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# Solution of Exponential Diophantine Equation $n^x + 43^y = z^2$, where $n \equiv 2(mod\ 129)$ and $n + 1$ is not a Perfect Square

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**Abstract**

Nowadays, researchers are very interested in studying various Diophantine equations due to their importance in Cryptography, Chemistry, Knot Theory, Astronomy, Geometry, Trigonometry, Biology, Algebra, Electrical Engineering, Economics, and Astrology. The present paper is about the non-negative integer solution of the exponential Diophantine equation $n^x + 43^y = z^2$, where $x, y, z$ are non-negative integers, $n$ is a positive integer with $n \equiv 2(mod\ 129)$ and $n + 1$ is not a perfect square. The authors use the famous Catalan conjecture for this purpose. Results of the present paper indicate that 2, 3, 0, and 3 are the only required values of $n, x, y$ and $z$, respectively, that satisfy the exponential Diophantine equation $n^x + 43^y = z^2$, where $x, y, z$ are non-negative integers, $n$ is a positive integer with $n \equiv 2(mod\ 129)$ and $n + 1$ is not a perfect square. The present technique of this paper proposes a new approach to solving the Diophantine equations, which is the main scientific contribution of this study, and it is very beneficial, especially for researchers, scholars, academicians, and people interested in the same field.

*Keywords*: Diophantine equation; Catalan conjecture; Perfect square; Solution; Mathematical induction method.


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## 1. Introduction

Various ancient time's famous puzzles such as the Monkey and coconut puzzle, Mahavira puzzles and problems of class numbers, factorization of polynomials, representation of natural numbers as sums of three cubes or four cubes or biquadrates, business investments, network flow, determining the rational points on the curve, balancing of chemical equations, analysis of the growth of microorganisms, pole placements, solving the quartic and quintic equations, preserving the data privacy depend for their solutions on the consideration of linear or non-linear Diophantine equations [1,2]. There is no universal technique available for addressing the Diophantine equations. So, researchers are keenly interested in developing new techniques for solving these equations. The Euler's method,

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decomposition method, continued fraction method, modular arithmetic method, parametric method, method of inequalities, method of infinite descent, and mathematical induction method are well-known methods for addressing the particular Diophantine equations and very well documented in the literature [3-5]. However, these methods are not enough for solving all types of Diophantine equations. Sroysang solved Diophantine equation $5^x + 63^y = z^2$ by using Catalan's conjecture method [6]. Suvarnamani [7] proved that $(x, y, z) = (1, 0, 2)$ is the only triplet of non-negative integers that satisfies the Diophantine equation $p^x + (p + 1)^y = z^2$ for $p = 3$.

Aggarwal [8,9] studied two Diophantine equations $193^x + 211^y = z^2$ and $(2^{2m+1} - 1) + (13)^n = z^2$ with the help of the modular arithmetic method and proved that these equations are not solvable in the set of non-negative integers. Hoque and Kalita [10] used Catalan's conjecture and studied the Diophantine equation $(p^q - 1)^x + p^{qy} = z^2$, where $q$ is a positive integer greater than one, and $p$ is a prime. Aggarwal and Sharma [11] proved that there is no non-negative integer value of unknowns $x, y$ and $z$ that satisfies the Diophantine equation $379^x + 397^y = z^2$. Bhatnagar and Aggarwal [12] examined the exponential Diophantine equation $421^p + 439^q = r^2$ and showed that there is no triplet of non-negative integers $p, q$ and $r$ that converts the equation $421^p + 439^q = r^2$ into an inequality. Aggarwal [13] studied Diophantine equations $323^x + 85^y = z^2$ with the help of Catalan's conjecture and proved that this Diophantine equation has a unique non-negative integer solution, namely $(x, y, z) = (1, 0, 18)$. Aggarwal et al. [14] applied the modular arithmetic method to the problem of the solution of Diophantine equation $223^x + 241^y = z^2$.

The Diophantine equation $143^x + 45^y = z^2$ was examined by Aggarwal et al. [15]. They showed that the triplet $(x, y, z) = (1, 0, 12)$ satisfies this equation, and it is the only non-negative integer solution. Tangjai and Chubthaisong [16] studied the Diophantine equation $3^x + p^y = z^2$, where $p \equiv 2(mod\ 3)$. The solution of exponential non-linear Diophantine equation $\beta^x + (\beta + 18)^y = z^2$, where $x, y, z$ are non-negative integers and $\beta, (\beta + 18)$ are primes such that $\beta$ has the form $6n + 1$ with the natural number $n$, as given by Aggarwal et al. [17]. Aggarwal and Kumar [18] studied the Diophantine equation $13^{2m} + (6r + 1)^n = z^2$. They proved that this equation has no solution in the whole number set. Biswas [19] discussed the solution of the Diophantine equation $3^x + 35^y = z^2$, where $x, y, z$ are whole numbers. He showed that this equation has two non-negative integer solutions, namely $(x, y, z) = (1, 0, 2)$ and $(x, y, z) = (0, 1, 6)$.

The aim of the present paper is to demonstrate the problem of the existence of the solution of exponential Diophantine equation $n^x + 43^y = z^2$, where $x, y, z$ are non-negative integers, $n$ is a positive integer with $n \equiv 2(mod\ 129)$ and $n + 1$ is not a perfect square. This study is significant because it provides a new approach to solving the Diophantine equation. The present approach provides a way of checking the existence of the solution of the particular Diophantine equations without doing complicated computational work. It has been noted that the present method accurately and precisely provides the solution (if it exists) for this study's particular Diophantine equation. A variety of problems in Algebra, Astronomy, Economics, Trigonometry, and Number

Theory may be solved using this approach in the future. The residual of this study is designed in five sections. Section 2 deals with the notation. Some preliminary concepts and Lemmas are given in Section 3, which will be used in developing our main results. Section 4 focuses on our study's main results with some interesting analogies. Finally, in Section 5, we summarized the conclusions of our study.

## 2. Notation

$\in$: Belongs to
$\notin$: Does not belong to
$Z^+$: The set of positive integers
$Z^+ \cup \{0\}$: The set of non-negative integers
$S$: The set of perfect squares
$\forall$: for all
$\equiv$: Congruent to
$P$: The set of prime numbers
$Z^+_{odd}$: The set of odd positive integers
$Z^+_{even}$: The set of even positive integers
$Z^+_{even} \cup \{0\}$: The set of non-negative even integers
$N$: The set of natural numbers

## 3. Preliminaries

**Proposition**: **3.1** Catalan's conjecture [20-21]: The Diophantine equation ${\omega_1}^{\alpha} - {\omega_2}^{\beta} = 1$, where $\omega_1, \omega_2, \alpha, \beta$ are integers such that $\min\{\omega_1, \omega_2, \alpha, \beta\} > 1$, has a unique solution, and it is given by $(\omega_1, \omega_2, \alpha, \beta) = (3, 2, 2, 3)$.
**LEMMA: 3.2** If $n \in Z^+$ such that $n \equiv 2 (mod\ 129)$, then $n \equiv 2 (mod\ 3)$ and $n \equiv 2 (mod\ 43)$.
**PROOF:** Suppose $n \in Z^+$ such that $n \equiv 2 (mod\ 129)$. Then 129 divides $n - 2$. So $n - 2 = 129t$ for some integer $t$. Thus $n - 2 = 3(43t)$ for some integers $t$ and $n - 2 = 43(3t)$ for some integer $t$. Hence $n \equiv 2 (mod\ 3)$ and $n \equiv 2 (mod\ 43)$.
**LEMMA: 3.3** There is no non-negative integer value of unknowns $y$ and $z$ that satisfy the Diophantine equation $43^y + 1 = z^2$, where $y, z \in Z^+ \cup \{0\}$.
**PROOF:** Suppose $y, z \in Z^+ \cup \{0\}$ such that $43^y + 1 = z^2$. If $y = 0$, then $z^2 = 43^0 + 1 = 2$, which is not possible due to the nature of $z$. Then $y \geq 1$. It follows that $z^2 = 43^y + 1 \geq 43^1 + 1 = 44$. Then $z \geq 7$. Now, we consider the equation $z^2 - 43^y = 1$. By Proposition 3.1, we have $y = 1$. We obtain that $z^2 = 44$. This is a contradiction due to the nature of $z$.

Hence, there is no non-negative integer value of unknowns $y$ and $z$ that satisfy the Diophantine equation $43^y + 1 = z^2$, where $y, z \in Z^+ \cup \{0\}$.
**LEMMA: 3.4** Suppose $n \in Z^+$ such that $n \equiv 2 (mod\ 129)$ and $n + 1 \notin S$. Then the unique non-negative integer value of unknowns $n, x, z$ is $2, 3, 3$ respectively that satisfy the Diophantine equation $n^x + 1 = z^2$, where $x, z \in Z^+ \cup \{0\}$.

**PROOF:** Suppose $x, z \in Z^+ \cup \{0\}$ and $n \in Z^+$ such that $n^x + 1 = z^2$, where $n \equiv 2 (mod 129)$ and $n + 1 \notin S$. If $x = 0$, then $z^2 = n^0 + 1 = 2$, which is not possible due to the nature of $z$. Now if $x = 1$, then $z^2 = n^1 + 1 = n + 1$, which is not possible due to the nature of $n + 1$. Now, if $x > 1$ then $n^x + 1 = z^2$ gives $z^2 = n^x + 1 > n^1 + 1 > 2$. Then $z > 1$. Now, we consider the equation $z^2 - n^x = 1$. By Proposition 3.1, we have $n = 2, x = 3, z = 3$.

Hence, the unique non-negative integer value of unknowns $n, x, z$ is $2, 3, 3$ respectively that satisfy the Diophantine equation $n^x + 1 = z^2$, where $x, z \in Z^+ \cup \{0\}$ and $n \in Z^+$ with $n \equiv 2 (mod\ 129)$ and $n + 1 \notin S$.

**LEMMA: 3.5** If $z \in Z^+ \cup \{0\}$, then $z^2 \equiv 0, 1\ (mod\ 3)$.

**PROOF:** Let $z \in Z^+ \cup \{0\}$. Then $z \equiv 0, 1, 2\ (mod\ 3)$.

**CASE 1:** $z \equiv 0 (mod\ 3)$. Then $z^2 \equiv 0\ (mod\ 3)$.

**CASE 2:** $z \equiv 1 (mod\ 3)$. Then $z^2 \equiv 1\ (mod\ 3)$.

**CASE 3:** $z \equiv 2 (mod\ 3)$. Then $z^2 \equiv 4\ (mod\ 3) \Rightarrow z^2 \equiv 1\ (mod\ 3)$.

Hence $z^2 \equiv 0, 1\ (mod\ 3)$, where $z \in Z^+ \cup \{0\}$.

**LEMMA: 3.6** If $z \in Z^+ \cup \{0\}$, then

$z^2 \equiv 0, 1, 4, 6, 9, 10, 11, 13, 14, 15, 16, 17, 21, 23, 24, 25, 31, 35, 36, 38, 40, 41 (mod\ 43)$.

**PROOF:** Let $z \in Z^+ \cup \{0\}$. Then

$z \equiv 0, 1, 2, \ldots \ldots, 41, 42 (mod\ 43)$.

Through a simple and direct check, the reader can easily verify that

$z^2 \equiv 0, 1, 4, 6, 9, 10, 11, 13, 14, 15, 16, 17, 21, 23, 24, 25, 31, 35, 36, 38, 40, 41 (mod\ 43)$, where $z \in Z^+ \cup \{0\}$.

**LEMMA: 3.7** If $x \in Z^+_{odd}$, then $2^x \equiv 2, 8, 22, 27, 32, 39, 42\ (mod\ 43)$.

**PROOF:** To prove this result, we will use the mathematical induction method and will establish that

$2^{2n-1} \equiv 2, 8, 22, 27, 32, 39, 42\ (mod\ 43)\ \forall\ n \in N$.

If $n = 1$, we have $2^1 \equiv 2\ (mod\ 43)$, which shows that the statement is true for $n = 1$.

Assume that the statement is true for $n = k \in N$. Then $2^{2k-1} \equiv 2, 8, 22, 27, 32, 39, 42\ (mod\ 43)$.

Now, we will prove the statement for $n = k + 1 \in N$.

**CASE 1:** $2^{2k-1} \equiv 2\ (mod\ 43)$. Then $2^{2(k+1)-1} \equiv 2^{2k+1} \equiv 8\ (mod\ 43)$.

**CASE 2:** $2^{2k-1} \equiv 8\ (mod\ 43)$. Then $2^{2(k+1)-1} = 2^{2k+1} \equiv 32\ (mod\ 43)$.

**CASE 3:** $2^{2k-1} \equiv 22\ (mod\ 43)$. Then $2^{2(k+1)-1} = 2^{2k+1} \equiv 88\ (mod\ 43) \Rightarrow 2^{2k+1} \equiv 2\ (mod\ 43)$.

**CASE 4:** $2^{2k-1} \equiv 27\ (mod\ 43)$. Then $2^{2(k+1)-1} = 2^{2k+1} \equiv 108\ (mod\ 43) \Rightarrow 2^{2k+1} \equiv 22\ (mod\ 43)$.

**CASE 5:** $2^{2k-1} \equiv 32\ (mod\ 43)$. Then $2^{2(k+1)-1} = 2^{2k+1} \equiv 128\ (mod\ 43) \Rightarrow 2^{2k+1} \equiv 42\ (mod\ 43)$.

**CASE 6:** $2^{2k-1} \equiv 39\ (mod\ 43)$. Then $2^{2(k+1)-1} = 2^{2k+1} \equiv 156\ (mod\ 43) \Rightarrow 2^{2k+1} \equiv 27\ (mod\ 43)$.

**CASE 7:** $2^{2k-1} \equiv 42\ (mod\ 43)$. Then $2^{2(k+1)-1} = 2^{2k+1} \equiv 168\ (mod\ 43) \Rightarrow 2^{2k+1} \equiv 39\ (mod\ 43)$.

Thus $2^{2k+1} \equiv 2, 8, 22, 27, 32, 39, 42\ (mod\ 43)$, which shows that the statement is true for $n = k + 1 \in N$. Hence by mathematical induction method
$2^{2n-1} \equiv 2, 8, 22, 27, 32, 39, 42\ (mod\ 43) \forall\ n \in N \Rightarrow 2^x \equiv$
$2, 8, 22, 27, 32, 39, 42\ (mod\ 43) \forall\ x \in Z^+_{odd}$.

## 4. Main Results

**THEOREM 4.1** The unique non-negative integer value of the unknowns $n, x, y, z$ is $2, 3, 0, 3$ respectively that satisfy the Diophantine equation $n^x + 43^y = z^2$, where $x, y, z \in Z^+ \cup \{0\}$, $n \in Z^+$ with $n \equiv 2(mod\ 129)$ and $n + 1 \notin S$.
**PROOF:** Let $n \in Z^+$ with $n \equiv 2(mod\ 129)$ and $n + 1 \notin S$. Further, let $x, y, z \in Z^+ \cup \{0\}$ such that $n^x + 43^y = z^2$.
If $y = 0$, then $n^x + 43^y = z^2 \Rightarrow n^x + 1 = z^2$, which has a unique non-negative integer value of unknowns $n, x, z$ by using Lemma 3.4, and this value is $2, 3, 3$, respectively.
If $y \geq 1$, then there are two cases for the value of $x$.
**CASE 1:** $x \in Z^+_{even} \cup \{0\}$. Since $n \equiv 2(mod\ 129)$ implies $n \equiv 2(mod\ 3)$ using Lemma 3.2.
Since $n \equiv 2(mod\ 3) \Rightarrow n \equiv -1(mod\ 3)$

$$\Rightarrow n^x \equiv (-1)^x (mod\ 3)\ \forall\ x \in Z^+_{even} \cup \{0\}$$

$$\Rightarrow n^x \equiv 1(mod\ 3)\ \forall\ x \in Z^+_{even} \cup \{0\} \quad (1)$$

$$\text{Also } 43^y \equiv 1(mod\ 3) \forall\ y \geq 1 \quad (2)$$

Using (1) and (2), we get
$n^x + 43^y \equiv 2(mod\ 3)$, when $x \in Z^+_{even} \cup \{0\}$ and $y \geq 1$.
Since $n^x + 43^y = z^2$ so $z^2 \equiv 2(mod\ 3)$, when $x \in Z^+_{even} \cup \{0\}$ and $y \geq 1$. This is a contradiction due to the Lemma 3.5.
**CASE 2:** $x \in Z^+_{odd}$. Since $n \equiv 2(mod\ 129)$ implies $n \equiv 2(mod\ 43)$ using Lemma 3.2.

$$\text{Now } n^x \equiv 2^x (mod\ 43)\ \forall\ x \in Z^+_{odd} \quad (3)$$

Using Lemma 3.7, if $x \in Z^+_{odd}$, then we have

$$2^x \equiv 2, 8, 22, 27, 32, 39, 42\ (mod\ 43) \quad (4)$$

By (3) and (4), if $x \in Z^+_{odd}$, then we get

$$n^x \equiv 2, 8, 22, 27, 32, 39, 42\ (mod\ 43) \quad (5)$$

$$\text{Also } 43^y \equiv 0(mod\ 43)\ \forall\ y \geq 1 \quad (6)$$

Using (5) and (6), we get
$n^x + 43^y \equiv 2, 8, 22, 27, 32, 39, 42\ (mod\ 43)$, when $x \in Z^+_{odd}$ and $y \geq 1$.
Since $n^x + 43^y = z^2$ so $z^2 \equiv 2, 8, 22, 27, 32, 39, 42\ (mod\ 43)$, when $x \in Z^+_{odd}$ and $y \geq 1$. This is a contradiction due to the Lemma 3.6.
Therefore, the unique non-negative integer value of the unknowns $n, x, y, z$ is $2, 3, 0, 3$ respectively that satisfy the Diophantine equation $n^x + 43^y = z^2$, where $x, y, z \in Z^+ \cup \{0\}$, $n \in Z^+$ with $n \equiv 2(mod\ 129)$ and $n + 1 \notin S$.

**COROLLARY 4.2** The unique non-negative integer value of the unknowns $x, y, z$ is $3, 0, 3$, respectively that satisfy the Diophantine equation $2^x + 43^y = z^2$, where $x, y, z \in Z^+ \cup \{0\}$.

**PROOF:** Let $x, y, z \in Z^+ \cup \{0\}$ such that $2^x + 43^y = z^2$. Since $2 \equiv 2 (mod\ 129)$ and $2 + 1 = 3$ is not a perfect square, by Theorem 4.1, the unique non-negative integer value of the unknowns $x, y, z$ is $3, 0, 3$ respectively that satisfy the Diophantine equation $2^x + 43^y = z^2$, where $x, y, z \in Z^+ \cup \{0\}$.

**COROLLARY 4.3** There is no non-negative integer value of unknowns $x, y, z$ that satisfy the Diophantine equation $131^x + 43^y = z^2$, where $x, y, z \in Z^+ \cup \{0\}$.

**PROOF:** Let $x, y, z \in Z^+ \cup \{0\}$ such that $131^x + 43^y = z^2$. Since $131 \equiv 2 (mod\ 129)$ and $131 + 1 = 132$ is not a perfect square, by Theorem 4.1, there is no non-negative integer value of unknowns $x, y, z$ that satisfy the Diophantine equation $131^x + 43^y = z^2$, where $x, y, z \in Z^+ \cup \{0\}$.

**COROLLARY 4.4** There is no non-negative integer value of unknowns $x, y, z$ that satisfy the Diophantine equation $260^x + 43^y = z^2$, where $x, y, z \in Z^+ \cup \{0\}$.

**PROOF:** Let $x, y, z \in Z^+ \cup \{0\}$ such that $260^x + 43^y = z^2$. Since $260 \equiv 2 (mod\ 129)$ and $260 + 1 = 261$ is not a perfect square, so by Theorem 4.1, there is no non-negative integer value of unknowns $x, y, z$ that satisfies the Diophantine equation $260^x + 43^y = z^2$, where $x, y, z \in Z^+ \cup \{0\}$.

**COROLLARY 4.5** There is no non-negative integer value of unknowns $x, y, z$ that satisfy the Diophantine equation $389^x + 43^y = z^2$, where $x, y, z \in Z^+ \cup \{0\}$.

**PROOF:** Let $x, y, z \in Z^+ \cup \{0\}$ such that $389^x + 43^y = z^2$. Since $389 \equiv 2 (mod\ 129)$ and $389 + 1 = 390$ is not a perfect square, by Theorem 4.1, there is no non-negative integer value of unknowns $x, y, z$ that satisfies the Diophantine equation $389^x + 43^y = z^2$, where $x, y, z \in Z^+ \cup \{0\}$.

## 5. Conclusion

In this paper, a new approach has been presented successfully for examining the exponential Diophantine equation $n^x + 43^y = z^2$, where $x, y, z \in Z^+ \cup \{0\}$, $n \in Z^+$ with $n \equiv 2 (mod\ 129)$ and $n + 1 \notin S$. The authors used a famous Catalan conjecture for this purpose and proved that $(n, x, y, z) = (2, 3, 0, 3)$ is a unique solution of this Diophantine equation. Corollary 4.4 depicts the particular case of the present problem when $n \notin P$. The main conclusion of this paper is that the proposed method is quite for examining Diophantine equations. The findings of this paper may provide a way of solving other interesting problems related to the Diophantine equation in the future.

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