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Mathematical Analysis of Fractional Diabetes Model via an Efficient Computational Technique

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Abstract

Diabetes is referred to a chronic metabolic disease signalized by elevated levels of blood glucose (also known as blood sugar level), which results over time in serious damage to the heart, blood vessels, eyes, kidneys, and nerves in the body. A mathematical assessment of the diabetes model using the Caputo fractional order derivative operator is given in this research paper. The concept of a Caputo fractional order derivative is a novel class of non-integer order derivative that has many applications in real-life scenarios. The proposed model is represented by a set of fractional ordinary differential equations. The authors employed the Sumudu Transform Homotopy Perturbation Method (STHPM**)** for finding the series solutions of the model being studied. By giving various numerical values to the respective model parameters, graphical analysis is also performed. It is observed in the numerical discussion that a decrease in both fractional order α and β leads to decrease in the number of diabetic people.

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1. Introduction

Every year millions of people suffer with diabetes and die from this disease throughout the world. According to the World Health Organization, 642 million people will have diabetes by the year 2040 globally; up from an estimated 422 million people today and diabetes-related deaths reach 1.5 million each year. Diabetes is steadily on the rise everywhere in the globe, but most noticeably in the middle-income nations [1]. One in six people with diabetes in the world is from India. Diabetes, usually referred to as diabetes mellitus, is a series of metabolic diseases. Insulin is required by our body to convert glucose into energy. In general, there are two forms of diabetes [2]. Type I diabetes (lack

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of insulin) occurs when the pancreas fails not make insulin. People with this type of diabetes who require daily insulin injections are taken into account. Type II diabetes (ineffective utilization of insulin) occurs when the pancreas cannot make enough insulin or it cannot be processed. In this type of diabetes, doctors recommend to the patients to follow particular diet chart, regular exercise, yoga etc.

In order to fully comprehend the dynamic behavior of the Diabetes system, mathematical modeling is used. There are many different types of mathematical models that have been suggested to describe the mechanics of diabetes. A mathematical model to track the growth of populations with and without complications from diabetes was provided by Boutayeb *et al*. [3]. Shah *et al*. [4] described a mathematical model for diabetic to be on dialysis. A mathematical model for the epidemiology of diabetes mellitus incorporating lifestyle and genetic variables was developed by Widyaningsih *et al*. [5]. A mathematical approach for identifying diabetes in the cape coast was proposed by Jacobs [6].

Fractional calculus has emerged as a vibrant and pivotal research area in contemporary times, capturing the imagination of researchers across diverse fields within the applied sciences. The allure of fractional calculus lies in its remarkable versatility and its myriad applications, which have become a magnet for scholarly attention. This captivating discipline has, in turn, facilitated the extension of its theory and fundamental concepts into the realm of real-world problems. One of the most remarkable facets of fractional calculus is its ability to transcend the limitations of classical integer-order calculus. Fractional differential equations have been applied in engineering, physics, biology, and biomedical processes to successfully model a range of real-world problems as reported in literature [7, 8-16]. Diabetes mellitus is a metabolic disease characterized by chronic hyperglycemia as a result of progressive loss of pancreatic cells, which could lead to several debilitating complications. A fractional mathematical model of diabetes was demonstrated by Dubey *et al.* [17] along with a discussion on various complications of diabetes.

The motive of the present paper is to develop a mathematical model for diabetes with fractional order in Caputo sense. The Sumudu transform homotopy perturbation technique (STHPM) is applied to discretize the proposed model and obtained the series solution of the same. The numerical results for different instances of fractional order parameters are reported for each of the sub-classes of the model i.e. $C(t)$ and $E(t)$ sub-classes. These $C(t)$ and $E(t)$ will reveal a great deal about number of person having diabetics with complications and size of population of diabetics at time t respectively.

2. Preliminaries and Basic Definitions

Definition 1 Fractional integral of Riemann-Liouville type of order $\xi \geq 0$, for a function $f(t) \in C_l, l \geq -1$ is defined [18], as

$$I^{\xi} f(t) = \begin{cases} \frac{1}{\Gamma(\xi)} \int_0^t (t-\tau)^{\alpha-1} f(\tau) d\tau, & \xi > 0 \\ f(t), & \xi = 0 \end{cases} \tag{1}$$

from (1), its yields

$$I^{\xi}t^{\varrho} = \frac{\Gamma(\varrho+1)}{\Gamma(\varrho+\xi+1)} t^{\alpha+\gamma}, \xi \geq 0, \varrho > -1. \quad (2)$$

Definition 2 As stated [19], the Caputo sense fractional derivative of f(t) is

$$D^{\xi}f(t) = I^{\varrho-\xi}D^{\varrho}f(t) = \frac{1}{\Gamma(\varrho-\xi)}\int_0^t \frac{f^{(\varrho)}(\tau)}{(t-\tau)^{\xi-\varrho+1}}d\tau, \qquad 0 \leq \varrho - 1 < \xi \leq \varrho,\ \varrho \in N, t > 0,$$
$$R(\xi) > 0. \quad (3)$$

Definition 3 Consider a set of functions

$$\Lambda = \left\{ f(t) | \exists M, k_j > 0, j = 1,2;\ |f(t)| \leq Me^{\frac{|t|}{k_j}} \text{ if } t \in (-1)^j \times [0,\infty) \right\}, \quad (4)$$

Sumudu transform of $f(t) \in \Lambda$ is defined [20].

$$S[f(t); u] = G(u) = \int_0^{\infty} \frac{1}{u} e^{-\frac{t}{u}} f(t)dt, \quad \forall\, t \geq 0 \quad u \in (-k_1, k_2), \quad (5)$$

More about Sumudu transform is available in literature [21-23].

Definition 4 As stated in literature [24], the Sumudu transform of Caputo fractional derivative (3) is

$$S[D^{\xi}f(t)] = u^{-\xi}S[f(t)] - \sum_{m=0}^{n-1} u^{-\xi+m} f^{(m)}(0),\ n-1 < \alpha \leq n, n \in N. \quad (6)$$

3. Mathematical Model for Diabetes with Fractional Order

In this section, authors explore the mathematical representation of the diabetic patient and the consequences that Boutayeb *et al.* [3] described. Both diabetes patients with complications and those without can be studied using this method of research. Authors employ various parameters in this model that are specified as

$A(t) -$ the incidence of diabetes mellitus, $B(t) -$ number of person having diabetics without complications, $C(t)$ −the total number of diabetics with complications, $E(t)$ −diabetic community size at moment t, $\delta -$ the probability of a person having diabetic and developing complications, $\varepsilon -$ the natural mortality rate, $\lambda -$ the rate at which complications are resolved, $\vartheta -$ the rate at which diabetic patients experience complications that result in severe disability, $\mu -$ the rate at which people die from diabetic complications.

In this model authors have considered $E(t) = B(t) + C(t)$. This model can be comprehended very well with the help of schematic representation as given in Fig. 1.

To define this proposed model; Boutayeb's *et al.* [3] used ordinary differential equations and gave the following mathematical model

$$\left.\begin{aligned} \frac{dB(t)}{dt} &= -(\delta+\varepsilon)B(t) + \lambda C(t) \\ \frac{dC(t)}{dt} &= A(t) - (\mu+\lambda+\varepsilon+\vartheta)C(t) + \delta B(t) \end{aligned}\right\} \quad (7)$$

$$E(t) = B(t) + C(t). \quad (8)$$

On using (8), then (7) reduces to

$$\left.\begin{aligned} \frac{dC(t)}{dt} &= -(\delta+\psi)C(t) + \delta E(t),\ t > 0 \\ \frac{dE(t)}{dt} &= A(t) - (\vartheta+\mu)C(t) - \varepsilon E(t),\ t > 0 \end{aligned}\right\} \quad (9)$$

where $\psi = \mu + \lambda + \varepsilon + \vartheta$ and initial conditions are

$$C(0) = C_0, E(0) = E_0. \quad (10)$$

For more details about this proposed Diabetes model are available in literature [3,17].
In this study, authors develop the proposed model in the form of linked fractional ordinary differential equations by extending the model (7) using the Caputo fractional derivative [19], as

$$\left.\begin{aligned} D_t^{\alpha}\big(C(t)\big) &= -(\delta+\psi)C(t)+\delta E(t),\ t>0,\ 0<\alpha\le 1,\\ D_t^{\beta}\big(E(t)\big) &= A(t)-(\vartheta+\mu)C(t)-\varepsilon E(t), t>0,\ 0<\beta\le 1, \end{aligned}\right\} \quad (11)$$

$$E(t) = B(t) + C(t), \quad (12)$$

with ICs are

$$C(0) = C_0, E(0) = E_0. \quad (13)$$

where $\mathrm{D}^{\alpha} = D_t^{\alpha} = \frac{d^{\alpha}}{dt^{\alpha}}$ is the fractional derivative of Caputo sense and $\psi = \mu + \lambda + \varepsilon + \vartheta$; the parameters we utilized here are defined in detail at the beginning of Section 3. In this research work A(t) the incidence of diabetes mellitus are considered as a constant. The incidence of diabetes mellitus is regarded as a constant in this study work $A(t)$. Authors will obtain the values of $C(t)$ and $E(t)$ after solving the system of coupled ODEs (11) using the method STHPM, reported in literature [25] and these values will reveal a great deal about number of persons having diabetics with complications and size of population of diabetics at time t respectively.

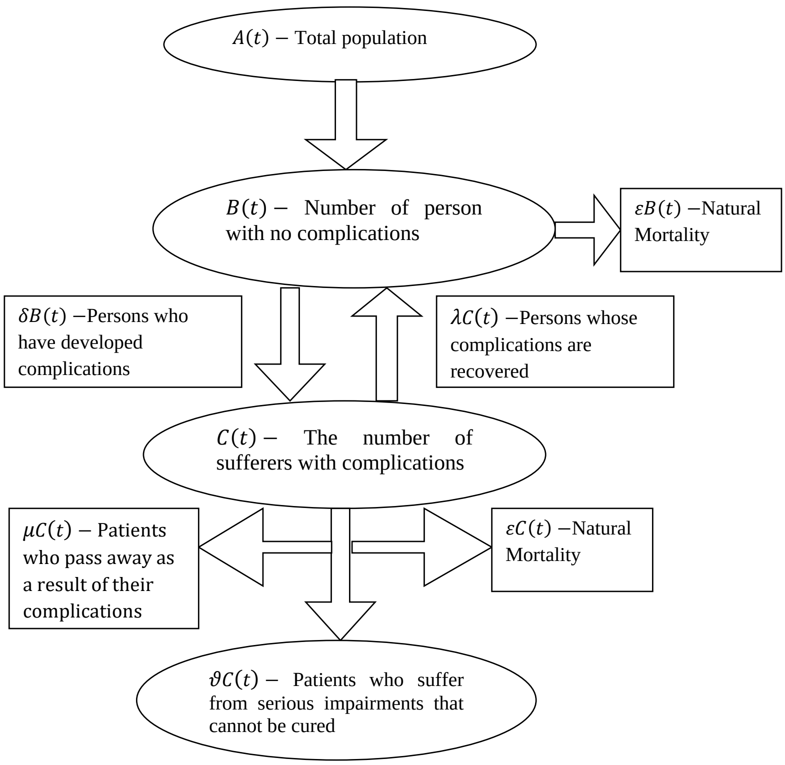

Fig. 1. Schematic representation of Diabetes model.

4. Implementation of STHPM [25] to Fractional Order Diabetes's Model

Applying the Sumudu transform to (11), gives

$$\left.\begin{aligned} S\left[D_t^{\alpha}\left(C(t)\right)\right] &= S[-(\delta+\psi)C(t)+\delta E(t)] \\ S\left[D_t^{\beta}\left(E(t)\right)\right] &= S[A(t)] + S[-(\vartheta+\mu)C(t)-\varepsilon E(t)] \end{aligned}\right\} \quad (14)$$

After a little simplification in (14), on applying (6) and (13), its yields

$$\left.\begin{aligned} S\left[\left(C(t)\right)\right] &= C_0 + \frac{1}{\mathrm{u}^{-\alpha}} S[-(\delta+\psi)C(t)+\delta E(t)] \\ S\left[\left(E(t)\right)\right] &= E_0 + \frac{A(t)}{\mathrm{u}^{-\beta}} + \frac{1}{\mathrm{u}^{-\beta}} S[-(\vartheta+\mu)C(t)-\varepsilon E(t)] \end{aligned}\right\} \quad (15)$$

Employing the inverse Sumudu transform on (15), its yields

$$\left.\begin{aligned} C(t) &= C_0 + \mathrm{S}^{-1}\left[\frac{1}{\mathrm{u}^{-\alpha}} S[-(\delta+\psi)C(t)+\delta E(t)]\right] \\ E(t) &= E_0 + A(t)\frac{t^{\beta}}{\Gamma(\beta+1)} + \mathrm{S}^{-1}\left[\frac{1}{\mathrm{u}^{-\beta}} S[-(\vartheta+\mu)C(t)-\varepsilon E(t)]\right] \end{aligned}\right\} \quad (16)$$

The following homotopy is obtained by using the homotopy perturbation approach on (16), as described in [25]

$$\left.\begin{aligned} \sum_{n=0}^{\infty} p^n C_n(t) &= C_0 + p\left(\mathrm{S}^{-1}\left[\frac{1}{\mathrm{u}^{-\alpha}} S\left[-(\delta+\psi)\sum_{n=0}^{\infty} p^n C_n(t) + \delta \sum_{n=0}^{\infty} p^n E_n(t)\right]\right]\right) \\ \sum_{n=0}^{\infty} p^n E_n(t) &= E_0 + A(t)\frac{t^{\beta}}{\Gamma(\beta+1)} + p\left(\mathrm{S}^{-1}\left[\frac{1}{\mathrm{u}^{-\beta}} S\left[-(\vartheta+\mu)\sum_{n=0}^{\infty} p^n C_n(t) - \varepsilon \sum_{n=0}^{\infty} p^n E_n(t)\right]\right]\right) \end{aligned}\right\} \quad (17)$$

The values of similar powers of p on each side of (17) are compared, and the results are

$$p^0: C_0(t) = C_0$$

$$p^0: E_0(t) = E_0 + A(t)\frac{t^{\beta}}{\Gamma(\beta+1)},$$

again

$$\begin{aligned} p^1: C_1(t) &= \mathrm{S}^{-1}\left[\frac{1}{\mathrm{u}^{-\alpha}} S[-(\delta+\psi)C_0(t)+\delta E_0(t)]\right], \\ &= \mathrm{S}^{-1}\left[\frac{1}{\mathrm{u}^{-\alpha}} S\left[-(\delta+\psi)C_0 + \delta E_0 + \delta A(t)\frac{t^{\beta}}{\Gamma(\beta+1)}\right]\right], \\ &= -(\delta+\psi)C_0\frac{t^{\alpha}}{\Gamma(\alpha+1)} + \delta E_0\frac{t^{\alpha}}{\Gamma(\alpha+1)} + \delta A(t)\frac{t^{\alpha+\beta}}{\Gamma(\alpha+\beta+1)}, \end{aligned}$$

$$\begin{aligned} p^1: E_1(t) &= \mathrm{S}^{-1}\left[\frac{1}{\mathrm{u}^{-\beta}} S[-(\vartheta+\mu)C_0(t)-\varepsilon E_0(t)]\right], \\ &= \mathrm{S}^{-1}\left[\frac{1}{\mathrm{u}^{-\beta}} S\left[-(\vartheta+\mu)C_0 - \varepsilon E_0 - \varepsilon A(t)\frac{t^{\beta}}{\Gamma(\beta+1)}\right]\right], \\ &= -(\vartheta+\mu)C_0\frac{t^{\beta}}{\Gamma(\beta+1)} - \varepsilon E_0\frac{t^{\beta}}{\Gamma(\beta+1)} - \varepsilon A(t)\frac{t^{2\beta}}{\Gamma(2\beta+1)}, \end{aligned}$$

similarly, $C_2(t)$ and $E_2(t)$, obtain as

$$p^2: C_2(t) = \mathrm{S}^{-1}\left[\frac{1}{\mathrm{u}^{-\alpha}} S[-(\delta+\psi)C_1(t)+\delta E_1(t)]\right],$$

$$\begin{aligned} = \mathrm{S}^{-1}\Bigg[\frac{1}{\mathrm{u}^{-\alpha}} S\Bigg[&-(\delta+\psi)\left\{-(\delta+\psi)C_0\frac{t^{\alpha}}{\Gamma(\alpha+1)} + \delta E_0\frac{t^{\alpha}}{\Gamma(\alpha+1)} + \delta A(t)\frac{t^{\alpha+\beta}}{\Gamma(\alpha+\beta+1)}\right\} + \\ &\delta\left\{-(\vartheta+\mu)C_0\frac{t^{\beta}}{\Gamma(\beta+1)} - \varepsilon E_0\frac{t^{\beta}}{\Gamma(\beta+1)} - \varepsilon A(t)\frac{t^{2\beta}}{\Gamma(2\beta+1)}\right\}\Bigg]\Bigg], \end{aligned}$$

$$= (\delta+\psi)^2 C_0\frac{t^{2\alpha}}{\Gamma(2\alpha+1)} - (\delta+\psi)\delta E_0\frac{t^{2\alpha}}{\Gamma(2\alpha+1)} - (\delta+\psi)\delta A(t)\frac{t^{2\alpha+\beta}}{\Gamma(2\alpha+\beta+1)} -$$

$$\delta(\vartheta+\mu)C_0\frac{t^{\alpha+\beta}}{\Gamma(\alpha+\beta+1)}-\varepsilon\,\delta E_0\frac{t^{\alpha+\beta}}{\Gamma(\alpha+\beta+1)}-\varepsilon\delta A(t)\frac{t^{\alpha+2\beta}}{\Gamma(\alpha+2\beta+1)},$$

$$p^2: E_2(t)=S^{-1}\left[\frac{1}{\mathrm{u}^{-\beta}}S[-(\vartheta+\mu)C_1(t)-\varepsilon E_1(t)]\right],$$

$$=S^{-1}\left[\frac{1}{\mathrm{u}^{-\beta}}S\left[-(\vartheta+\mu)\left\{-(\delta+\psi)C_0\frac{t^{\alpha}}{\Gamma(\alpha+1)}+\delta E_0\frac{t^{\alpha}}{\Gamma(\alpha+1)}+\delta A(t)\frac{t^{\alpha+\beta}}{\Gamma(\alpha+\beta+1)}\right\}-\right.\right.$$
$$\left.\left.\varepsilon\left\{-(\vartheta+\mu)C_0\frac{t^{\beta}}{\Gamma(\beta+1)}-\varepsilon E_0\frac{t^{\beta}}{\Gamma(\beta+1)}-\varepsilon A(t)\frac{t^{2\beta}}{\Gamma(2\beta+1)}\right\}\right]\right],$$
$$=(\vartheta+\mu)(\delta+\psi)C_0\frac{t^{\alpha+\beta}}{\Gamma(\alpha+\beta+1)}-\delta E_0(\vartheta+\mu)\frac{t^{\alpha+\beta}}{\Gamma(\alpha+\beta+1)}-\delta A(t)(\vartheta+$$
$$\mu)\frac{t^{\alpha+2\beta}}{\Gamma(\alpha+2\beta+1)}+\varepsilon(\vartheta+\mu)C_0\frac{t^{2\beta}}{\Gamma(2\beta+1)}+\varepsilon^2E_0\frac{t^{2\beta}}{\Gamma(2\beta+1)}+\varepsilon^2A(t)\frac{t^{3\beta}}{\Gamma(3\beta+1)},$$

The remaining components can likewise be found by using the same process, such as $C_3(t)$, $E_3(t)$ and $C_4(t)$, $E_4(t)$ and so on. However, mentioning these values would take plenty of space.

On plugging these obtained values in the following equation (18), the series solutions of the system of two linked differential equations (11), are finally found

$$\left.\begin{aligned} C(t)&=\textstyle\sum_{n=0}^{\infty}C_n(t)=C_0(t)+C_1(t)+C_2(t)+\cdots,\\ E(t)&=\textstyle\sum_{n=0}^{\infty}E_n(t)=E_0(t)+E_1(t)+E_2(t)+\cdots,\end{aligned}\right\}\quad(18)$$

Its yields

$$\left.\begin{aligned} C(t)&=C_0-(\delta+\psi)C_0\frac{t^{\alpha}}{\Gamma(\alpha+1)}+\delta E_0\frac{t^{\alpha}}{\Gamma(\alpha+1)}+\delta A(t)\frac{t^{\alpha+\beta}}{\Gamma(\alpha+\beta+1)}+(\delta+\psi)^2C_0\frac{t^{2\alpha}}{\Gamma(2\alpha+1)}-\\ &\quad(\delta+\psi)\delta E_0\frac{t^{2\alpha}}{\Gamma(2\alpha+1)}-(\delta+\psi)\delta A(t)\frac{t^{2\alpha+\beta}}{\Gamma(2\alpha+\beta+1)}-\delta(\vartheta+\mu)C_0\frac{t^{\alpha+\beta}}{\Gamma(\alpha+\beta+1)}-\\ &\quad\varepsilon\,\delta E_0\frac{t^{\alpha+\beta}}{\Gamma(\alpha+\beta+1)}-\varepsilon\delta A(t)\frac{t^{\alpha+2\beta}}{\Gamma(\alpha+2\beta+1)}-\cdots,\\ E(t)&=E_0+A(t)\frac{t^{\beta}}{\Gamma(\beta+1)}-(\vartheta+\mu)C_0\frac{t^{\beta}}{\Gamma(\beta+1)}-\varepsilon E_0\frac{t^{\beta}}{\Gamma(\beta+1)}-\varepsilon A(t)\frac{t^{2\beta}}{\Gamma(2\beta+1)}+\\ &\quad(\vartheta+\mu)(\delta+\psi)C_0\frac{t^{\alpha+\beta}}{\Gamma(\alpha+\beta+1)}-\delta E_0(\vartheta+\mu)\frac{t^{\alpha+\beta}}{\Gamma(\alpha+\beta+1)}-\delta A(t)(\vartheta+\mu)\frac{t^{\alpha+2\beta}}{\Gamma(\alpha+2\beta+1)}+\\ &\quad\varepsilon(\vartheta+\mu)C_0\frac{t^{2\beta}}{\Gamma(2\beta+1)}+\varepsilon^2E_0\frac{t^{2\beta}}{\Gamma(2\beta+1)}+\varepsilon^2A(t)\frac{t^{3\beta}}{\Gamma(3\beta+1)}\ldots\ .\end{aligned}\right\}\quad(19)$$

5. Numerical Results and Discussion

The numerical findings and explanation of (19) i.e. the series solution of (11) are presented in this section by giving specific values [3,17] to the parameters/ICs involved therein.

The number of incidence of diabetes mellitus $A(t)=6\times10^7$, initially the size of population of diabetics $E_0=6.11\times10^7$, initially the number of person having diabetics with complications $C_0=4.7\times10^7$, $\mu=0.05, \lambda=0.08$, $\varepsilon=0.02, \vartheta=0.05$ and $\delta=0.66$.

On putting the precise values of the parameters and ICs indicated above in (19) to obtain the approximate solution (series solution) of (11) in the manner given below

$$\left.\begin{aligned} C(t) &= 47000000 - 94000\frac{t^{\alpha}}{\Gamma(\alpha+1)} + 35691480\frac{t^{\alpha+\beta}}{\Gamma(\alpha+\beta+1)} + 80840\frac{t^{2\alpha}}{\Gamma(2\alpha+1)} \\ &\quad -34056000\frac{t^{2\alpha+\beta}}{\Gamma(2\alpha+\beta+1)} - 792000\frac{t^{\alpha+2\beta}}{\Gamma(\alpha+2\beta+1)} - \cdots, \\ E(t) &= 61100000 + 54078000\frac{t^{\beta}}{\Gamma(\beta+1)} + 9400\frac{t^{\alpha+\beta}}{\Gamma(\alpha+\beta+1)} - 1081560\frac{t^{2\beta}}{\Gamma(2\beta+1)} \\ &\quad -3960000\frac{t^{\alpha+2\beta}}{\Gamma(\alpha+2\beta+1)} + 24000\frac{t^{3\beta}}{\Gamma(3\beta+1)} - \cdots . \end{aligned}\right\} \tag{20}$$

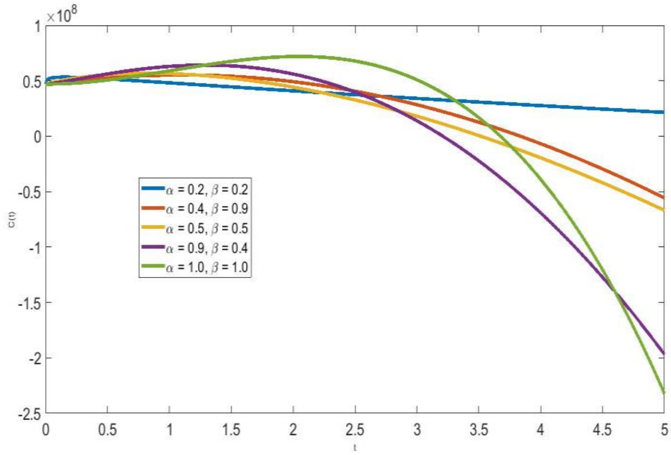

Fig. 2. Plots of $C(t)$ versus t, for (20) at different values of α and β.

From the graph it is observed that the $C(t)$ i.e. number of person having diabetics with complications will be decreased as the order of fractional derivative of the proposed model decreases.

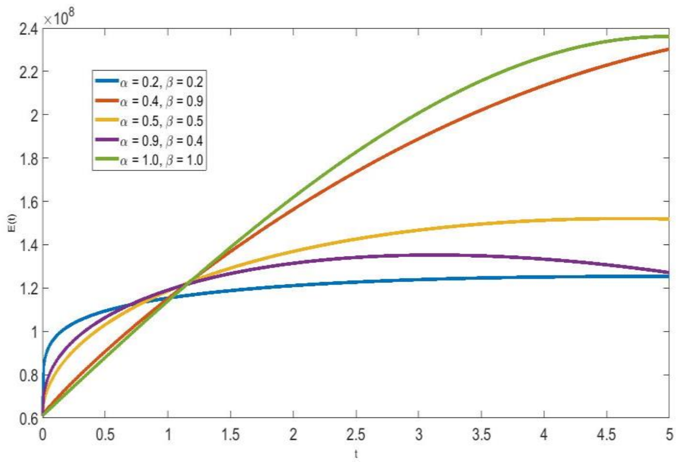

Fig. 3. Plots of $E(t)$ versus t, for (20) at different values of α and β.

From the graph it is observed that the $E(t)$ i.e. size of population of diabetics will be decreased as the order of fractional derivative of the proposed model decreases.

6. Conclusion

The fractional model of diabetes and its consequences has been presented herein. It extends to the field of fractional calculus, which authors have studied using Caputo's fractional derivative. Here, the mathematical solution of fractional diabetes model is obtained by using STHPM. The main findings of this work indicated that the STHPM is a very efficient computational technique for obtaining the exact or analytical solution of fractional coupled differential equation mathematical model without doing tedious and large computational work. The basic characters of this proposed method have been presented in detail manner. It is observed from graphs in the numerical discussion that a decrease in both fractional order α and β leads to decrease in the number of diabetic people. Extension of the fractional operator to model more complex scenarios in engineering and biomedical processes is left for future work.

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