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Anisotropic L. R. S. Bianchi type-V Cosmological Models with Bulk Viscous String within the Framework of Saez-Ballester Theory in Five-Dimensional Spacetime

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Abstract

When the source for the energy-momentum tensor is a bulk viscous fluid containing one-dimensional cosmic strings, a spatially homogeneous and anisotropic Bianchi type-V cosmological model is considered in this paper in a scalar-tensor theory of gravitation proposed by Saez-Ballester, we derive a determinate solution utilizing the plausible physical conditions: (i) the scalar of spacetime expansion is proportional to the shear scalar; (ii) the baratropic equation of state for pressure and density; and (iii) the bulk viscous pressure is related to the energy density. The model's physical and kinematic features are also discussed.

Keywords: Five dimensions; Saez-Ballester theory; Bulk viscous fluid; Bianchi type-V; String model.

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1. Introduction

Higher-dimensional cosmology models are crucial in many early-stage cosmological problems, and they are one of the most promising research areas for unifying gravity with other natural forces. Higher-dimensional spacetime research suggests that our universe was much smaller at the beginning of evolution than it is now. In addition to the four previously recognized dimensions, extra dimensions have been detected in contemporary research.

Cosmological models with more than four dimensions have attracted much attention recently. The Kaluza-Klein theory [1,2], in which they demonstrated that gravitation and electromagnetism might be combined in a single geometrical framework, has greatly enriched the subject of cosmology. Chodos and Detweiler developed a higher-dimensional cosmological model in which an extra dimension contracts, implying that the contraction results from cosmological evolution [3]. Extra dimensions produce large amounts of entropy during the contraction phase, according to Guth, Alvarez, and Gavela, which presents an alternative solution to the flatness and horizon difficulties compared to the

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standard inflationary scenario [4,5]. Several authors [6–12] found Saez-Ballester field equation solutions for higher-dimensional spacetimes containing a variety of matter fields. Some authors have demonstrated in their research that the four-dimensional spacetimes expand while the fifth dimension contracts or remains constant. Recently [13-15], several authors have studied higher dimensions using different cosmological Models.

Bulk viscosity is crucial in cosmology because it plays a significant role in the fast expansion of the universe, often known as the inflationary phase. There are numerous scenarios in which bulk viscosity could emerge during the evolution of the universe [16]. Viscosity appears when neutrinos disconnect from the cosmic fluid [17], during galaxies formation, and during particle synthesis in the early universe. Several authors [18–23] have studied bulk viscous cosmological models in general relativity. In Einstein's theory of gravitation [24–29], many authors have examined Bianchi-type cosmological models in the presence of cosmic strings and bulk viscosity. Johri and Sudharsan have studied bulk viscous cosmological models in the Brans-Dicke theory of gravitation [30]. Several researchers [31–37] have recently explored the Bianchi type and Kaluza-Klein bulk viscous string cosmological models in the Saez-Ballester theory and the f (R, T) modified theory of gravity.

It is commonly known that topological defects such as cosmic strings, domain barriers, and monopoles emerged shortly after the Big Bang due to spontaneous symmetry breaking in basic particle physics. Strings, line-like structures with particles linked to them, are important in cosmology because they are thought to be possible seeds for galaxy formation in the early stages of the universe's existence. As a result, researchers are increasingly interested in string cosmological models, both in general relativity and alternative theories of gravitation. [6,19,38–46] are some authors who have studied various aspects of string cosmological models in general relativity.

In recent years, there has been a lot of interest in building cosmological models in general relativity and alternative theories of gravitation to understand the development of the universe's structure and the early stages of evolution. Scalar-tensor theories of gravity are notable among the alternative theories of gravity proposed by Brans and Dicke [47] and Saez-Ballester [48]. In Brans-Dicke's theory, a long-range scalar field interacts similarly with all kinds of matter (excluding electromagnetism). In contrast, in Saez-Ballester scalar-tensor theory, the metric is simply coupled with a dimensionless scalar field. This coupling adequately describes weak fields. Despite the scalar field's dimensionless nature, an antigravity regime emerges. In non-flat F.R.W. cosmologies, this theory also proposes a solution to overcome the "missing" matter problem.

Saez and Ballester's field equations for coupled scalar and tensor fields are as follows:

$$R_{ij} - \frac{1}{2} g_{ij} R - \omega \phi^m \left(\phi_{,i} \phi_{,j} - \frac{1}{2} g_{ij} \phi_{,k} \phi^{,k} \right) = -8\pi T_{ij} \tag{1}$$

The scalar field ϕ also fulfills the equation

$$2\phi^m \phi^{;i}_{;i} + n\phi^{n-1} \phi_{,k} \phi^{,k} = 0 \tag{2}$$

And also

$$T^{ij}_{;j} = 0 \quad (3)$$

Here, T_{ij} is the matter's energy-momentum tensor, R_{ij} is the Ricci tensor, R is the Ricci scalar, and the comma and semicolon, respectively signify partial and covariant derivatives. $\boldsymbol{\omega}$ and n are constants.

Many fascinating characteristics of scalar-tensor theories have been extensively addressed in the literature. The most productive area of its application is cosmology, where the scalar field is frequently used as a quintessence field to drive the universe's accelerating phase. The investigation of large-scale cosmic microwave background fluctuations shows that our current physical universe is isotropic, homogeneous, and expanding and that the F.R.W. model accurately represents it. Other analyses, however, uncover some inconsistencies. Analysis of WMAP data sets reveals that the cosmos may have a preferred path. As a result, studying anisotropic Bianchi models is crucial. The study of Bianchi-type models in scalar-tensor gravity theories is also fuelling interest in anisotropic cosmological models of the universe. Several authors have studied Bianchi models and various parts of the Saez-Ballester theory of gravitation [49-59].

We studied the anisotropic L.R.S. five-dimensional Bianchi type-V bulk viscous string cosmological model in the Saez-Ballester scalar-tensor theory of gravity in response to the foregoing discussion and investigations in alternative theories of gravity. Bianchi type-V spacetimes are interesting because they have a more complex structure than physically and geometrically F.R.W. models. These are F.R.W. cosmological models that are open. The following is the structure of this paper: In Section 2, explicit field equations in this theory of gravity are constructed in the presence of bulk viscous fluid with one-dimensional cosmic strings using the Bianchi type-V metric. The solution of the field equations and the model are discussed in Section 3. The model's main physical and kinematic features are described in Section 4. In Section 5, we discuss the physical and geometrical interpretations. Conclusions are included in the last section.

2. Metric and Field Equations

We assume the spatially homogeneous and anisotropic Bianchi type-V spacetime defined by the line element.

$$ds^2 = -dt^2 + A^2dx^2 + e^{2ax}B^2(dy^2 + dz^2) + e^{2ax}C^2d\psi^2 \quad (4)$$

A, B and C are functions of cosmic time t, respectively, and "a" is a constant.

The energy-momentum tensor for a bulk viscous fluid containing one-dimensional cosmic strings is calculated as follows.

$$T_{ij} = (\rho + \bar{p})u_iu_j + \bar{p}g_{ij} - \lambda x_ix_j \quad (5)$$

$$\bar{p} = p - 3\zeta H \quad (6)$$

where ρ is the system's rest energy density, $\boldsymbol{\zeta}$ is the coefficient of bulk viscosity, $\bar{p}$ is the coefficient of bulk viscous pressure, H is Hubble's parameter, and $\boldsymbol{\lambda}$ is the string tension density.

Also u^i denotes the fluid's five velocity vectors and x^i is the direction of the string which satisfies the following conditions

$$u^i u_i = -x^i x_i = -1 \text{ and } u^i x_i = 0 \quad (7)$$

$$u^i = (0,0,0,0,1) \text{ and } x^i = (\tfrac{1}{A}, 0,0,0,0) \quad (8)$$

Here, we assume , $\bar{p}$, and $\boldsymbol{\lambda}$ as functions of time t only.

The field equations (1)–(3) for the metric (4) generate the following independent field equations using co-moving coordinates and Eqs. (5) - (8)

$$2\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + 2\frac{\dot{B}}{B}\frac{\dot{C}}{C} + \frac{\dot{B}^2}{B^2} - 3\frac{a^2}{A^2} - \omega\phi^m\frac{\dot{\phi}^2}{2} = -8\pi(\bar{p} - \lambda) \quad (9)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} + \frac{\dot{A}}{A}\frac{\dot{C}}{C} - 3\frac{a^2}{A^2} - \omega\phi^m\frac{\dot{\phi}^2}{2} = -8\pi\bar{p} \quad (10)$$

$$\frac{\ddot{A}}{A} + 2\frac{\ddot{B}}{B} + 2\frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\dot{B}^2}{B^2} - 3\frac{a^2}{A^2} - \omega\phi^m\frac{\dot{\phi}^2}{2} = -8\pi\bar{p} \quad (11)$$

$$2\frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{C}}{C} + 2\frac{\dot{B}}{B}\frac{\dot{C}}{C} - 6\frac{a^2}{A^2} + \frac{\dot{B}^2}{B^2} + \omega\phi^m\frac{\dot{\phi}^2}{2} = 8\pi\rho \quad (12)$$

$$3\frac{\dot{A}}{A} - 2\frac{\dot{B}}{B} - \frac{\dot{C}}{C} = 0 \quad (13)$$

$$\ddot{\phi} + \dot{\phi}\left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) + \frac{m}{2}\frac{\dot{\phi}^2}{2} = 0 \quad (14)$$

Here, an overhead dot denotes differentiation with respect to t.
The scale factor for the metric (4) and the spatial volume are defined respectively by

$$b = (AB^2C)^{\frac{1}{4}} \quad (15)$$

$$V^4 = AB^2C \quad (16)$$

The important physical quantities are the Hubble parameter H, Expansion scalar θ, anisotropy parameter Δ, and the shear scalar σ^2 are defined by as follows

$$H = \frac{1}{4}\left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) \quad (17)$$

$$\Theta = 4H = \left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) \quad (18)$$

$$\Delta = \frac{1}{4}\sum_{i=1}^{4}\left(\frac{H_i - H}{H}\right)^2 \quad (19)$$

$$2\sigma^2 = \sigma_{ij}\sigma^{ij} = \sum_{i=1}^{4} H_i^2 - 4H^2 = 4\Delta H^2 \quad (20)$$

3. Solutions and the Model

The field equations (9)–(14) are reduced to the independent equations as given below

$$\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} - \frac{\ddot{A}}{A} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} - \frac{\dot{A}}{A}\frac{\dot{B}}{B} - \frac{\dot{A}}{A}\frac{\dot{C}}{C} = 8\pi\lambda \quad (21)$$

$$\frac{\ddot{B}}{B}-\frac{\ddot{C}}{C}+\frac{\dot{A}}{A}\frac{\dot{B}}{B}-\frac{\dot{B}}{B}\frac{\dot{C}}{C}-\frac{\dot{A}}{A}\frac{\dot{C}}{C}+\frac{\dot{B}^2}{B^2}=0 \tag{22}$$

$$2\frac{\dot{A}}{A}\frac{\dot{B}}{B}+\frac{\dot{A}}{A}\frac{\dot{C}}{C}+2\frac{\dot{B}}{B}\frac{\dot{C}}{C}-6\frac{a^2}{A^2}+\frac{\dot{B}^2}{B^2}+\omega\phi^m\frac{\dot{\phi}^2}{2}=8\pi\rho \tag{23}$$

$$A^3=lB^2C \tag{24}$$

$$\dot{\phi}\phi^{\frac{m}{2}}AB^2C=\phi_0 \tag{25}$$

where l and ϕ_0 are integration constants. Without sacrificing generality, the constant l can be chosen as unity, resulting in Eq (24),

$$A^3=B^2C \tag{26}$$

Now eqs. (21) - (25) are a system of five independent equations with seven unknowns A, B, C, p, ρ, ϕ, and $\boldsymbol{\lambda}$. The equations are also extremely non-linear. As a result, we apply the following physically feasible requirements to define a determinate solution:

(i) Collins et al.[60] claim that the shear scalar σ^2 is proportional to scalar expansion θ

$$B=C^k \quad \text{where } k\neq 0 \text{ is a constant} \tag{27}$$

(ii) The combined effect of the appropriate pressure and the bulk viscous pressure on a barotropic fluid can be represented as

$$\bar{p}=p-3\zeta H=\varepsilon\rho \tag{28}$$

Where

$$\varepsilon=\varepsilon_0-\beta(0\leq\varepsilon_0\leq 1), \qquad p=\varepsilon_0\rho \tag{29}$$

Here ε_0 and $\boldsymbol{\beta}$ are constants.

From equation (22), (26) and (27), we define the expressions for the metric coefficients as

$$C=\left[\frac{4(2k+1)}{3}(C_0t+d)\right]^{\frac{3}{4(2k+1)}} \tag{30}$$

$$B=\left[\frac{4(2k+1)}{3}(C_0t+d)\right]^{\frac{3k}{4(2k+1)}} \tag{31}$$

$$A=\left[\frac{4(2k+1)}{3}(C_0t+d)\right]^{\frac{1}{4}} \tag{32}$$

where C_0 and d are integration constants.

The metric (4) can be expressed as (by an appropriate choice of coordinates and constants, such as $C_0=1$ and $d=0$) using Eqs. (30) - (32).

$$ds^2=-dt^2+\left[\frac{4(2k+1)}{3}t\right]^{\frac{1}{2}}dx^2+e^{2ax}\left[\frac{4(2k+1)}{3}t\right]^{\frac{3k}{2(2k+1)}}(dy^2+dz^2)+ e^{2ax}\left[\frac{4(2k+1)}{3}t\right]^{\frac{3}{2(2k+1)}}d\psi^2 \tag{33}$$

4. Physical Properties of the Model

Equation (33) depicts the Bianchi type-V bulk viscous string cosmological model in the Saez-Ballester scalar-tensor theory of gravitation, with the physical and kinematical characteristics listed below, which are important in cosmology discussions:
The Volume is

$$V^4 = \left[\frac{4(2k+1)}{3} t\right] \tag{34}$$

The Hubble parameter is given by

$$H = \frac{1}{4t} \tag{35}$$

Scalar expansion is obtained as

$$\Theta = \frac{1}{t} \tag{36}$$

The mean anisotropy parameter is given by

$$\Delta = \frac{3(k^2-2k+1)}{2(2k+1)^2} \tag{37}$$

Shear scalar is

$$\sigma^2 = \frac{3(k^2-2k+1)}{16t^2(2k+1)^2} \tag{38}$$

The string tension density

$$\lambda = 0 \tag{39}$$

The energy density is given by

$$8\pi\rho = \frac{3\left(14k^2+20k+3\omega\phi_0^2+2\right)}{32t^2(2k+1)^2} - 6a^2\left[\frac{4(2k+1)}{3} t\right]^{-\frac{1}{2}} \tag{40}$$

Pressure is obtained as

$$8\pi p = \varepsilon_0\left[\frac{3\left(14k^2+20k+3\omega\phi_0^2+2\right)}{32t^2(2k+1)^2} - 6a^2\left\{\frac{4(2k+1)}{3} t\right\}^{-\frac{1}{2}}\right] \tag{41}$$

The coefficient of bulk viscosity is

$$8\pi\zeta = \frac{t}{3}(\varepsilon_0 - \varepsilon)\left[\frac{3\left(14k^2+20k+3\omega\phi_0^2+2\right)}{32t^2(2k+1)^2} - 6a^2\left\{\frac{4(2k+1)}{3} t\right\}^{-\frac{1}{2}}\right] \tag{42}$$

The scalar field is given by

$$\phi = \left[\frac{3\phi_0(m+2)}{8(2k+1)}\log\frac{t}{t_0}\right]^{\frac{2}{m+2}} \tag{43}$$

Here t_0 is the integration constant.

The deceleration parameter is given by

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = 3 \tag{44}$$

Also, from equation (36) and (38), we obtain,

$$\lim_{t\to\infty} \frac{\sigma^2}{\theta^2} = \frac{3(k^2-2k+1)}{16(2k+1)^2} \neq 0 \tag{45}$$

The following Graphs have been drawn by choosing $k = \boldsymbol{\varepsilon} = .5,\ \omega = 500,\ \phi_0 = a = \varepsilon_0 = m = t_0 = 1$.

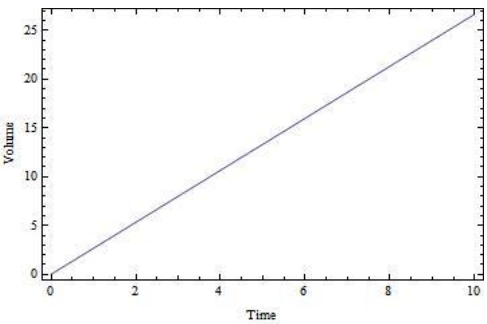

Fig. 1. Volume Vs. Time.

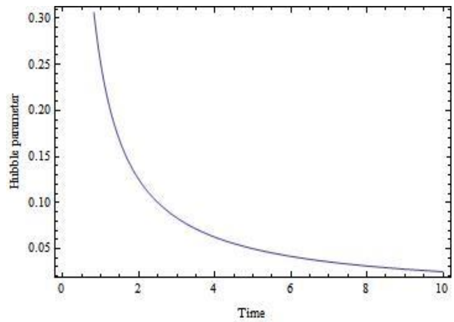

Fig. 2. Hubble Parameter H vs. Time.

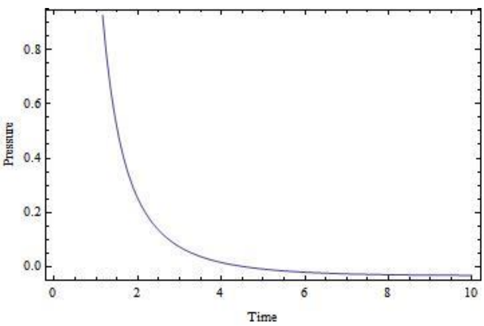

Fig. 3. Scalar expansion θ Vs. Time.

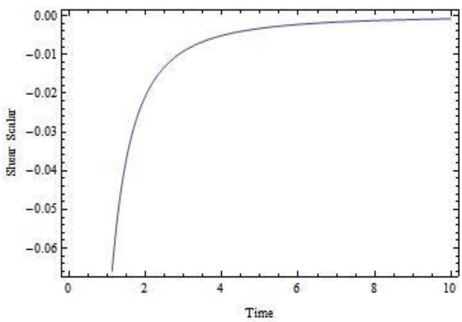

Fig. 4. Shear stress σ^2 Vs. Time.

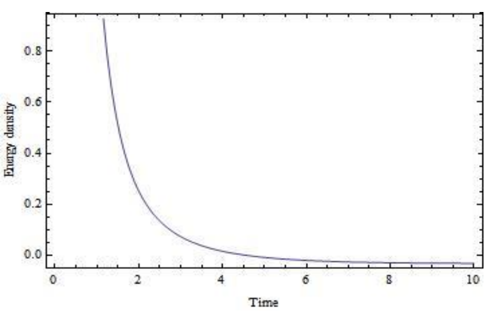

Fig. 5. Energy density ρ vs. time.

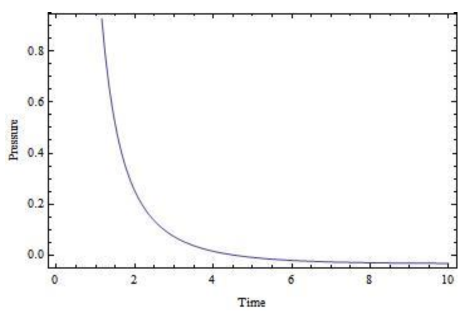

Fig. 6. Pressure p vs. time

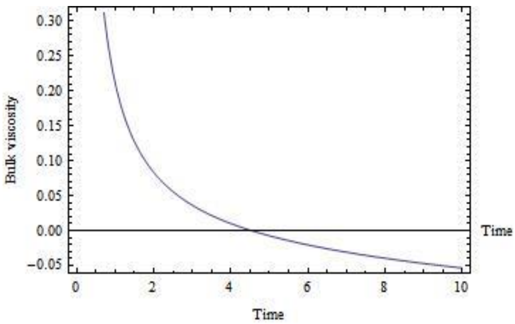

Fig. 7. Bulk viscosity ζ vs. time.

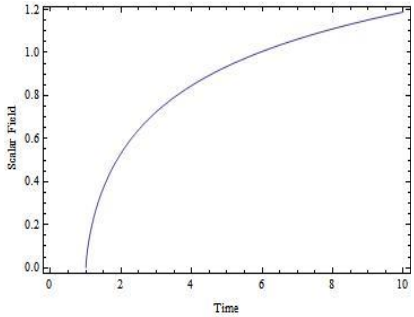

Fig. 8. Scalar field ϕ vs. time.

5. Physical and Geometrical Interpretation

The above results are useful in debating the behavior of the Saez-Ballester cosmological model, which is provided by Eq. (33). From equation (34), we get that, at the initial singularity, the appropriate volume V approaches zero. In the limit of $t \to \infty$, the universe approaches an infinitely large volume as time passes. As a result, the model grows over time. The model does not have an initial singularity, i.e., at time t = 0, as shown in Fig. 1. It is interesting to note from Eq. (39) that strings in this model do not survive. Moreover, it is also obtained that the Hubble parameters H, scalar expansion θ, shear scalar σ^2, energy density ρ, pressure p, and the coefficient of bulk viscosity ζ decreases when time increases. As shown in Figs. 2-7, they all become infinitely large at $t = 0$. The scalar field ϕ, on the other hand, approaches zero at $t \to \infty$, as shown in Fig. 8. The behavior of bulk viscosity follows the well-known trend of decreasing with time, resulting in an inflationary model [23]. Furthermore, from equation (45), we get that the model is anisotropic throughout, which aids in a better understanding of the universe's early stages of evolution. The deceleration parameter in the model turns out to be positive, as shown by equation (44). It is commonly known that if q is positive, the model decelerates normally, whereas if it is negative, the model accelerates. As a result, in this scenario, the model decelerates in the standard way.

6. Conclusion

We explored the spatially homogenous and anisotropic five-dimensional Bianchi type-V cosmological model in the presence of bulk viscous fluid with one-dimensional cosmic strings in Saez and Ballester's scalar-tensor theory of gravitation. It is discovered that the resultant model is singularity-free, shearing, non-spinning, and remains anisotropic throughout the universe's evolution. The physical parameters are infinite at $t = 0$ and tend to be zero at $t \to \infty$. As t becomes infinitely large, the scalar field approaches zero. In this situation, the model decelerates in the standard way. With cosmic time, bulk viscosity falls, leading to an inflationary scenario. In this universe, cosmic strings do not survive. Considering the fact that scalar fields and bulk viscosity have a vital role in describing the early universe, In the context of the Saez-Ballester scalar-tensor theory of gravitation, the model presented here will aid in a better understanding of the universe's evolution in five-dimensional Bianchi type-V spacetime.

7. References

1. T. Kaluza, Zum Unit¨atsproblem der Physik, Sitzungsberichte der Preussischen Akademie der Wissenschaften (Berlin, Germany, 1921) pp. 966.
2. O. T. Klein, Zeitschrift f¨ur Physik. **37**, 895 (1926). https://doi.org/10.1007/BF01397481
3. A. Chodosand and S. Detweiler, Phys. Rev. D **21**, 2167 (1980). https://doi.org/10.1103/PhysRevD.21.2167
4. A. H. Guth, Phys. Rev. D **23**, 347 (1981). https://doi.org/10.1103/PhysRevD.23.347
5. E. Alvarez and M. B. Gavela, Phys. Rev. Lett. **51,** 931 (1983). https://doi.org/10.1103/PhysRevLett.51.931
6. D. R. K. Reddy and R. L. Naidu, Astron. Astrophys. **307**, 395 (2007). https://doi.org/10.1007/s10509-007-9387-x
7. D. R. K. Reddy, P. K. Rao, and R. L. Naidu, Int. J. Theor. Phys. **47**, 2966 (2008). https://doi.org/10.1007/s10773-008-9731-0
8. S. D. Katore, K. S. Adhav, A. Y. Shaikh, and N. K. Sarkate, Int. J. Theor. Phys. **49**, 2358 (2010). https://doi.org/10.1007/s10773-010-0422-2
9. D. R. K. Reddy and R. L Naidu, Astron. Astrophys. **339**, 401 (2012). https://doi.org/10.1007/s10509-012-1007-8
10. G. C. Samanta, S. K. Biswal, and P. K. Sahoo, Int. J. Theor. Phys. **52**, 1504 (2013). https://doi.org/10.1007/s10773-012-1470-6
11. V. U. M. Rao, D. C. P, Rao, and D. R. K. Reddy, Astron. Astrophys. **357**, 1 (2015). https://doi.org/10.1007/s10509-015-2378-4
12. J. Daimary and R. R. Baruah, J. Math. Comput. Sci. **12**, 1 (2022). https://doi.org/10.28919/jmcs/7280
13. K. P. Singh, J. Baro, and A. J. Meitei, Front. Astron. Space Sci. **8,** 1 (2021). https://doi.org/10.3389/fspas.2021.777554
14. P. K. Singh and J. Baro, Indian J. Sci. Technol., **14**, 1239 (2021).
15. V. G. Mete and V. S. Deshmukh, J. Sci. Res. **15**(2), 351 (2023). http://dx.doi.org/10.3329/jsr.v15i2.61442
16. G. F. R. Ellis, In General Relativity and Cosmology, R.K. Sachs (ed) (Academic Press, New York, 1971) **47**, pp. 104.
17. C. W. Misner, Astrophys. J. **51,** 431 (1968). https://doi.org/10.1086/149448
18. J. D. Barrow, Phys. Lett. B **180**, 335 (1986). https://doi.org/10.1016/0370-2693(86)91198-6
19. V. U. M. Rao, G. S. D. Kumari, and K.V.S. Sireesha, Astrophys. Space Sci. **335**, 635 (2011). https://doi.org/10.1007/s10509-011-0776-9
20. R. Maartens, Class. Quantum Gravity **12**, 1455 (1995). https://doi.org/10.1088/0264-9381/12/6/011
21. G. Mohanty and B. D. Pradhan, Int. J. Theor. Phys. **31**, 151 (1992). https://doi.org/10.1007/BF00674348
22. J. K. Singh and S. Ram, Astrophys. Space Sci. **236**, 277 (1996). https://doi.org/10.1007/BF00645149
23. T. Padmanabhan and S. M. Chitre, Phys. Lett. A **120**, 433 (1987). https://doi.org/10.1016/0375-9601(87)90104-6
24. X. X. Wang, Chin. Phys. Lett. **21**, 1205 (2004). https://doi.org/10.1088/0256-307X/21/7/006
25. X. X. Wang, Chin. Phys. Lett. **22**, 29 (2005).
26. X. X. Wang, Chin. Phys. Lett. **23**, 1702 (2006). https://doi.org/10.1088/0256-307X/23/7/013
27. R. Bali and S. Dave, Astrophys. Space Sci. **282**, 461 (2002). https://doi.org/10.1023/A:1020834610024

28. R. Bali and A. Pradhan, Chin. Phys. Lett. **24**, 585 (2007). https://doi.org/10.1088/0256-307X/24/2/079
29. S. K. Tripathy, D. Behra, and T.R. Routray, Astrophys. Space Sci. **325**, 93 (2010). https://doi.org/10.1007/s10509-009-0169-5
30. V. B. Johri and R Sudharsan, Aust. J. Phys. **42**, 215 (1989). https://doi.org/10.1071/PH890215
31. D. R. K. Reddy, C. P. Rao, T. Vidyasagar, and R. B. Vijaya, Adv. High Energy Phys. **2013**, ID 609807 (2013). https://doi.org/10.1155/2013/609807
32. D. R. K. Reddy, R. L. Naidu, K. S. Babu, and K. D. Naidu, Astrophys. Space Sci. **349**, 473 (2014). https://doi.org/10.1007/s10509-013-1619-7
33. M. V. Santhi, T. C. Naidu, and D. C. Paparao, J. Phys. Conf. Ser. **1344**, ID 012036 (2019). https://doi.org/10.1088/1742-6596/1344/1/012036
34. B. P. Brahma and M. Dewri, J. Sci. Res. **14**, 721 (2022). https://doi.org/10.3329/jsr.v14i3.56416
35. K. P. Singh and J. Boro, Adv. Math.: Sci. J. **9**, 4907 (2020). https://doi.org/10.37418/amsj.9.7.55
36. Y. V. Rathod, G. R. Avachar, and S. W. Samdurkar, J. Phys. Conf. Ser. **1913,** ID 012106 (2021). https://doi.org/10.1088/1742-6596/1913/1/012106
37. B. P. Brahma and M. Dewri, Front. Astron. Space Sci. **9**, 1 (2022). https://doi.org/10.3389/fspas.2022.831431
38. P. S. Letelier, Phys. Rev. D. **28**, 2414 (1983). https://doi.org/10.1103/PhysRevD.28.2414
39. M. Kiran and D. R. K. Reddy, Astrophys. Space Sci. **28**, 521 (2013). https://doi.org/10.1007/s10509-013-1459-5
40. D. R. K. Reddy, Astrophys. Space Sci. **286**, 359 (2003). https://doi.org/10.1023/A:1026370732619
41. D. R. K. Reddy and R. L. Naidu, Astrophys. Space Sci. **312**, 99 (2007). https://doi.org/10.1007/s10509-007-9657-7
42. A. Pradhan and D. S. Chouhan, Astrophys. Space Sci. **331**, 697 (2011). https://doi.org/10.1007/s10509-010-0478-8
43. N. K. Sharma and J. K. Singh, Int. J. Theor. Phys. **53**, 4132 (2014). https://doi.org/10.1007/s10773-014-2164-z
44. P. K. Sahoo, B. Mishra, P. Sahoo, and S. K. J. Pacif, Eur. Phys. J. Plus. **131**, 333 (2016). https://doi.org/10.1140/epjp/i2016-16333-x
45. D. R. K. Reddy, K. D. Raju, and V. U. M. Rao, Prespacetime, **10**, 700 (2019).
46. S. Sharma, L. Poonia, and V. Maheshwari, Conf. Ser.: Earth Environ. Sci. ID 785012006 (2021).
47. C. Brans and R. H. Dicke, Phys.Rev. **124**, 925 (1961). https://doi.org/10.1103/PhysRev.124.925
48. D. Saez and V. Ballester, J. Phys. A. **113**, 467 (1986). https://doi.org/10.1016/0375-9601(86)90121-0
49. C. P. Singh and S. Ram, Astrophys. Space Sci. **284**, 1199 (2003). https://doi.org/10.1023/A:1023637627922
50. D. R. K. Reddy and N. V. Rao, Astrophys. Space Sci. **277**, 461 (2001). https://doi.org/10.1023/A:1012543323667
51. D. R. K. Reddy, M. V. S. Rao, and G. K. Rao, Astrophys. Space Sci. **306**, 171 (2006). https://doi.org/10.1007/s10509-006-9210-0
52. V. U. M. Rao, T. Vinutha, and M. V. Shanthi, Astrophys. Space Sci. **312**, 189 (2007). https://doi.org/10.1007/s10509-007-9674-6
53. G. Ramesh and S. Umadevi, Astrophys. Space Sci. **361**, 50 (2016). https://doi.org/10.1007/s10509-015-2588-9
54. V. U. M. Rao and U. Y. D. Prasanthi, Eur. Phys. J. Plus **132**, 64 (2017).
55. R. K. Mishra and H. Dua, Astrophys. Space Sci. **364**, 195 (2019). https://doi.org/10.1007/s10509-019-3689-7

56. R. K. Mishra and A. Chand, Astrophys. Space Sci. **365**, 76 (2020). https://doi.org/10.1007/s10509-020-03790-w
57. R. K. Mishra and H. Dua, Astrophys. Space Sci. **366**, 47 (2021). https://doi.org/10.1007/s10509-021-03952-4
58. T. Vinutha and K. V. Vasavi, Eur. Phys. J. Plus **137**, 1294 (2022). https://doi.org/10.1140/epjp/s13360-022-03477-x
59. P. S. Singh and K. P. Singh, Universe **8**, 1 (2022). https://doi.org/10.3390/universe8100539
60. C. B. Collins, Gen. Relativ. Gravit. **12**, 805 **(**1980). https://doi.org/10.1007/BF00763057