

Higher Dimensional Bianchi Type-III String Cosmological Models in Lyra Geometry

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Received 8 June 2023, accepted in final revised form 26 November 2023

Abstract

Here, we studied Bianchi type-III string cosmological models generated using a cloud of strings with particles connected to them in the framework of Lyra geometry considering five-dimensional spacetime. We assume that the shear scalar and scalar expansion are proportional to obtain the exact solutions of the survival field equations here. Secondly, we adopt the assumption considering the Reddy string condition. From the different cases obtained here, the first case gives the Bianchi type-III string cosmology in Lyra geometry in five-dimensional spacetime, and the second case gives the five-dimensional vacuum model in general relativity. The various parameters of the model universe, which are very important for the description of the Universe, are obtained, and their properties are studied and compared with the recent observational findings. The model universe obtained here starts with the Big Bang, and as time progresses, both particle density and energy density decrease with the expansion of our Universe.

Keywords: Five dimensions; Cloud string; Bianchi type-III spacetime; Lyra geometry; Expansion scalar and big-bang.

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doi: <https://dx.doi.org/10.3329/jsr.v16i2.66568> J. Sci. Res. **16** (2), 369-380 (2024)

1. Introduction

It is still an interesting area of research to discover its unknown phenomenon that has yet to be observed to study the ultimate fate of the universe. So, cosmologists have taken considerable interest in understanding the future evolution of the Universe, comprehension of the past and present state of the Universe, and understanding the universe in the modern era. Recently, there has been considerable interest in studying string cosmology because of the important role of cosmic strings in studying the origin and evolution of the universe. But these days, we cannot make a very last assertion about its origin and evolution with strong evidence. So, more and more investigations are very important to discover the unknown phenomenon of the whole universe. The string theory was developed to describe the events of the early stage of the origin and evolution of the

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universe. Generally, Stachel [1] and Letelier [2] started the string study. Because of the key function of strings in describing the evolution of the early stage of our universe, in recent times, many prominent authors have been interested in cosmic strings in general relativity [3,4] and give rise to density perturbations leading to the formation of large Scale Structure (galaxies) of the universe [5,6]. The strings are nothing but crucial topological defects because of the phase transition, which occurs at a temperature lower than some critical point at the beginning of our universe according to grand unified theories [7,8]. Also, a large network of strings in the early universe does not oppose present-day observational findings of the universe. It results in anisotropy in spacetime, though the strings are not observable in the present-day universe. Those strings have stress energy and may be coupled with the gravitational field. The Strings don't cause any harm to the cosmological models; rather, they can lead to very interesting astrophysical phenomena.

Among the modified theories of gravity, Lyra geometry is also one of the important theories in which the displacement field can be considered as a component of the total energy that can play the role of dark energy. Lyra [9] suggested a modification to Riemannian geometry by using a gauge function φ_μ into the structureless geometry, which removed the nonintegrability condition to H. Weyl's geometry, and this modified Riemannian geometry is known as Lyra's Geometry. Some of the prominent researchers in the area of Lyra geometry are Rahaman and Bera [10], Rahaman *et al.* [11], Baro *et al.* [12], Mohanty *et al.* [13,14], Mohanty and Mahanta [15], Singh and Mollah [16], Brahma and Dewri [17], Mete and Deshmukh [18] who constructed various cosmological models universe in the context of Lyra geometry.

Many researchers have studied different string cosmological models in different dimensions. It is believed that the Universe had some higher dimensional era at its early stage of evolution, which motivates us to investigate the cosmological model universe in five-dimensional spacetime. Nowadays, studying string cosmology in five-dimensional space times in the framework of general relativity and Lyra geometry is interesting. Rahaman *et al.* [19] obtained realistic solutions of the field equations in Lyra geometry for a five-dimensional spacetime when the gravitational sources are massive strings. Mohanty and Samanta [20] constructed five-dimensional string cosmological models in the Lyra manifold with a massive scalar field and obtained that the models avoid initial singularity in the presence of the massive scalar field. Yadav *et al.* [21] have investigated Bianchi type-III anisotropic universes with a cloud of strings within the framework of Lyra geometry. Samanta *et al.* [22] constructed some Bianchi type-III cosmological models in the framework of general relativity considering five-dimensional spacetime with massive string as a source of gravitational field. Adhav *et al.* [23] and Kandalkar *et al.* [24] constructed string cosmological models by considering the Reddy string condition ($\rho+\lambda=0$) for solving the field equations. Sahoo *et al.* [25] investigated the Bianchi type-III String cosmological model with bulk viscous fluid in the Lyra manifold. Reddy [26], Baro & Singh [27], Khadekar [28], Rao [29], Yadav [30], Goswami [31], and Singh [32] are some authors who studied different cosmological models with various modified theories

of relativity in a different context with different Bianchi type spacetimes. Not only the above-mentioned authors, recently Tripathi [33], Singh [34], Pradhan [35], Tiwari *et al.* [36], Ram *et al.* [37], Mollah *et al.* [38] constructed string cosmological models in various contexts and in various spacetimes.

Motivated by the above discussions and investigations in Einstein's theory and its alternative theories of gravity, here we have studied the five-dimensional string cosmological models in Bianchi type-III spacetime considering Lyra geometry. The model presented here is somewhat different from the earlier findings in many areas and is as physically realistic as the present-day observational data. The paper is planned as follows: Section 1 briefly introduces strings, its importance, and a review of Lyra Geometry. In Section 2, the five-dimensional Bianchi type-III metric is presented, and the field equations in the framework of Lyra geometry are derived; Section 3, deals with the determinate solutions of the field equations determined by using some physical plausible conditions and Physical and geometrical properties of our model universe with the help of graph are presented in Section 4; In Section 5 conclusions are given.

2. The Metric and the Field Equations

We consider Bianchi type-III metric in five-dimension as

$$ds^2 = a^2 dx^2 + b^2 e^{-2x} dy^2 + c^2 dz^2 + D^2 dm^2 - dt^2, \tag{1}$$

Here a, b, c, and D all are the functions of 't'. Here, the extra coordinate 'm' is taken to be space-like, and the fifth coordinate is taken to be time-like. For the above metric, let

$$x^1 = x, \quad x^2 = y, \quad x^3 = z, \quad x^4 = m \quad \text{and} \quad x^5 = t, \tag{2}$$

The field equations in the Lyra manifold given by Sen[39] with $8\pi G = 1, C=1$ is

$$R_{ij} - \frac{1}{2} R g_{ij} + \frac{3}{2} \phi_i \phi_j - \frac{3}{4} g_{ij} \phi_k \phi^k = -T_{ij}, \tag{3}$$

Here ϕ_i is the displacement vector field and is defined by

$$\phi_i = (0,0,0,0, \beta(t)), \tag{4}$$

For a cloud string, the energy-momentum tensor is

$$T_{ij} = \rho u_i u_j - \lambda x_i x_j, \tag{5}$$

Here, $\rho = \lambda + \rho_p$ is the energy density for a cloud of string with particles connected to them, ρ_p is the particle density and λ is the tension density of a cloud of string. The coordinates are co-moving, u^i is the five velocity vector, and x^i is the unit vector (space-like) which represents the direction of the strings, and they are given by

$$u^i = (0,0,0,0,1) \quad \text{and} \quad x^i = (0,0,c^{-1},0,0), \tag{6}$$

$$u_i u^i = -1 = -x_i x^i \quad \text{and} \quad u_i x^i = 0, \tag{7}$$

The field equation (3) with the equations (4)-(7) for the metric (1) takes the form

$$\frac{\ddot{b}}{b} + \frac{\ddot{c}}{c} + \frac{\ddot{D}}{D} + \frac{\dot{b}\dot{c}}{bc} + \frac{\dot{b}\dot{D}}{bD} + \frac{\dot{c}\dot{D}}{cD} + \frac{3}{4}\beta^2 = 0, \tag{8}$$

$$\frac{\ddot{a}}{a} + \frac{\ddot{c}}{c} + \frac{\ddot{D}}{D} + \frac{\dot{a}\dot{c}}{ac} + \frac{\dot{a}\dot{D}}{aD} + \frac{\dot{c}\dot{D}}{cD} + \frac{3}{4}\beta^2 = 0, \tag{9}$$

$$\frac{\ddot{a}}{a} + \frac{\ddot{b}}{b} + \frac{\ddot{D}}{D} + \frac{\dot{a}\dot{b}}{ab} + \frac{\dot{a}\dot{D}}{aD} + \frac{\dot{b}\dot{D}}{bD} + \frac{3}{4}\beta^2 - \frac{1}{a^2} = \lambda, \tag{10}$$

$$\frac{\ddot{a}}{a} + \frac{\ddot{b}}{b} + \frac{\ddot{c}}{c} + \frac{\dot{a}\dot{b}}{ab} + \frac{\dot{a}\dot{c}}{ac} + \frac{\dot{b}\dot{c}}{bc} + \frac{3}{4}\beta^2 - \frac{1}{a^2} = 0, \tag{11}$$

$$\frac{\dot{a}\dot{b}}{ab} + \frac{\dot{a}\dot{c}}{ac} + \frac{\dot{a}\dot{D}}{aD} + \frac{\dot{b}\dot{c}}{bc} + \frac{\dot{b}\dot{D}}{bD} + \frac{\dot{c}\dot{D}}{cD} - \frac{3}{4}\beta^2 - \frac{1}{a^2} = \rho, \tag{12}$$

$$\frac{\dot{a}}{a} - \frac{\dot{b}}{b} = 0, \tag{13}$$

Here the overhead dots denote the order of differentiation w.r.t. time 't'.

3. Solutions of the Field Equations

Solving (13), we get $a = lb$, (14)

Here, l is the integration constant. Without affecting the generality, we take $l = 1$. And using it, (14) can be written as,

$$a = b, \tag{15}$$

Thus, using relation (15), the field equations (8) to (12) reduces to

$$\frac{\ddot{b}}{b} + \frac{\ddot{c}}{c} + \frac{\ddot{D}}{D} + \frac{\dot{b}\dot{c}}{bc} + \frac{\dot{b}\dot{D}}{bD} + \frac{\dot{c}\dot{D}}{cD} + \frac{3}{4}\beta^2 = 0, \tag{16}$$

$$2\frac{\ddot{b}}{b} + \frac{\ddot{D}}{D} + \frac{\dot{b}^2}{b^2} + 2\frac{\dot{b}\dot{D}}{bD} + \frac{3}{4}\beta^2 - \frac{1}{b^2} = \lambda, \tag{17}$$

$$2\frac{\ddot{b}}{b} + \frac{\ddot{c}}{c} + \frac{\dot{b}^2}{b^2} + 2\frac{\dot{b}\dot{c}}{bc} + \frac{3}{4}\beta^2 - \frac{1}{b^2} = 0, \tag{18}$$

$$\frac{\dot{b}^2}{b^2} + 2\frac{\dot{b}\dot{c}}{bc} + 2\frac{\dot{b}\dot{D}}{bD} + \frac{\dot{c}\dot{D}}{cD} - \frac{3}{4}\beta^2 - \frac{1}{b^2} = \rho, \tag{19}$$

We have 4 independent equations (16)-(19) with six unknown variables b, c, D, β, λ and ρ . For a deterministic solution, we considered the following physically plausible conditions:

Here, we assume that the shear and the expansion scalars are proportional to each other, $\sigma\alpha\theta$ which leads to

$$D = c^n, \quad n \neq 0 \text{ is a constant.} \tag{20}$$

The above assumption is based on the investigation of velocity and red-shift relative for an extragalactic source, which predicted that the expansion of the Hubble Parameter is 30 % isotropic, which is supported by the works of Thorne [40], Kantowski and Sachs [41], Kristian and Sachs [42]. In particular, it can be said that $\frac{\sigma}{H} \leq 0.30$, where σ and H are respectively shear scalar and Hubble constant. Also, Collins *et al.* [43] have shown that if the normal to the spatially homogeneous line element is congruent to the homogeneous hyper-surface, then $\frac{\sigma}{\theta}$ constant, θ being the expansion factor.

We also adopt the assumption considering the Reddy String Condition [44-47].

$$\rho + \lambda = 0, \tag{21}$$

This is the most suitable condition to explore the physically meaningful solutions of the above field equations.

Now subtracting (16) from (18) yields

$$\frac{\dot{b}}{b} - \frac{\dot{D}}{D} + \frac{b^2}{b^2} + \frac{b\dot{c}}{bc} - \frac{b\dot{D}}{bD} - \frac{c\dot{D}}{cD} - \frac{1}{b^2} = 0, \tag{22}$$

Adding (17) and (19) and using (21) yields,

$$2\frac{\dot{b}}{b} + \frac{\dot{D}}{D} + 2\frac{b^2}{b^2} + 2\frac{b\dot{c}}{bc} + 4\frac{b\dot{D}}{bD} + \frac{c\dot{D}}{cD} - 2\frac{1}{b^2} = 0, \tag{23}$$

Equations (22) and (23) yields

$$\frac{\ddot{D}}{D} + \frac{b\dot{D}}{bD} + \frac{c\dot{D}}{cD} = 0, \tag{24}$$

By using (20) in (24), we get

$$\frac{\dot{c}}{c} \left[\frac{\ddot{c}}{\dot{c}} + n \frac{\dot{c}}{c} + 2 \frac{\dot{b}}{b} \right] = 0, \tag{25}$$

This yields the following three cases:

CaseI: $\left(\frac{\ddot{c}}{\dot{c}} + n \frac{\dot{c}}{c} + 2 \frac{\dot{b}}{b}\right) = 0$, **CaseII:** $\frac{\dot{c}}{c} = 0$ and **CaseIII:** $\frac{\dot{c}}{c} = 0$ & $\left(\frac{\ddot{c}}{\dot{c}} + n \frac{\dot{c}}{c} + 2 \frac{\dot{b}}{b}\right) = 0$,

We intend to determine the cosmic string cosmological models for three different cases mentioned above separately.

CaseI: $\left(\frac{\ddot{c}}{\dot{c}} + n \frac{\dot{c}}{c} + 2 \frac{\dot{b}}{b}\right) = 0$, solving this we get,

$$c = [(n + 1) \left(\int \frac{K}{b^2} dt + K_1\right)], \quad n \neq 1, \tag{26}$$

From this, it is clear that c(t) can be found for any given function b(t). So, the solutions are not unique. So, for further investigation, we consider [Mohanty and Mahanta [15].

$$\frac{\ddot{c}}{\dot{c}} + n \frac{\dot{c}}{c} = -2 \frac{\dot{b}}{b} = k \text{ (Constant),} \tag{27}$$

Now, solving (27) and using (15) and (20), we get,

$$a = b = k_1 e^{-\frac{kt}{2}}, c = [(n + 1)\left(\frac{k_2}{k} e^{kt} + k_3\right)^{\frac{1}{n+1}}] \& D = [(n + 1)\left(\frac{k_2}{k} e^{kt} + k_3\right)^{\frac{n}{n+1}}], \quad (28)$$

Here, k_1, k_2 and $k_3 (\neq 0)$ are integration constants and $n \neq -1$.

The following metric describes the geometry of the model.

$$ds^2 = k_1^2 e^{-kt}(dx^2 + e^{-2x} dy^2) + [(n + 1)\left(\frac{k_2}{k} e^{kt} + k_3\right)^{\frac{2}{n+1}} dz^2 + [(n + 1)\left(\frac{k_2}{k} e^{kt} + k_3\right)^{\frac{2n}{n+1}} dm^2 - dt^2 \quad (29)$$

Using (28) in the equation (16) yields

$$\frac{3}{4} \beta^2 = \frac{nk_2^2 e^{2kt}}{(n+1)^2 \left(\frac{k_2}{k} e^{kt} + k_3\right)^2} - \frac{kk_2 e^{kt}}{2\left(\frac{k_2}{k} e^{kt} + k_3\right)^2} - \frac{k^2}{4}, \quad (30)$$

The energy density is obtained by using (28) and (30) in the equation (19)

$$\rho = \frac{k^2}{2} - \frac{kk_2 e^{kt}}{2\left(\frac{k_2}{k} e^{kt} + k_3\right)^2} - \frac{e^{kt}}{k_1^2}, \quad (31)$$

From (21), the tension density is given by

$$\lambda = \frac{kk_2 e^{kt}}{2\left(\frac{k_2}{k} e^{kt} + k_3\right)^2} + \frac{e^{kt}}{k_1^2} - \frac{k^2}{2}, \quad (32)$$

The particle density is given by

$$\rho_p = k^2 - \frac{kk_2 e^{kt}}{\left(\frac{k_2}{k} e^{kt} + k_3\right)^2} - \frac{2e^{kt}}{k_1^2}, \quad (33)$$

The volume is obtained as

$$V = \frac{(n+1)k_1^2 \left(\frac{k_2}{k} e^{kt} + k_3\right)}{e^{kt}}, \quad (34)$$

The scalar expansion is

$$\theta = -\frac{k_3 k^2}{k_2 e^{kt} + k k_3} - k, \quad (35)$$

The Hubble parameter is

$$H = -\frac{k_3 k^2}{4(k_2 e^{kt} + k k_3)} - \frac{k}{4}, \quad (36)$$

The deceleration parameter is

$$q = -\left[\frac{4kk_2 k_3 e^{kt}}{(k_2 e^{kt} + 2kk_3)^2} + 1\right], \quad (37)$$

The shear scalar of the model is

$$\sigma^2 = \left[\frac{k^2(12kk_2 k_3 e^{kt}(1+n)^2 + 4k^2 k k_3^2 (1+n)^2 + k_2^2 e^{2kt}(11+14n+11n^2))}{8(n+1)^2 (k_2 e^{kt} + k k_3)^2}\right], \quad (38)$$

CaseII: $\frac{\dot{c}}{c} = 0$

Integrating we get

$$c = k_4, \quad k_4 \text{ is an integration constant.} \quad (39)$$

Equation (20) yields

$$D = k_4^n, \tag{40}$$

Now field equations (16)-(19) with the help of (39) and (40) reduces to

$$\frac{\dot{b}}{b} + \frac{3}{4}\beta^2 = 0, \tag{41}$$

$$\frac{\dot{b}}{b} + \frac{\dot{b}^2}{b^2} + \frac{3}{4}\beta^2 - \frac{1}{b^2} = \lambda, \tag{42}$$

$$\frac{\dot{b}}{b} + \frac{\dot{b}^2}{b^2} + \frac{3}{4}\beta^2 - \frac{1}{b^2} = 0, \tag{43}$$

$$\frac{\dot{b}^2}{b^2} - \frac{3}{4}\beta^2 - \frac{1}{b^2} = \rho, \tag{44}$$

Using (41) and (43) and solving, we get

$$a = b = t + k_5, \quad k_5 \text{ is the constant of integration.} \tag{45}$$

Equations (42), (43) and (21) give

$$\lambda = 0, \quad \rho = 0 \text{ and } \rho_p = 0, \tag{46}$$

This leads to a vacuum model universe in five dimensions in general relativity.

Thus, the following metric describes the geometry of the model.

$$ds^2 = (t + k_5)^2(dx^2 + e^{-2x}dy^2) + k_4^2 dz^2 + k_4^{2n} dm^2 - dt^2, \tag{47}$$

CaseIII: $\frac{\dot{c}}{c} = 0$ and $\left(\frac{\dot{c}}{c} + n\frac{\dot{c}}{c} + 2\frac{\dot{b}}{b}\right) = 0$

The second equation is invalid for $\frac{\dot{c}}{c} = 0$. So, we neglect this one case.

4. Discussion

In case I, we have obtained a five-dimensional Bianchi type-III string cosmological model universe in Lyra geometry given by the equation (29). The geometrical and physical behavior of the model (29) are discussed as

- i. The metric (29) becomes flat at the initial epoch $t = 0$, and as time increases and $-1 < n < 0$, the model expands along the axis but contracts along the x and y axes. The extra dimension (coordinate) contracts and becomes nonobservable at $t \rightarrow \infty$.
- ii. The model obtained here satisfies the energy density conditions $\rho \geq 0$ and $\rho_p \geq 0$. From Fig. 1, it is seen that the evolution of energy density ρ is infinite at the time $t = 0$, and it decreases as the time t increases, and finally, energy density is constant at $t \rightarrow \infty$. Therefore, our model universe expands with high energy density initially but slows down as time progresses.
- iii. From Fig. 1, it is also observed that, $\rho > 0$, $\lambda < 0$ and $\rho_p > 0$, which shows that particles exist with positive ρ_p and strings exist with negative λ . So, the present Universe is particle-dominated which is in accordance with recent observational data.

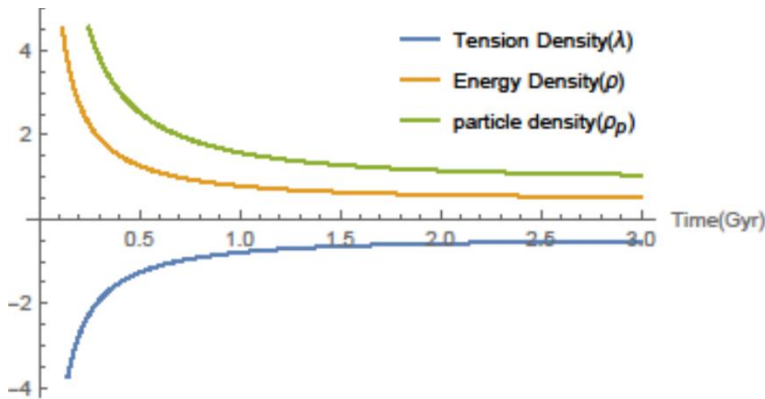


Fig. 1. Variation of λ, ρ, ρ_p Vs. Time for $k = -1, k_1 = k_2 = k_3 = 2, n = -0.5$.

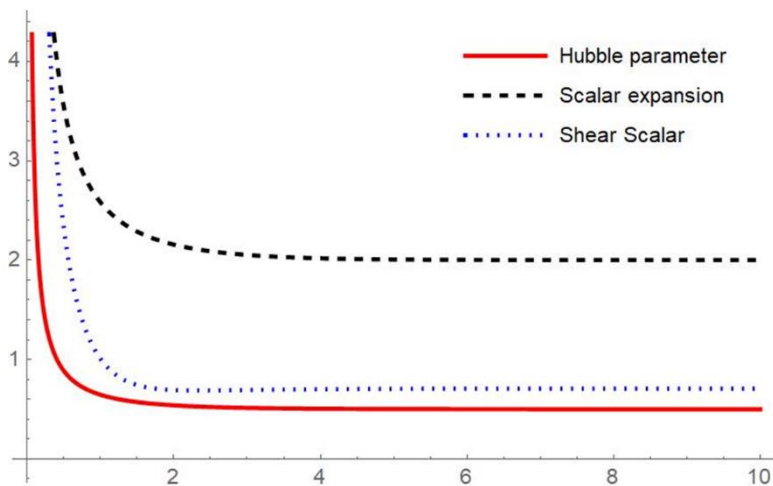


Fig. 2. Variation of θ, H, σ Vs. Time for $k = -1, k_1 = k_2 = k_3 = 2, n = -0.5$.

- iv. Also, from the Fig. 1, it is observed that the particle density ρ_p has a large value at $t = 0$. As the evolution of time decreases, it becomes a constant finite value at $t \rightarrow \infty$, which corresponds to a total constant finite number of particles in the late Universe.
- v. From the expansion scalar expression of equation (35) for the model (29), when $t = 0$, the expansion scalar θ is infinitely large, and as time progresses gradually, it decreases and finally θ becomes constant when $t \rightarrow \infty$, as shown in Fig. 2. Hence the model shows the expanding nature of Universe with the progress of time however the expansion rate slower with respect to time. According to equation (36), the Hubble parameter (H) is infinitely large when $t \rightarrow 0$, but it decreases with increasing time, and when $t \rightarrow \infty$, H remains constant, as shown by Fig. 2. The Universe has been expanding at its fastest rate due to an increased Hubble parameter value. According to

our model, the Universe expands at its fastest rate at the beginning and slows down at the end.

- vi. Our model represents a shearing universe, as shown in Fig. 2, which indicates that $\frac{\sigma^2}{\theta^2} \rightarrow \infty$ at $t = 0$. Since $\frac{\sigma^2}{\theta^2} = \text{const}(\neq 0)$ as $t \rightarrow \infty$, and hence, our model universe obtained here is an anisotropic one. Though the anisotropy is included, it does not contradict recent observational findings that the Universe is isotropic. This is because, during the evolution of our Universe, the initial anisotropy disappears after some epoch and approaches the isotropy in the late time universe.

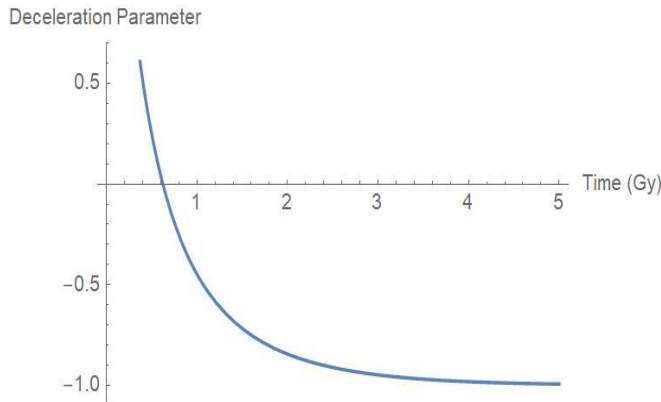


Fig. 3. Variation of DP' q' Vs. Time for $k = -1, k_2 = k_3 = 2$.

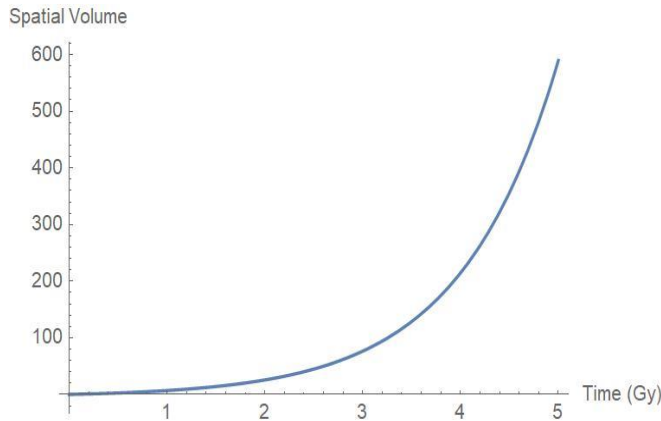


Fig. 4. Variation of Volume' V' Vs. Time for $k = -1, k_1 = k_2 = k_3 = 2, n = -0.5$.

- vii. According to equation (37), deceleration parameter q decreases as time increases and becomes negative after a certain period, as shown in Fig. 3, and $q = -1$ at $t \rightarrow \infty$, which indicates that our model universe is accelerating at late time.
- viii. The spatial volume V of the equation (34) in the model (29) indicates that the volume(V) is 0(zero) at $t = 0$, and as $t \rightarrow \infty$, the volume of the Universe becomes

infinitely large and also shows that our model universe is expanding as shown as shown in Fig. 4. Hence, our model has a singularity at the beginning.

In case II, since $\rho = \lambda = 0$, the strings don't survive, and this leads to five-dimensional vacuum models in Einstein's theory of relativity. So, the model (47) represents a five-dimensional Bianchi type -III vacuum cosmological model in general relativity.

5. Conclusion

Here, a five-dimensional Bianchi type-III String cosmological model has been constructed in the framework of Lyra geometry. The physical and geometrical parameters, which are very important in the discussion of cosmological models, have been obtained and discussed. The model universe obtained here is expanding, anisotropic, shearing, and accelerating. It is observed that our model is an inflationary model universe that decelerates at the initial epoch and accelerates after some finite time, indicating inflation in the model after an epoch of deceleration, which is in accordance with the present-day observational scenario of the accelerated expansion of our Universe as type Ia supernovae [48,49]. The model has a cosmological significance since it can explain the early stage of the Universe at the early epoch. Though the model universe is an anisotropic one in the early stage, it does not contradict the recent observational findings that there is a discrepancy in the measurement of intensities of microwaves coming from different directions of the sky, which urges us to study the Universe with anisotropy Bianchi type-III metric in such a way to describe our Universe in a more realistic situation. Also, several cosmic microwave background(CMB) anomalies, such as inconsistency of the temperature anisotropies in the CMB with exact homogeneous and isotropic FRW model measured by COBE(Cosmic Background Explorer)/WMAP (Wilkinson Microwave Anisotropy Probe) satellites, foregrounds, and exotic topologies are in evidence that we are living in a globally anisotropic universe. During the inflation, the shear scalar decreases, and in due course, it turns into an anisotropic phase with a very small value. As expected, in the present models, the energy density and particle density remain positive. So, it satisfies the energy conditions, showing that our models are as physically realistic as the present observational data. Thus, the solutions and interpretations presented in this paper may put some ideas on understanding the evolution of the early Universe in further research.

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