N. M. Vernet, S. Hoang, and M. Moncuquet, J. Geophys. Res. **98**, 21163 (1993). https://doi.org/10.1029/93JA02587
15. D. J. Gershman, J. E. P. Connerney, S. Levin, G. A. DiBraccio, *et al.*, Geophys. Res. Lett. **46**, 7157 (2019). https://doi.org/10.1029/2019GL082951
16. R. L. Lysak, Auroral Zone Plasma Physics (University of Minnesota, U.S.A. 2001).
17. P. N. Deka and S. J. Gogoi, J. Sci. Res. **11**, 339 (2019). https://doi.org/10.3329/jsr.v11i3.40982
18. M. Nambu, Phys. Rev. Lett. **34**, 387 (1975). https://doi.org/10.1103/PhysRevLett.34.387
19. M. Nambu, J. Phys. Soc. Japan. **50**, 11(1981). https://doi.org/10.1143/JPSJ.50.11
20. P. N. Deka and P. Senapati, Far East J. Math. Sci. **108**, 73 (2018). https://doi.org/10.17654/MS108010073
21. J. K. Deka, Ph. D. Thesis, Dibrugarh University, Dibrugarh, India (2022).
22. D. J. Southwood, Space Sci. Rev. **16**, 413 (1974). https://doi.org/10.1007/BF00171566
23. C. Uberoy, J. Indian Inst. Sci. **75**, 495 (1995).

24. V. Yukhimuk, Y. Voitenko, V. Fedun, and A. Yukhimuk, J. Plasma Phys. **60**, 485 (1998). https://doi.org/10.1017/S0022377898006874
25. N. A. Krall and A. W. Trivelpiece, Principles of Plasma Physics (McGraw Hill Kogakusha Ltd. New York, 1973). https://doi.org/10.1119/1.1987587
26. C. K. Goertz and R. W. Boswell, Geophys. Res. **84**, 7239 (1979). https://doi.org/10.1029/JA084iA12p07239
27. B. K. Saikia, Ph.D. Thesis, Gauhati University, Guwahati. India (1993).
28. A. Hasegawa and K. Mima, J. Geophys. Res., Space Phys. **83**, 1117 (1978). https://doi.org/10.1029/JA083iA03p01117
29. J. Wu, G. L. Huang, and D. Y. Wang, Phys. Rev. Lett. **77**, 4346 (1996). https://doi.org/10.1103/PhysRevLett.77.4346
30. G. V. Lizunov, Y. Khotyantsev, and K. Stasiewiez, Adv. Space Res. **28**, 1649 (2001). https://doi.org/10.1016/S0273-1177(01)00485-9