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Improved Distribution-free Control Chart for the Joint Monitoring of Location and Scale

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Abstract

The nonparametric (distribution-free) Shewhart-type control chart based on the Pettitt test has recently been introduced in the literature for joint monitoring of the location and the scale parameters of a continuous process. The chart is based on a simple random sampling (SRS) technique (denoted as SP-SRS chart). In the literature, the ranked set sampling (RSS) technique is preferred over the SRS technique as it reduces the variability and improves the performance of the control chart. The aim of this paper is to develop a distribution-free control chart based on the Pettitt test using the RSS technique (denoted as SP-RSS Chart) further to enhance the joint monitoring of location and scale. The run length performance of the proposed SP-RSS chart is compared with the SP-SRS chart. The comparison revealed that the proposed SP-RSS chart outperforms the SP-RSS chart for joint monitoring of the location and scale of a continuous process.

Keywords: Average run length; Joint monitoring; Location parameter; Scale parameter; Ranked set sampling; Simple random sampling.

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1. Introduction

Control charts are the most important statistical process monitoring (SPM) tools used to monitor manufacturing processes to detect any change in process parameters that may affect the output quality. Shewhart $\bar{X}$ and $R or S$ control charts are the most popular control charts for monitoring process mean and process variability. These control charts are easy to implement but are based on the fundamental assumption that the distribution of quality characteristics is normal. In real applications, there are many situations in which process data come from a non-normal distribution. In such situations, it is desirable to use distribution-free control charts. The main advantage of a distribution-free control chart is that it does not assume any probability distribution for the characteristic of interest. A formal definition of a nonparametric or distribution-free control chart is given in terms of its run-length distribution. The number of samples that need to be collected before a chart

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gives the first out-of-signal is a random variable called the run-length; the probability distribution of the run-length is referred to as the run-length distribution. If the in-control run-length distribution is the same for every continuous distribution, then the chart is called distribution-free or nonparametric. The location and scale of a process are the two main parameters most often monitored in distribution-free control charts. The existing distribution-free control charts are designed for monitoring location and scale by using separate control charts. Using two separate charts for monitoring location and scale can sometimes be difficult in practice for interpreting signals because the effect of changes in one of the parameters can affect the changes in another. The joint monitoring scheme with a single chart has received more attention in the recent literature due to its simplicity and clarity. A single control chart uses a statistic that combines two separate statistics, one each for mean and variance. Joint monitoring of a process involves two parameters, the mean (location) and variance (scale), and typically uses an efficient statistic for monitoring each parameter. Distribution-free joint monitoring scheme is an important area for research, and literature in this area is currently very limited and thus presents a great opportunity for further research.

2. Review of Literature

The problem of monitoring the location of a process is important in many applications. The location parameter could be the distribution's mean, median, or some percentiles. In the literature of distribution-free control charts, several charts are proposed for monitoring the location of univariate and multivariate processes. Bakir [1] developed a nonparametric control chart based on Wilcoxon signed-rank statistics. This chart was later modified by Chakraborti and Eryilmaz [2] by using several run-rules. Khilare and Shirke [3] developed the nonparametric Shewhart-type synthetic control chart based on sign statistics to monitor the location of a univariate process. Pawar and Shirke [4] developed the nonparametric Shewhart-type synthetic control chart based on the signed-rank statistics to monitor the location of a univariate process. Zombade and Ghute [5] developed a distribution-free control chart based on run statistics for monitoring the process location of a continuous process. Khilare and Shirke [6] proposed a nonparametric group-run control chart using sign statistics for monitoring the shift in the process location. Das [7] developed a multivariate nonparametric control chart based on a bivariate sign test in the multivariate process monitoring. Boone and Chakraborti [8] proposed two Shewhart-type multivariate nonparametric control charts based on multivariate forms of sign and signed-rank tests. Ghute and Shirke [9] proposed a nonparametric synthetic control chart based on the bivariate signed-rank test to monitor changes in the center of the bivariate process. Ghute and Shirke [10] also developed a nonparametric synthetic control chart based on a bivariate sign test to monitor changes in the center of the bivariate process.

The majority of existing distribution-free control charts focus on monitoring of location parameter. The problem of monitoring the scale parameter of a process is also important in many applications. For monitoring scale parameter of a process very few nonparametric control charts are available in the literature. Amin *et al.* [11] proposed a sign chart based on quartiles for process variation. Das [12] proposed a nonparametric control chart for controlling variability based on the squared rank test. Das [13] also developed a nonparametric control chart based on the rank test. Das and Bhattacharya [14] proposed a control chart for controlling variability based on some nonparametric tests. Murakami and Matsuki [15] developed a nonparametric control chart based on Mood statistics for dispersion. Khilare and Shirke [16] developed a nonparametric synthetic control chart based on sign statistics for process variability. Zombade and Ghute [17] provided nonparametric control charts for process variation based on Sukhatme's and Mood's tests. Shirke and Barale [18] proposed a nonparametric cumulative sum control chart for process dispersion using in-control deciles. Chakraborti *et al*. [19-21] presented extensive literature overviews on nonparametric control charts and discussed their advantages. Recently, Chakraborti and Graham [22] presented an updated overview and some results on distribution-free control charts.

The literature on distribution-free joint monitoring schemes is currently very limited. A few distribution-free joint monitoring schemes are available in the literature. Mukherjee and Chakraborti [23] proposed a single distribution-free control chart for joint monitoring based on Lepage [24] statistic, which constitutes a quadratic form that combines Wilcoxon rank-sum statistic for location and Ansari-Bradely statistic for scale. Chowdhury *et al*. [25] proposed a single distribution-free chart for joint monitoring based on the Cucconi test for equality of location and scale parameters of two populations. Zombade [26] developed a single distribution-free control chart based on Pettitt's [27] test statistic for joint monitoring of the location and scale parameters of a continuous process. Ghadage and Ghute [28] developed a distribution-free control chart for joint monitoring of location and scale based on a modified Lepage test proposed by Neuhäuser [29]. The test combines the Baumgartner and Ansari-Bradely statistics to detect location and scale changes jointly. All these control charts are based on the SRS method.

To enhance the efficiency of process monitoring charts, various sampling schemes have been implemented by researchers in SPM literature. RSS is one of the important sampling schemes suggested by McIntyre [30], which is highly beneficial and superior to the SRS scheme. Many authors have developed control charts for monitoring the process mean using RSS or its modification schemes. Salazar and Sinha [31] first developed a control chart for monitoring process mean using the RSS scheme. It was shown that the control chart based on the RSS scheme is superior to that of SRS. Muttlak and Al-Sabah [32] proposed several improved Shewhart-type mean charts for the process mean based on different RSS schemes. It was shown that all these charts perform better than the classical SRS control charts for means.

Some distribution-free control charts are also developed by researchers using the RSS scheme to monitor the location parameter of the process. Tapang *et al*. [33] proposed three nonparametric control charts based on the RSS scheme. Abid *et al*. [34] developed a nonparametric EWMA control chart based on a sign test using RSS to monitor the possible small shifts in the process mean. Abid *et al*. [35] developed a nonparametric

EWMA control chart based on Wilcoxon signed-rank statistics using the RSS scheme. Abid *et al*. [36] also proposed a nonparametric CUSUM control chart based on sign statistics using the RSS technique. Asghari *et al*. [37] proposed an RSS-based nonparametric sign control chart for monitoring process centers. Abbas *et al*. [38] proposed a DEWMA chart based on the Wilcoxon signed rank test under SRS and RSS techniques for efficient monitoring of the process location. Rasheed *et al*. [39] presented an RSS-based nonparametric double homogeneously weighted moving average (DHWMA) control chart based on Wilcoxon signed-rank statistic for enhanced monitoring of process location shift. Almanjahie *et al*. [40] proposed a nonparametric homogeneously weighted moving average based on the Wilcoxon signed-rank test with an RSS scheme for detecting shifts in the process location of a continuous and symmetric distribution.

The use of the RSS scheme for control charts leads to substantial improvements over the traditional control charts based on SRS. Based on a thorough literature review, it is observed that no one has developed a distribution-free control chart based on an RSS scheme for joint monitoring of location and scale. This is a research gap that needs to be explored. The purpose of this paper is to contribute to the research on the distribution-free joint monitoring scheme under the RSS scheme. In this paper, a distribution-free Shewhart-type control chart based on Pettitt's [27] test statistic is developed by using the RSS scheme for joint monitoring of location and scale parameters of a continuous process distribution. The test combines the Wilcoxon rank-sum and Mood statistics to detect location and scale changes jointly. The run-length performance of the proposed control chart is evaluated under normal and Laplace distributions through average run length (ARL), standard deviation of run length (SDRL), median, and some percentiles, including the first and third quartiles.

3. Pettitt Test for L and Scale

In this section, we briefly discuss the nonparametric test for location parameters, scale parameters, and joint location and scale parameters.

3.1. *Wilcoxon rank sum test for location*

Let $X = (X_1, X_2, \ldots, X_m)$ and $Y = (Y_1, Y_2, \ldots, Y_n)$ be two samples of size m and n from two populations with continuous distribution functions described as $F(x)$ and $G(y) = F(\theta + \lambda y)$ $\lambda > 0, -\infty < \theta < \infty$ respectively, where F is some unknown continuous distribution function. The constants θ and λ represent the unknown location and scale parameter. It is assumed that tie does not occur. The combined sample of size $N = m + n$ is given as $(X_1, X_2, \ldots, X_m, Y_1, Y_2, \ldots, Y_n)$. Based on the combined sample, an indicator variable Z_k and two sample test statistic W are defined respectively as

$$Z_k = \begin{cases} 1, & \text{when } k^{th} \text{ order statistic of the combined } N \text{ observations is } Y \\ 0, & \text{when } k^{th} \text{ order statistic of the combined } N \text{ observations is } X \end{cases} \quad (1)$$

$$W = \sum_{k=1}^{N} k\, Z_k \tag{2}$$

The mean and variance of statistic W are given as

$$E(W) = \frac{n(N+1)}{2} \text{ and } Var(W) = \frac{mn(N+1)}{12} \tag{3}$$

Clearly, W is the sum of the ranks of $Y_j's$ in the combined sample and represents the well-known Wilcoxon rank-sum (WRS) test statistic for location.

3.2. *Mood test for scale*

The Mood test is a two-sample scale test. In the combined sample of N observations with no ties, the average rank is the mean of the first N integers $\frac{N+1}{2}$. The number of deviations of the observations' ranks about this mean indicates relative spread. The test statistic based on the sum of squares of the deviation of ranks of $Y's$ from the average combined rank is the Mood test statistic for scale and is given as

$$M = \sum_{k=1}^{N} \left(k - \frac{N+1}{2}\right)^2 Z_k \tag{4}$$

The mean and variance of the statistic M is given as

$$E(M) = \frac{n(N^2-1)}{12} \text{ and } Var(M) = \frac{mn(N+1)(N^2-4)}{180} \tag{5}$$

A large value of M would imply that $Y's$ are more widely dispersed since it gives large weights to the tails of the arrangement.

3.3. *Pettitt test for location and scale*

Pettitt [27] test statistic for jointly testing location-scale parameters constitutes a quadratic form combining Wilcoxon rank-sum statistic for location and Mood statistic for scale. The Pettitt test statistic is given by

$$T = \left(\frac{W-E(W)}{\sqrt{Var(W)}}\right)^2 + \left(\frac{M-E(M)}{\sqrt{Var(M)}}\right)^2 \tag{6}$$

where W is the Wilcoxon rank sum statistic for location shift, and M is the Mood statistic for scale shift.

The Pettitt test statistic T given in Equation (6) is considered a control chart statistic for the proposed Shewhart-type distribution-free control chart for simultaneous monitoring of the location and scale of a continuous process. The chart indicates that a shift in location and/or scale has occurred if $T > H$, where H is an upper control limit of the chart.

4. Distribution-Free Control Chart for Joint Monitoring of Location and Scale

In this section, we develop a distribution-free control chart based on the Pettitt test statistic for simultaneous monitoring of the location and scale parameters of a continuous process. The single plotting statistic T for the joint monitoring of location and scale is given in Equation (6), and the chart is called the Shewhart-Pettitt chart (denoted as SP chart). To adopt the idea of two-sample tests for control chart implementation, m independent observations from an in-control process are used as a reference sample and compared to future sample subgroups of n independent observations. Let us consider $X = (X_1, X_2, ..., X_m)$ as reference sample of size m from an in-control process and that $Y = (Y_1, Y_2, ..., Y_n)$ be an arbitrary test sample of size n .The working mechanism of the proposed SP chart under the SRS scheme and RSS scheme is described below:

4.1. *Charting procedure of proposed SP chart under the SRS scheme*

The charting procedure of the proposed SP chart under the SRS scheme (denoted as SP-SRS chart) is as follows:

Step1: Collect Phase-I reference sample $X = (X_1, X_2, \ldots, X_m)$ of size m using SRS from an in-control process.

Step2: Let $Y = (Y_1, Y_2, \ldots, Y_n)$ be $j^{th}, j = 1,2,3, \ldots$ Phase-II (test) sample of size n using SRS scheme.

Step 3: Calculate W_j and M_j using Equations (2) and (4) for j^{th} test sample.

Step 4: Compute means and standard deviations of W and M statistics, respectively.

Step 5: Calculate the standardized W and M statistics respectively as

$$T_{1j} = \left(\frac{W - E_O(W)}{\sqrt{Var_0(W)}}\right) \text{and} T_{2j} = \left(\frac{M - E_O(M)}{\sqrt{Var_0(M)}}\right).$$

Step 6: Calculate the control chart statistic of the SP chart as $T_j = T_{1j}^2 + T_{2j}^2, j = 1,2,3, \ldots$

Step 7: Plot T_j against an upper control limit, $H > 0$.

Step 8: If T_j exceed H, the process is out of control at the j^{th} test sample. If not, the process is thought to be in control, and testing continues to the next sample.

4.2. *Charting procedure of proposed SP chart under RSS scheme*

In the RSS scheme, samples obtained will be ranked using another variable that relates to the variable of interest or variable to be the actual measurement. The procedure for the selection of a sample of size n using the RSS scheme is given below:

i. Select n^2 units with an SRS scheme from the target population.
ii. Randomly allocate these n^2 units in n groups each of size n.
iii. Rank the units in each group in ascending order of magnitude by personal judgment or visual inspection or by using some auxiliary variable.
iv. Select the smallest value from the first group and the second smallest value from the second group.

v. This procedure will continue, and the last sample unit corresponds to the largest value from the n^{th} group.

The charting procedure of the proposed SP chart under the RSS scheme (denoted as the SP-RSS chart) is the same as that of the SP-SRS chart. Only the SRS sample is replaced by the RSS sample in the computation of control chart statistic T.

Step1: Collect a reference sample of size m using RSS from an in-control process$X_{rss} = (X_{rss,_1}, X_{rss,_2}, \ldots, X_{rss,_m})$.

Step2: Collect a $j^{th}, j = 1,2,3, \ldots$ Phase-II (test) sample of size n using RSS as $Y_{rss} = (Y_{rss,_1}, Y_{rss,_2}, \ldots, Y_{rss,_n})$ …

Use Step 3 to Step 8 in the procedure of the SRS scheme using RSS samples instead of SRS samples.

5. Performance Evaluation and Analysis of the SP Chart

Implementation of the proposed SP chart requires the upper control limit H. Typically, in practice, it is determined for specified in-control average run length (ARL_0), say, 500 under SRS and RSS schemes. A Monte-Carlo simulation approach based on a sufficiently large number of possible samples is used to determine H. For a given pair of (m, n) values, a search is conducted with different values of H, and that value of H is obtained for which ARL_0 is equal to nominal (target) value. In the present study, the values of reference sample size and test sample size are selected respectively as $m = 30,50,100,150$and $n = 5,11,25$. The target value of in-control ARL is fixed as $ARL_0 = 500$. The values of upper control limit H for combinations (m, n) values under SRS scheme and RSS scheme are presented in Table 1.

Table 1. Charting constant *H* for the proposed SP chart under SRS and RSS schemes.

Reference sample size	Test sample size	Charting constant (upper control limit): *H*	
m	n	SRS	RSS
30	5	9.3410	5.2512
30	11	7.9432	3.0041
30	25	7.1904	1.2547
50	5	10.4690	5.4997
50	11	8.8690	3.1767
50	25	8.2412	1.3994
100	5	12.2002	5.9940
100	11	10.7845	3.8659
100	25	9.8005	1.9057
150	5	12.4158	6.2020
150	11	11.1169	3.8621
150	25	10.1898	1.8664

The performance of a control chart is generally studied through its run length distribution. If the run length distribution is skewed to the right, it is useful to come across various measures such as ARL, SDRL, and several percentiles, including the first and third quartiles, to characterize the distribution. The performance of the proposed SP chart both under in-control and out-of-control setups is studied under both SRS and RSS schemes. For the in-control setup, both the reference and the test samples are generated from a standard normal distribution. For a given pair of (m, n) values, the upper control limit H is obtained for nominal (target) ARL_0 of 500, and different characteristics of the in-control run-length distribution are obtained. The simulation results are presented in Tables 2 and 3 under SRS and RSS schemes, respectively.

Table 2. In-control performance characteristics of the SP chart under the SRS scheme.

m	n	H	ARL_0	$SDRL_0$	5th Percentile	1st Quartile	Median	3rd Quartile	95th Percentile
30	5	9.3410	500.9	500.4	27	145	348	692	1499
30	11	7.9432	499.5	499.0	25	144	348	690	1500
30	25	7.1904	501.2	500.7	26	144	347	694	1505
50	5	10.4690	500.5	500.0	26	142	350	697	1494
50	11	8.8690	500.6	500.1	26	144	350	694	1509
50	25	8.2412	499.6	499.1	25	142	346	696	1499
100	5	12.2002	499.4	498.9	25	143	347	693	1501
100	11	10.7845	499.9	499.4	26	145	347	692	1487
100	25	9.8005	500.4	499.9	26	145	346	693	1508
150	5	12.4158	500.6	500.1	26	144	346	695	1495
150	11	11.1169	500.0	499.5	25	144	348	694	1493
150	25	10.1898	499.6	499.1	26	144	348	691	1505

Table 3. In-control performance characteristics of the SP chart under RSS schemes.

m	n	H	ARL_0	$SDRL_0$	5th Percentile	1st Quartile	Median	3rd Quartile	95th Percentile
30	5	5.2512	499.8	499.3	26	145	348	689	1499
30	11	3.0041	500.6	500.1	27	144	347	698	1486
30	25	1.2547	500.5	500.0	26	145	344	695	1493
50	5	5.4997	501.0	500.5	27	145	347	698	1506
50	11	3.1767	499.7	499.2	26	144	348	697	1488
50	25	1.3994	500.7	500.2	26	146	349	694	1498
100	5	5.9940	500.1	499.6	26	144	349	696	1482
100	11	3.8659	501.0	500.5	26	145	347	695	1494
100	25	1.9057	499.9	499.4	25	144	348	691	1501
150	5	6.2020	500.2	499.7	26	144	349	69	1489
150	11	3.8621	500.9	500.4	26	144	346	693	1502
150	25	1.8664	502.6	502.1	26	144	349	694	1510

From Tables 2 and 3, it is observed that the target $ARL_0 = 500$ is much larger than the median for all (m, n) combinations. Hence, the in-control run-length distribution of the SP chart is highly skewed to the right. To investigate the out-of-control performance of the proposed SP chart under SRS and RSS schemes, the underlying process distributions considered in the study are normal and Laplace distributions. The Laplace distribution is considered a process distribution to study the effect of heavy-tailed distribution on the performance of the SP chart. The distribution of observations from the process is considered to have mean zero and variance one for both the process distributions under study.

5.1. *Performance comparison of SP chart under SRS and RSS scheme for normal distribution*

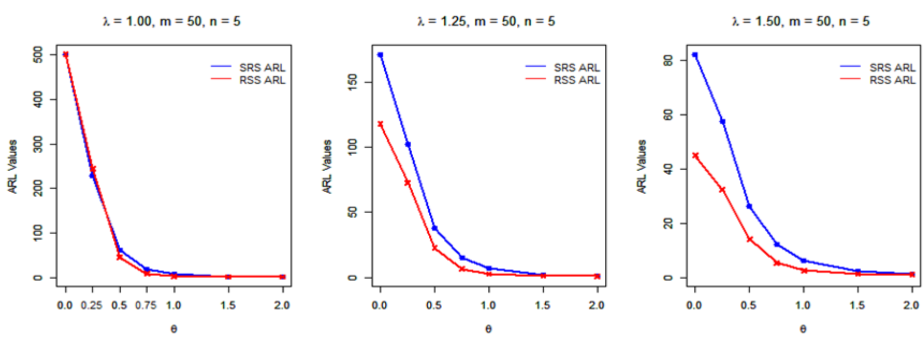
In order to investigate the out-of-control performance comparison of the proposed SP chart under SRS and RSS schemes, the normal distribution is considered as the underlying process distribution. Samples are generated from $N(\theta,\lambda)$ distribution, with in-control samples coming from $N(0,1)$ distribution. To examine the effects of shifts in the process mean and the process variability, 30 combinations of (θ,λ) values are considered with $\theta = 0.0,0.25,0.5,1.0,1.5,2.0$and $\lambda = 1.0,1.25,1.5,1.75,2.0$. Tables 4, 5, and Fig. 1 present the performance characteristics of the SP-SRS and SP-RSS charts when the underlying process distribution is normal with combinations of the reference and test sample sizes $m = 50,100$ and $n = 5$.

Table 4. Performance comparisons of SP chart under SRS and RSS scheme for the $N(\theta,\lambda)$ distribution.

$n = 50, m = 5$															
θ	λ	SP-SRS							SP-RSS						
		ARL	SDRL	P5	Q1	Q2	Q3	P95	ARL	SDRL	P5	Q1	Q2	Q3	P95
0.0	1.0	500.5	500.0	26	142	350	697	1494	501.0	500.5	27	145	347	698	1506
0.25	1.0	229.1	228.6	12	66	158	316	688	244.6	244.1	13	71	171	342	727
0.5	1.0	61.4	60.9	4	18	42	85	184	44.4	43.9	3	13	31	61	133
1.0	1.0	7.4	6.8	1	2	5	10	21	2.6	2.1	1	1	2	3	7
1.5	1.0	2.1	1.5	1	1	2	3	5	1.1	0.3	1	1	1	1	2
2.0	1.0	1.2	0.5	1	1	1	1	2	1.0	0.0	1	1	1	1	1
0.0	1.25	171.2	170.7	9	50	119	237	510	117.7	117.2	7	34	81	163	350
0.25	1.25	102.5	102.0	6	30	71	143	307	72.8	72.3	4	21	51	101	218
0.5	1.25	37.9	37.4	2	11	26	52	113	22.7	22.2	2	7	16	31	67
1.0	1.25	6.7	6.2	1	2	5	9	19	2.6	2.0	1	1	2	3	7
1.5	1.25	2.2	1.7	1	1	2	3	6	1.1	0.4	1	1	1	1	2
2.0	1.25	1.3	0.6	1	1	1	1	2	1.0	0.1	1	1	1	1	1
0.0	1.5	82.3	81.8	5	24	57	113	247	45.0	44.5	3	13	31	62	133
0.25	1.5	57.5	57.0	3	17	40	80	171	32.4	31.9	2	10	23	45	96
0.5	1.5	26.3	25.8	2	8	18	36	77	14.1	13.6	1	4	10	19	41
1.0	1.5	6.2	5.6	1	2	4	8	17	2.5	1.9	1	1	2	3	6
1.5	1.5	2.3	1.7	1	1	2	3	6	1.2	0.4	1	1	1	1	2
2.0	1.5	1.3	0.7	1	1	1	2	3	1.0	0.1	1	1	1	1	1
0.0	1.75	47.7	47.2	3	14	33	65	142	23.3	22.8	2	7	16	32	68
0.25	1.75	36.4	35.9	2	11	25	50	108	18.1	17.6	1	5	13	25	53
0.5	1.75	20.0	19.5	1	6	14	27	59	9.6	9.1	1	3	7	13	28
1.0	1.75	5.8	5.3	1	2	4	8	16	2.4	1.8	1	1	2	3	6
1.5	1.75	2.4	1.8	1	1	2	3	6	1.2	0.5	1	1	1	1	2
2.0	1.75	1.4	0.8	1	1	1	2	3	1.0	0.1	1	1	1	1	1
0.0	2.0	31.5	31.0	2	9	22	43	93	14.3	13.8	1	4	10	20	42
0.25	2.0	25.5	25.0	2	8	18	35	75	11.9	11.4	1	4	8	16	35
0.5	2.0	15.8	15.2	1	5	11	22	46	7.3	6.8	1	2	5	10	21
1.0	2.0	5.4	4.9	1	2	4	7	15	2.3	1.8	1	1	2	3	6
1.5	2.0	2.4	1.9	1	1	2	3	6	1.2	0.5	1	1	1	1	2
2.0	2.0	1.5	0.8	1	1	1	2	3	1.0	0.2	1	1	1	1	1

Table 5. Performance comparisons of SP chart under SRS and RSS scheme for the $N(\theta, \lambda)$ distribution.

		$n = 100, m = 5$													
θ	λ	SP-SRS							SP-RSS						
		ARL	SDRL	P5	Q1	Q2	Q3	P95	ARL	SDRL	P5	Q1	Q2	Q3	P95
0.0	1.0	499.4	498.9	25	143	347	693	1501	500.1	499.6	26	144	349	696	1482
0.25	1.0	285.3	284.8	15	84	197	396	850	284.4	283.9	15	82	198	394	849
0.5	1.0	75.6	75.1	4	22	53	104	226	54.7	54.2	3	16	38	75	163
1.0	1.0	8.4	7.9	1	3	6	11	24	3.0	2.4	1	1	2	4	8
1.5	1.0	2.3	1.7	1	1	2	3	6	1.1	0.3	1	1	1	1	2
2.0	1.0	1.2	0.5	1	1	1	1	2	1.0	0.0	1	1	1	1	1
0.0	1.25	181.5	181.0	10	52	126	252	548	119.1	118.6	7	34	83	164	358
0.25	1.25	122.9	122.4	7	36	85	171	367	82.3	81.8	5	24	57	114	246
0.5	1.25	45.3	44.8	3	14	32	63	135	26.8	26.3	2	8	19	37	80
1.0	1.25	7.6	7.1	1	3	5	10	22	2.9	2.3	1	1	2	4	8
1.5	1.25	2.4	1.8	1	1	2	3	6	1.1	0.4	1	1	1	1	2
2.0	1.25	1.3	0.6	1	1	1	1	3	1.0	0.1	1	1	1	1	1
0.0	1.5	88.6	88.1	5	26	62	123	263	46.5	46.0	3	14	33	64	139
0.25	1.5	66.6	66.1	4	20	46	92	200	36.1	35.6	2	11	25	50	106
0.5	1.5	31.0	30.5	2	9	22	43	92	16.0	15.5	1	5	11	22	47
1.0	1.5	6.9	6.4	1	2	5	9	20	2.8	2.2	1	1	2	4	7
1.5	1.5	2.5	1.9	1	1	2	3	6	1.2	0.5	1	1	1	1	2
2.0	1.5	1.4	0.7	1	1	1	2	3	1.0	0.1	1	1	1	1	1
0.0	1.75	52.5	52.0	3	15	37	72	156	23.8	23.3	2	7	17	33	71
0.25	1.75	42.6	42.1	3	13	30	59	127	20.0	19.5	2	6	14	27	59
0.5	1.75	23.0	22.5	2	7	16	32	68	10.9	10.4	1	3	8	15	32
1.0	1.75	6.4	5.9	1	2	5	9	18	2.6	2.1	1	1	2	3	7
1.5	1.75	2.5	2.0	1	1	2	3	7	1.2	0.5	1	1	1	1	2
2.0	1.75	1.5	0.8	1	1	1	2	3	1.0	0.1	1	1	1	1	1
0.0	2.0	34.6	34.1	2	10	24	48	103	14.8	14.3	1	5	10	20	44
0.25	2.0	29.5	29.0	2	9	21	41	87	12.8	12.3	1	4	9	18	38
0.5	2.0	18.0	17.5	1	5	13	25	53	8.0	7.5	1	3	6	11	23
1.0	2.0	6.0	5.4	1	2	4	8	17	2.5	1.9	1	1	2	3	6
1.5	2.0	2.6	2.0	1	1	2	3	7	1.2	0.6	1	1	1	1	2
2.0	2.0	1.5	0.9	1	1	1	2	3	1.0	0.2	1	1	1	1	1



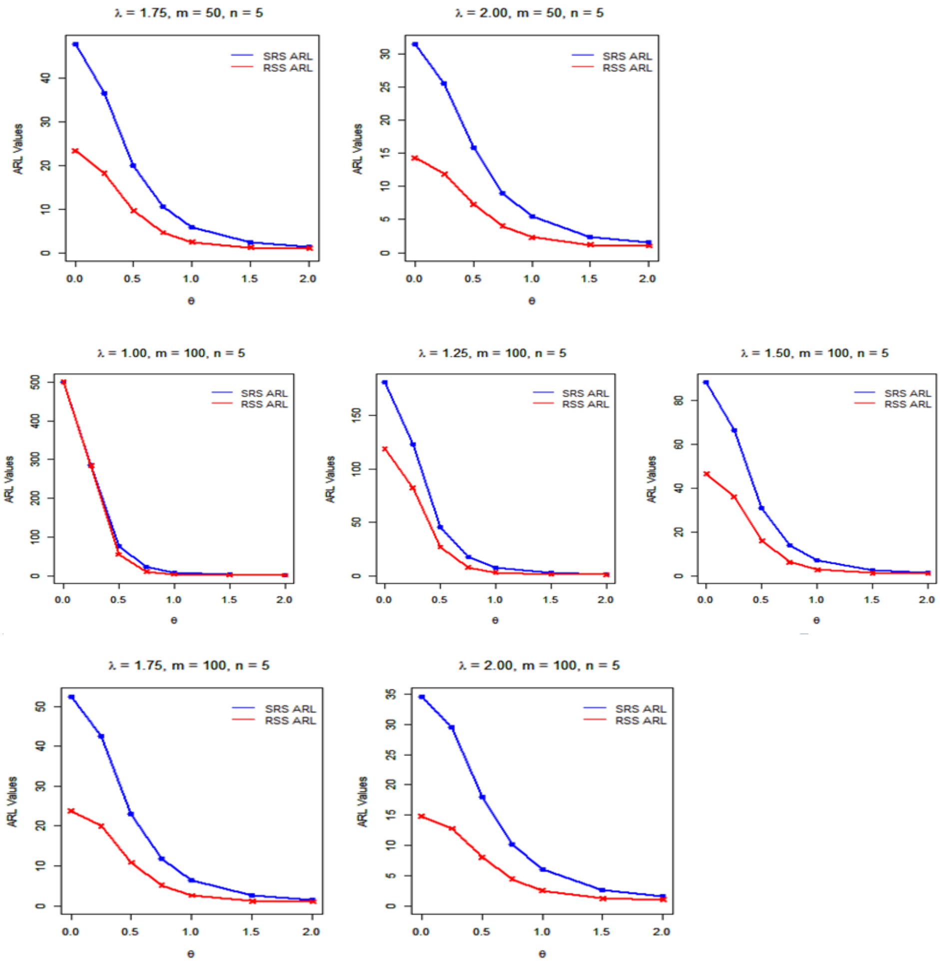


Fig. 1. ARL performance of SP chart under SRS and RSS schemes for $N(\theta, \lambda)$ distribution.

The results in Tables 4, 5, and Fig. 1 clearly indicate that the out-of-control run-length distributions are also skewed to the right. It is observed that, for a fixed m, n and a given ARL_0The out-of-control ARL values and the percentiles all decrease sharply with increasing shifts in the location and the increasing shift in the scale. It indicates that the proposed SP chart effectively detects shifts in location and/or scale. The proposed SP chart under the SRS and RSS scheme detects a shift in the scale more quickly than that in the location. For example, from Table 4, we observe that for a 25% increase in location when the scale is in-control, the ARL decreases by 54% under the SRS scheme and decreases by 51% under the RSS scheme, whereas for a 25% increase in a scale when the location is in-control, ARL decreases by 66% under SRS scheme and decreases by 76% under RSS scheme. Finally, when location and scale increase by 25%, the ARL decreases by 79% under the SRS scheme and decreases by 85% under the RSS scheme. The pattern is the same for SDRL; it decreases for an increase in the shift in both parameters but decreases more for a shift in scale. For example, from Table 4, for a 25 % increase in location, the SDRL decreases by 54 % under the SRS scheme and 51 % under the RSS scheme, but for a 25 % increase in scale, the SDRL decreases by 66 % under the SRS scheme, and decreases 76 % under RSS scheme. Further, Table 4 shows that for normal process distribution, the SP-RSS chart clearly outperforms the SP-SRS chart if there is a shift in the scale parameter along with some shift in location. As the location parameter increases, the SP-RSS chart shows better performance than the SP-SRS chart. If there is a small shift in the location along with the scale parameter, the in-control SP-RSS chart shows better performance than the SP-RSS chart. In addition, Fig. 1 illustrates that the proposed SP-RSS control chart performs better than the SP-SRS chart.

5.2. *Performance comparison of SP chart under SRS and RSS scheme for Laplace distribution*

Laplace distribution is included in the study as an ailed process distribution to study the effect of heavy-tailed distribution on the performance of the proposed SP chart under the SRS and RSS scheme, Laplace distribution is included in the study as heavy tailed process distribution. The performance characteristics of the run-length were evaluated when the in-control sample is from a $L(0,1)$ distribution that has a mean of 0 and a variance of 2. Test samples are generated from the Laplace distribution with mean θ and standard deviation λ

To examine the effect of shifts in location and scale, as in normal cases, 30 combinations of (θ, λ) values are considered. Tables 6, 7, and Fig. 2 present the performance characteristics of proposed SP-SRS and SP-RSS charts when the underlying process distribution is the Laplace distribution with combinations of reference and test samples of size $m = 50{,}100$ and $n = 5$.

Table 6. Performance comparisons of SP chart under SRS and RSS scheme for the $L(\theta, \lambda)$ distribution.

		$n = 50, m = 5$													
θ	λ	SP-SRS							SP-RSS						
		ARL	SDRL	P5	Q1	Q2	Q3	P95	ARL	SDRL	P5	Q1	Q2	Q3	P95
0.0	1.0	500.5	500.0	27	143	345	694	1501	499.3	498.8	26	144	346	692	1490
0.25	1.0	442.0	441.5	22	126	306	613	850	295.3	294.8	16	85	203	408	888
0.5	1.0	191.6	191.1	10	56	134	266	226	70.1	69.6	4	21	49	97	212
1.0	1.0	25.3	24.8	2	8	18	35	24	4.7	4.1	1	2	3	6	13
1.5	1.0	5.0	4.5	1	2	4	7	6	1.4	0.7	1	1	1	2	3
2.0	1.0	2.0	1.4	1	1	1	3	2	1.0	0.2	1	1	1	1	1
0.0	1.25	120.8	120.3	7	35	84	168	548	71.3	70.8	4	21	50	99	212
0.25	1.25	110.2	109.7	6	32	76	152	367	52.8	52.3	3	16	37	73	156
0.5	1.25	66.2	65.7	4	19	46	92	135	24.2	23.7	2	7	17	33	72
1.0	1.25	15.9	15.4	1	5	11	22	22	4.1	3.5	1	2	3	5	11
1.5	1.25	4.6	4.1	1	2	3	6	6	1.5	0.8	1	1	1	2	3
2.0	1.25	2.1	1.6	1	1	2	3	3	1.1	0.3	1	1	1	1	2
0.0	1.5	45.9	45.3	3	14	32	63	263	21.9	21.4	2	7	15	30	64
0.25	1.5	43.8	43.3	3	13	31	60	200	18.2	17.7	1	6	13	25	53

0.5	1.5	31.2	30.7	2	9	22	43	92	11.3	10.8	1	4	8	15	33
1.0	1.5	11.3	10.8	1	4	8	15	20	3.4	2.8	1	1	2	4	9
1.5	1.5	4.3	3.8	1	2	3	6	6	1.5	0.9	1	1	1	2	3
2.0	1.5	2.2	1.6	1	1	2	3	3	1.1	0.4	1	1	1	1	2
0.0	1.75	23.1	22.6	2	7	16	32	156	10.2	9.6	1	3	7	14	29
0.25	1.75	22.2	21.7	2	7	15	31	127	8.9	8.4	1	3	6	12	26
0.5	1.75	17.9	17.4	1	6	13	25	68	6.5	6.0	1	2	5	9	18
1.0	1.75	8.5	7.9	1	3	6	11	18	2.8	2.3	1	1	2	4	7
1.5	1.75	3.9	3.4	1	1	3	5	7	1.5	0.9	1	1	1	2	3
2.0	1.75	2.2	1.7	1	1	2	3	3	1.1	0.4	1	1	1	1	2
0.0	2.0	13.8	13.3	1	4	10	19	103	6.0	5.5	1	2	4	8	17
0.25	2.0	13.6	13.1	1	4	9	19	87	5.5	5.0	1	2	4	7	16
0.5	2.0	11.7	11.1	1	4	8	16	53	4.4	3.9	1	2	3	6	12
1.0	2.0	6.6	6.1	1	2	5	9	17	2.4	1.9	1	1	2	3	6
1.5	2.0	3.6	3.0	1	1	3	5	7	1.5	0.9	1	1	1	2	3
2.0	2.0	2.2	1.7	1	1	2	3	3	1.2	0.4	1	1	1	1	2

Table 7. Performance comparisons of SP chart under SRS and RSS scheme for the $L(\theta,\lambda)$ distribution.

$n = 100, m = 5$															
θ	λ	SP-SRS							SP-RSS						
		ARL	SDRL	P5	Q1	Q2	Q3	P95	ARL	SDRL	P5	Q1	Q2	Q3	P95
0.0	1.0	500.8	500.3	26	144	347	695	1513	500.4	499.9	26	144	347	694	1505
0.25	1.0	259.8	259.3	14	75	180	360	780	330.8	330.3	18	96	229	460	985
0.5	1.0	96.0	95.5	5	28	67	133	286	94.2	93.7	5	28	66	131	281
1.0	1.0	13.6	13.1	1	4	10	19	40	4.9	4.4	1	2	4	7	14
1.5	1.0	3.4	2.8	1	1	2	4	9	1.3	0.7	1	1	1	2	3
2.0	1.0	1.6	1.0	1	1	1	2	4	1.0	0.2	1	1	1	1	1
0.0	1.25	120.2	119.7	7	35	84	167	360	68.5	68.0	4	20	48	95	204
0.25	1.25	81.4	80.9	4	24	57	113	242	54.7	54.2	3	16	38	76	164
0.5	1.25	41.9	41.4	3	12	29	58	124	27.1	26.6	2	8	19	37	81
1.0	1.25	10.0	9.5	1	3	7	14	29	4.1	3.6	1	2	3	6	11
1.5	1.25	3.4	2.8	1	1	2	4	9	1.5	0.8	1	1	1	2	3
2.0	1.25	1.7	1.1	1	1	1	2	4	1.1	0.3	1	1	1	1	2
0.0	1.5	45.9	45.4	3	13	32	64	136	20.7	20.2	2	6	15	28	61
0.25	1.5	36.4	35.9	2	11	25	50	108	18.0	17.5	1	6	13	25	53
0.5	1.5	23.0	22.5	2	7	16	32	68	11.7	11.2	1	4	8	16	34
1.0	1.5	7.9	7.4	1	3	6	11	23	3.4	2.9	1	1	2	4	9
1.5	1.5	3.3	2.8	1	1	2	4	9	1.5	0.9	1	1	1	2	3
2.0	1.5	1.8	1.2	1	1	1	2	4	1.1	0.4	1	1	1	1	2
0.0	1.75	23.2	22.7	2	7	16	32	69	9.5	9.0	1	3	7	13	28
0.25	1.75	20.0	19.4	1	6	14	27	59	8.8	8.3	1	3	6	12	25
0.5	1.75	14.5	14.0	1	5	10	20	42	6.7	6.2	1	2	5	9	19
1.0	1.75	6.5	6.0	1	2	5	9	18	2.8	2.3	1	1	2	4	7
1.5	1.75	3.2	2.6	1	1	2	4	8	1.5	0.9	1	1	1	2	3
2.0	1.75	1.9	1.3	1	1	1	2	5	1.1	0.4	1	1	1	1	2
0.0	2.0	14.2	13.7	1	4	10	20	41	5.7	5.2	1	2	4	8	16
0.25	2.0	12.6	12.1	1	4	9	17	37	5.4	4.8	1	2	4	7	15
0.5	2.0	10.1	9.6	1	3	7	14	29	4.4	3.9	1	2	3	6	12
1.0	2.0	5.4	4.9	1	2	4	7	15	2.4	1.8	1	1	2	3	6
1.5	2.0	3.0	2.4	1	1	2	4	8	1.5	0.8	1	1	1	2	3
2.0	2.0	1.9	1.3	1	1	1	2	5	1.2	0.4	1	1	1	1	2

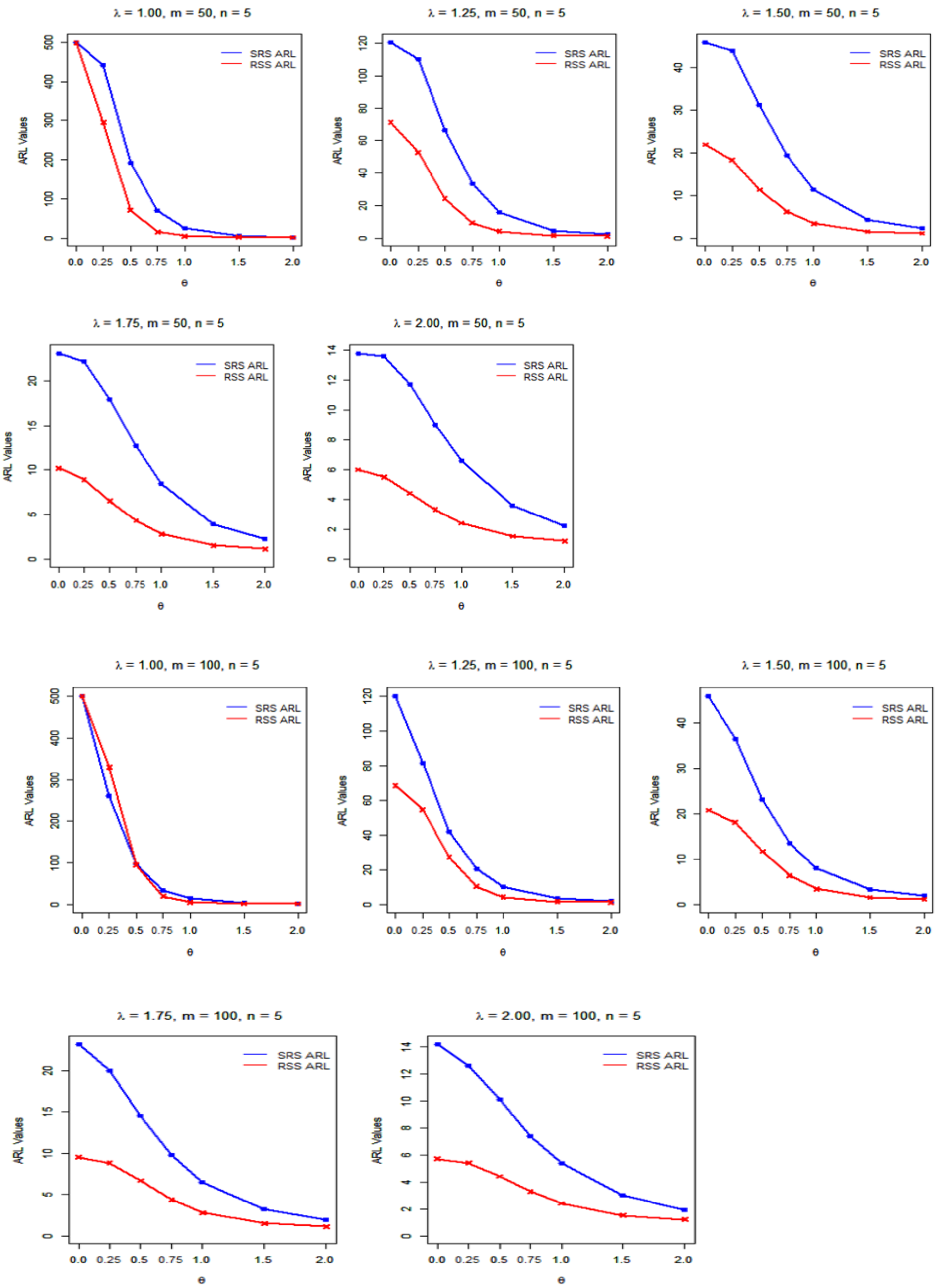


Fig. 2. ARL performance of SP chart Under SRS and RSS schemes for $L(\theta, \lambda)$ distribution.

From Tables 6, 7, and Fig 2, it is observed that when the underlying distribution is Laplace, the general pattern of run-length characteristics remains the same as in the case of normal distribution. However, the out-of-control ARL values for detecting a shift in a location under Laplace distribution are larger than that of the ARL values under normal process distribution. The out-of-control ARL values for detecting small shifts in location and/or scale under Laplace distribution are smaller than the ARL values under normal process distribution, whereas the out-of-control ARL values for detecting large shifts in location and/or variability under Laplace distribution are larger than that of ARL values under normal process distribution under both SRS and RSS schemes.

6. Conclusion

In this paper, a single distribution-free Shewhart-type control chart based on the Pettitt test is developed to closely monitor the location and scale parameters of a continuous process distribution. Both in-control and out-of-control performance of the proposed SP chart are studied under SRS and RSS schemes for normal and heavy-tailed Laplace distributions. The various performance characteristics are examined, such as mean, standard deviation, median, and some percentiles of the run-length distribution. It is observed that the proposed SP chart under SRS and RSS schemes maintains its designed in-control ARL under the considered process distributions. The chart is more efficient under the RSS scheme than the SRS scheme for normal and Laplace distributions.

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