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Nano Fuzzy Z - Open Sets and its Application in Nano Fuzzy Topological Space

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Abstract

This research aims to define and investigate the properties of Nano fuzzy Z-open explicitly sets defined in Nano fuzzy topological spaces. Also, there is an attempt to define Nano fuzzy Z-closure Nano fuzzy Z-interior in Nano fuzzy topological spaces. The work has grown by incorporating Nano fuzzy δ open sets, Nano fuzzy δ semi-open sets, Nano fuzzy δ semi-open sets, and Nano fuzzy δ pre-open sets. Also, the work has been concluded with a numerical application of the Nano fuzzy score function in the medical field (to check the proper diagnosis of disease and drug combinations given to the patient).

Keywords: Nano fuzzy Z-open sets; Nano fuzzy Z-continuity; Nano fuzzy Z- interior (closure); Nano fuzzy δ open sets; Nano fuzzy δ semi-open sets; Nano fuzzy δ semi-open sets; Nano fuzzy δ pre-open sets.

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1. Introduction

When discussing Nano topological spaces, Thivagar's [1,2] contribution and involvement in this field becomes evident. His noteworthy theory has expanded the horizons of small and finite topological spaces. Because Nano topological spaces consist of limited elements, they are highly practical in various applications. When we consider topological spaces defined by approximations and their spatial coverage within the universe, this concept extends the principles of rough set theory. This theory is given by Pawlak [3]. Subsequent researchers have further developed the theory of Nano topological spaces, aiming to advance the theory itself and discover numerous practical applications. A Z-open set in a Nano topological space is described and defined in 2021 by Selvaraj and Balakrishna [4]. Pankajam *et al.* [5] developed δ-Nano topological space; Vadivel *et al.* [6] built δ-open sets in Neutrosophic topological space.

The revolutionary fuzzy theory given by Zadeh [7] in 1965 has motivated the research associated with uncertainty and vagueness. This work has become more

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significant in the study of topologies since Chang [8] introduced fuzzy topology. After this, the research was more associated with the fuzzification of different topological structures. δ-open sets were introduced in fuzzy topological spaces by Saha [9].

Centered on this core concept, Navalakhe *et al*. [10] have recently explored a novel notation known as Nano fuzzy topology. In addition to this theory, Rajwade *et al*. [11,12] have proposed Nano fuzzy semi-open sets, Nano fuzzy α-open sets, Nano fuzzy pre-open sets, and Nano fuzzy regular open sets. Also, Nano fuzzy b-open sets [13] were defined and studied. So, the theory of Nano fuzzy topological spaces is in progress to be developed.

This paper's objective is to elucidate the concepts of Nano fuzzy Z-open sets and Nano fuzzy δ open sets in Nano fuzzy topological spaces. To support our practical demonstration, a numerical case study is examined. In this scenario, data on administering various drug combinations to different patients to treat a specific illness is analyzed. We can determine whether the drugs have been administered accurately to genuinely infected patients using the Nano fuzzy score function. Through this analysis, more effective drug options and combinations is proposed. Since a single disease can have multiple underlying causes and drug combinations can vary widely, indiscernibility is common, thus justifying the use of Nan fuzzy topological spaces in this context.

2. Preliminary Definitions and Results

In this section, some definitions and results that are prerequisites for defining Nano fuzzy Z-open sets is included.

Definition 2.1 [14, 15, 16] To construct a subset A of a topological space (U, τ_R (X)) you say:

1. A Z - open set if $A \subseteq NCl\big(NInt_{\delta}(A)\big) \cup NInt\big(NCl(A)\big)$;
2. A Z - closed set if $NInt\big(NCl_{\delta}(A)\big) \cap NCl\big(NInt(A)\big) \subseteq A$

The set of all Z - open (resp. Z - closed) subsets of a space (U, τ_R (X)) shall be denoted by NZO(X), as it always has been.

Definition 2.2 [6, 9] Let $\big(U, \tau_R(X)\big)$ be a Nano topological space and $A \subseteq U$. Then A is said to be

1. Nano semi-open if $A \subseteq NCl\big(NInt(A)\big)$.
2. Nano pre-open if $A \subseteq NInt\big(NCl(A)\big)$.
3. Nano α-open if $A \subseteq NInt\Big(NCl\big(NInt(A)\big)\Big)$.

***Definition* 2.3 [9]** Let $\big(U, \tau_R(X)\big)$ be a Nano topological space and $A \subseteq U$. Then

1. δ semi interior of A, or $N\delta sInt(A)$ for short, is the union of all Nano δ semi-open sets that are inside A.
2. The definition of the semi-closure of A (abbreviated $N\delta sCl(A)$) is the intersection of all Nano δ semi-closed sets that include A.

3. The pre-interior of A (abbreviated $NpInt(A)$) is defined as the union of all Nano pre-open sets inside of A.
4. The intersection of all Nano pre-open sets containing A is represented by $NpCl(A)$, which designates the pre-closure of A.

Definition 2.4 [10] Let (X, R) be an approximation space and $\frac{X}{R} = \{\lambda_1, \lambda_{2,\ldots,}\lambda_n\}$. Then, for any $\lambda \leq F(X)$, where $F(X)$ is a set of all fuzzy subsets of X, $\underline{R}(\lambda)$ and $\underline{R}(\lambda)$ are the lower and upper approximations of λ with respect to R, respectively, and they are the fuzzy sets in $\frac{X}{R}$. That is, $\underline{R}(\lambda)$, $\underline{R}(\lambda)$: $\frac{X}{R} \rightarrow [0,1]$, such that

1. $\underline{R}(\lambda)(\lambda_j) = inf_{y \leq \lambda_j} \lambda(y)$ and
2. $\underline{R}(\lambda)(\lambda_j) = sup_{y \leq \lambda_j} \lambda(y)$, for all $j = 1, 2, 3, \ldots\ldots, n$.

Definition 2.5 [10] In a fuzzy topological space X, let a fuzzy set be denoted by λ. Then, we may characterize the fuzzy bounds of λ as

1. $Bd(\lambda) = Cl(\lambda) \wedge Cl(\lambda^c)$
2. $Bd(\lambda) = Cl(\lambda) - Int(\lambda)$.

Definition 2.6 [10] Fuzzy subset X will be denoted by λ. Then, the following are exchangeable:

1. The fuzzy closure operator, represented by, $\underline{R}(\lambda)$ is an upper approximation of λ.
2. The fuzzy interior operator, represented by, $\underline{R}(\lambda)$ is the lower approximation of λ.

Also,
$Bd(\lambda) = Cl(\lambda) - Int(\lambda) = \underline{R}(\lambda) - \underline{R}(\lambda)$

2.7. *Properties of Fuzzy approximation space [10]*

Let R be an arbitrary relation from X to Y. The lower and upper approximation operators of a fuzzy set are denoted by $\underline{\mathrm{R}}$ and $\overline{\mathrm{R}}$ respectively, satisfies the following properties: for all $\alpha, \beta \in F(X)$,

(FL1) $\underline{R}(\alpha) = (\underline{R}(\alpha^C))^C$
(FU1) $\underline{R}(\alpha) = (\underline{R}(\alpha^C))^C$
(FL2) $\underline{R}(\alpha \wedge \beta) = \underline{R}(\alpha) \wedge \underline{R}(\beta)$
(FU2) $\underline{R}(\alpha \vee \beta) = \underline{R}(\alpha) \vee \underline{R}(\beta)$
(FL3) $\alpha \leq \beta \Rightarrow \underline{R}(\alpha) \leq \underline{R}(\beta)$
(FU3) $\alpha \leq \beta \Rightarrow \underline{R}(\alpha) \leq \underline{R}(\beta)$
(FL4) $\underline{R}(\alpha \vee \beta) = \underline{R}(\alpha) \vee \underline{R}(\beta)$
(FL4) $\underline{R}(\alpha \wedge \beta) = \underline{R}(\alpha) \wedge \underline{R}(\beta)$

Definition 2.8 [10] Let X be a non-empty finite set, R be an equivalence relation on X, $\lambda \leq X$ be a fuzzy subset, and $\tau_R(\lambda) = \{1_\lambda, 0_\lambda, \underline{R}(\lambda), \underline{R}(\lambda), Bd(\lambda)\}$. Then, by property (2.7), $\tau_R(\lambda)$satisfies the following axioms

1. $0_{\lambda}, 1_{\lambda} \in \tau_{(R)}(\lambda)$ where $0_{\lambda}: \lambda \to I$ denotes the null fuzzy sets and $1_{\lambda}: \lambda \to I$ denotes the whole fuzzy set.
2. Arbitrary union of members of $\tau_{(R)}(\lambda)$ is a member of $\tau_{(R)}(\lambda)$.
3. Finite intersection of members of $\tau_{(R)}(\lambda)$ is a member of $\tau_{(R)}(\lambda)$.

A topology on X termed the Nano fuzzy topology on X with regard to λ is denoted by $\tau_{(R)}(\lambda)$. $\left(X, \tau_{(R)}(\lambda)\right)$ is referred to as the Nano fuzzy topological space (NFTS). Nano fuzzy open sets are subsets of the topological space $\tau_{(R)}(\lambda)$, while Nano fuzzy closed sets are subsets of the space $[\tau_{(R)}(\lambda)]^C$

Definition 2.9 [10] The basis for the Nano fuzzy topology $\tau_{(R)}(\lambda)$ with respect to λ is given by $B = \left\{1_{\lambda}, \underline{R}(\lambda)(x), \underline{R}(\lambda)(x)\right\}$.

Definition 2.10 [10] $NfInt(\mu)$ stands for the union of all Nano fuzzy open subsets of a set, which is the definition of the Nano fuzzy interior of a set μ. Assume that there exists a Nano fuzzy topological space denoted by $(X, \tau(R))\ (\mu))$, where $\lambda \leq X$, and if $\mu \leq X$, then the Nano fuzzy interior of is the union of all Nano fuzzy open subsets of. So, it's the largest Nano fuzzy open subset in μ.

The intersection of all Nano fuzzy closed sets that include is equivalent to the definition of the Nano fuzzy closure of $\mu. NfCl(\mu)$ stands for the smallest Nano fuzzy closed set that contains μ.

Definition 2.11 [17] Nano Fuzzy score function For decision-making in any practical example, we need to define Nano fuzzy score function. As it is based on a methodical approach, it will be easier for us to calculate a parameter on which our decision depends.

Let S and M be two non-empty sets and $S: M \longrightarrow [0, 1]$. The fuzzy score function (in short, $NFSF$) is $S(M) = \frac{1}{k}\sum_{i=1}^{k} \mu_{M_i}$ that represents the average of positivity of the fuzzy component μ_M. The following fundamental steps propose the specific technique for selecting the correct qualities and alternatives in a decision-making situation utilizing fuzzy sets.

Step 1: Problem field selection: Consider the universe of discourse (set of objects) *m*, the set of alternatives *n*, and the set of decision attributes *p*.

Step 2: Construct a fuzzy matrix of alternative versus objects and object versus decision attributes. Calculation Part:

Step 3: Frame the in-discernibility relation *R* on *m*.

Step 4: Construct the fuzzy Nano topologies τ_j and ϑ_k.

Step 5: Find the score values by Definition of each of the entries of the Nano fuzzy topological spaces. Conclusion part:

Step 6: Organize the fuzzy score values of the alternatives $\tau_1 \leq \tau_2 \leq \tau_3 \leq \cdots \leq \tau_n$ and the attributes $\vartheta_1 \leq \vartheta_2 \leq \vartheta_3 \leq \cdots \leq \vartheta_p$. Choose the attribute ϑ_p for the alternative τ_1 and ϑ_{p-1} for the alternative τ_2 etc. If $n < p$, then ignore ϑ_k where $k = 1,2,3, \dots, n - p$.

3. Nano Fuzzy Z-Open Sets

In this paper, $\left(X, \tau_{(R)}(\lambda)\right)$ represents Nano fuzzy topological space with respect to λ where $\lambda \leq X$ (a fuzzy subset of X) and R is an equivalence relation on X, where X/R denotes the family of equivalence classes of X by R.

Definition 3.1 Let α be a fuzzy subset of Nano fuzzy topological space $\left(X, \tau_{(R)}(\lambda)\right)$. It is said to be

1. Nano fuzzy Z-open set (briefly, NfZ-open) if
   $\alpha \leq NfCl\big(NfInt_\delta(\alpha)\big) \vee NfInt\big(NfCl_\delta(\alpha)\big)$
2. Nano fuzzy Z-closed set (briefly, NfZ-closed) if
   $NfInt\big(NfCl_\delta(\alpha)\big) \wedge NfCl\big(NfInt_\delta(\alpha)\big) \leq \alpha$

Definition 3.2 The Nano fuzzy Z-closure of a fuzzy set μ, denoted by $NfCl_Z(\mu)$, is the intersection of Nano fuzzy Z -closed sets contain μ. The Nano fuzzy Z-interior of a set μ, is denoted by $NfInt_Z(\mu)$, and it is the union of Nano fuzzy Z-open sets contained in μ.

Remark 3.3 Let μ be a subset of Nano fuzzy topological space $\left(X, \tau_{(R)}(\lambda)\right)$. Then $\big(NfCl_Z(\mu)\big)^C = NfInt_Z(\mu^C)$, $\big(NfInt_Z(\mu)\big)^C = NfCl_Z(\mu^C)$.

Definition 3.4 Let $\left(X, \tau_{(R)}(\lambda)\right)$ be a Nano fuzzy topological space and $\alpha \leq X$. Then

1. Nano fuzzy δ semi interior of α (briefly $Nf\delta sInt(\alpha)$) is defined as a union of all Nano fuzzy δ semi-open sets contained in α.
2. Nano fuzzy δ semi closure of α (briefly $Nf\delta sCl(\alpha)$) is defined as the intersection of all Nano fuzzy δ semi-closed sets which contain α.
3. Nano fuzzy Pre-interior of α (briefly $NfpInt(\alpha)$) is defined as a union of all Nano fuzzy pre-open sets contained in α.
4. Nano fuzzy Pre-closure of α (briefly $NfpCl(\alpha)$) is defined as the intersection of all Nano fuzzy pre-open sets which contain α.

Theorem 3.5 The following statements are true. Every

I. Nano fuzzy δ-open set (resp. Nano fuzzy δ-closed set) is Nano fuzzy open (resp. Nano fuzzy closed) set.
II. Nano fuzzy open set (resp. Nano fuzzy closed set) is Nano fuzzy pre-open (resp. Nano fuzzy pre-closed) set.
III. Nano fuzzy δ-open set (resp. Nano fuzzy δ-closed set) is Nano fuzzy δ-semi open (resp. Nano fuzzy δ semi-closed) set.
IV. Nano fuzzy δ-semi open (resp. Nano fuzzy δ semi-closed) set is the Nano fuzzy Z-open (resp. Nano fuzzy Z-closed) set.
V. Nano fuzzy pre-open (resp. Nano fuzzy pre-closed) set is the Nano fuzzy Z-open (resp. Nano fuzzy Z-closed) set.

But are not conversely.

Proof:

(i) Let α is a Nano fuzzy δ-open set, then $\alpha = NfInt_\delta(\alpha) \leq NfInt(\alpha)$. Therefore α is a Nano fuzzy open.

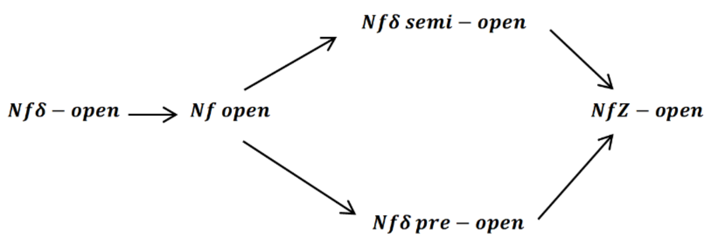
(ii) Let α is a Nano fuzzy open set, then $\alpha = NfInt(\alpha) \leq NfInt(NfCl(\alpha))$. Therefore α is a Nano fuzzy pre-open.

(iii) Let α is a Nano fuzzy open set, then $\alpha = NfInt(\alpha) \leq NfCl(NfInt_\delta(\alpha))$. Therefore α is a Nano fuzzy δ semi-open.

(iv) If α is a Nano fuzzy δ semi-open set, then $\alpha \leq NfCl(NfInt_\delta(\alpha))$ and so $\alpha \leq NfCl(NfInt_\delta(\alpha)) \leq NfCl(NfInt_\delta(\alpha)) \vee NfInt(NfCl(\alpha))$. Therefore α is a Nano fuzzy Z-open.

(v) If α is a Nano fuzzy pre-open set, then $\alpha \leq NfInt(NfCl(\alpha))$ and so $\alpha \leq NfInt(NfCl(\alpha)) \leq NfCl(NfInt_\delta(\alpha)) \vee NfInt(NfCl(\alpha)). \leq NfCl(NfInt_\delta(\alpha))$. Therefore α is a Nano fuzzy Z-open.

Remark 3.6 In a Nano fuzzy topological space $\left(X, \tau_{(R)}(\lambda)\right)$, the following diagram holds good for different types of Nano fuzzy open sets:



Preposition 3.7 Let $\left(X, \tau_{(R)}(\lambda)\right)$ be a Nano fuzzy topological space and $\alpha \leq \left(X, \tau_{(R)}(\lambda)\right)$. Then for any fuzzy subset β of α, the following hold:

(i) $NfInt_\delta(\beta) \leq Nf\delta sInt(\beta)) \leq NfInt_Z(\beta)$

(ii) $NfInt_\delta(\beta) \leq NfInt(\beta) \leq NfpInt(\beta) \leq NfInt_Z(\beta)$

(iii) $NfCl_\delta(\beta) \geq Nf\delta sCl(\beta)) \geq NfCl_Z(\beta)$

(iv) $NfCl_\delta(\beta) \geq NfCl(\beta) \geq NfpCl(\beta) \geq NfCl_Z(\beta)$

Example 3.8 Let $X = (a, b, c, d)$ and $X/R = \{(a, d), (b), (c)\}$.
Let $\lambda = \{(a, 0.2), (b, 0.3), (c, 0.4), (d, 0.1)\}$ be fuzzy subset of X. Then
$\underline{R}(\lambda)(x) = \{(a, 0.1), (b, 0.3), (c, 0.4), (d, 0.1)\}$
$\overline{R}(\lambda)(x) = \{(a, 0.2), (b, 0.3), (c, 0.4), (d, 0.2)\}$
$Bd(\lambda) = \{(a, 0.1), (b, 0.0), (c, 0.0), (d, 0.1)\}$.
Thus $\tau_R(\lambda) = \left\{ 1_\lambda, 0_\lambda, \underline{R}(\lambda), \overline{R}(\lambda), Bd(\lambda)\right\}$ is Nano fuzzy topology. Then

1. $\{(a, 0.2), (b, 0.3), (c, 0.6), (d, 0.2)\}$ is Nano fuzzy Z-open but not Nano fuzzy pre-open set.
2. $\{(a, 0.3), (b, 0.4), (c, 0.7), (d, 0.3)\}$ is Nano fuzzy Z-open but not Nano fuzzy δ-semi open set.

Lemma 3.9 Let $\left(X,\tau_{(R)}(\lambda)\right)$ be a Nano fuzzy topological space. Then, the union (resp. intersection) of arbitrary Nano fuzzy Z-open sets is Nano fuzzy Z-open set (resp. Nano fuzzy Z-closed set).
Proof: Let $\{\alpha_i; i \in I\}$ be a family of Nano fuzzy Z-open sets. Then
$\alpha_i \leq NfCl\big(NfInt_\delta(\alpha_i)\big) \vee NfInt\big(NfCl_\delta(\alpha_i)\big)$ and hence
$\vee_i \alpha_i \leq \vee_i \Big(NfCl\big(NfInt_\delta(\alpha_i)\big) \vee NfInt\big(NfCl_\delta(\alpha_i)\big)\Big) \leq$
$NfCl\big(NfInt_\delta(\vee_i \alpha_i)\big) \vee NfInt\big(NfCl_\delta(\vee_i \alpha_i)\big), \forall i \in I$. Thus $\vee_i \alpha_i$ is Nano fuzzy Z-open. The other is comparable.

3.10 Remark The intersection of any two Nano fuzzy Z-open sets is not Nano fuzzy Z-open, as shown in the following example.

Example 3.11 In example (3.8)
let $\alpha = \{(a, 0.33), (b, 0.53), (c, 0.53), (d, 0.33)\}$ and
$\beta = \{(a, 0.11), (b, 0.22), (c, 0.77), (d, 0.11)\}$ are Nano fuzzy Z-open sets but $\alpha \wedge \beta = \{(a, 0.11), (b, 0.22), (c, 0.53), (d, 0.11)\}$ is not Nano fuzzy Z-open.

Theorem 3.12 If α is Nano fuzzy Z - closed set in $\left(X,\tau_{(R)}(\lambda)\right)$ and $\alpha \leq \beta \leq NfCl_Z(\alpha)$, then β is also Nano fuzzy Z-closed in $\left(X,\tau_{(R)}(\lambda)\right)$.
Proof: Let α is Nano fuzzy closed set in $\left(X,\tau_{(R)}(\lambda)\right)$, with $\alpha \leq \beta \leq NfCl_Z(\alpha)$. Let $\beta \leq \gamma$ be the Nano fuzzy open set in X, and γ be the Nano fuzzy open. Since $\alpha \leq \beta$, $\beta \leq \gamma$ follows that $NfCl_Z(\alpha) \leq \gamma$, since α is Nano fuzzy Z-closed. According to the hypothesis $\beta \leq NfCl_Z(\alpha)$, $NfCl_Z(\beta) \leq NfCl_Z(NfCl_Z(\alpha)) = NfCl_Z(\alpha) \leq \gamma$, which implies $NfCl_Z(\beta) \leq \gamma$. Therefore β is Nano fuzzy Z-closed in $\left(X,\tau_{(R)}(\lambda)\right)$.

Theorem 3.13 If a fuzzy subset α of X is Nano fuzzy Z - closed set, then
$NfCl(\{x_r\}) \wedge \alpha \neq 0_\lambda \ \forall x_r \in NfCl_Z(\alpha)$.
Proof: Assume that α is a Nano fuzzy Z - closed set and that $x_r \in NfCl_Z(\alpha)$. If possible, $NfCl(\{x_r\}) \wedge \alpha = 0_\lambda$. Then $\alpha \leq 1_\lambda - NfCl(\{x_r\})$ and $1_\lambda - NfCl(\{x_r\})$ is a Nano fuzzy open set containing α. Since α is Nano fuzzy Z-closed set, implies $NfCl_Z(\alpha) \leq 1_\lambda - NfCl(\{x_r\})$ which is a contradiction to $x_r \in NfCl_Z(\alpha)$. Therefore, $NfCl(\{x_r\}) \wedge \alpha \neq 0_\lambda$.

Preposition 3.14 The closure of a Nano fuzzy Z-open set is Nano fuzzy δ-semi open.
Proof: Let α be a Nano fuzzy Z-open set in $\left(X,\tau_{(R)}(\lambda)\right)$. Then
$NfCl(\alpha) \leq NfCl(Nf\ 〚Int〛\_\delta\ (\alpha)) \vee NfInt(NfCl(\alpha)) \leq NfCl(Nf\ 〚Int〛\_\delta\ (\alpha)) \vee NfCl(NfInt(NfCl(\alpha)) = NfCl\big(NfInt_\delta(\alpha)\big)$. Therefore, $NfCl(\alpha)$ is Nano fuzzy δ-semi open.

Preposition 3.15 Let α be a Nano fuzzy Z-open set of a Nano fuzzy topological space $\left(X,\tau_{(R)}(\lambda)\right)$ and $NfInt_\delta(\alpha) = \phi$. Then α is Nano fuzzy pre-open.

Theorem 3.16 Let α and β be two fuzzy subsets of a Nano fuzzy topological space

$X, \tau_{(R)}(\lambda))$. The following holds true:

1. $NfCl_Z(1_\lambda - \alpha)) = 1_\lambda - NfInt_Z(\alpha)$,
2. $NfInt_Z(1_\lambda - \alpha)) = 1_\lambda - NfCl_Z(\alpha)$,
3. If $\alpha \leq \beta$, then $NfCl_Z(\alpha) \leq NfCl_Z(\beta)$ and $NfInt_Z(\alpha) \leq NfInt_Z(\beta)$.

Proof: By applying definition (3.2), the result follows directly.

Preposition 3.17 Let α and β be a fuzzy subset in a Nano fuzzy topological space $\left(X, \tau_{(R)}(\lambda)\right)$. Then:

(i) $NfpCl(\alpha) = \alpha \vee NfCl\left(NfInt(\alpha)\right), NfpInt(\alpha) = \alpha \wedge NfInt\left(NfCl(\alpha)\right)$,

(ii) $Nf_\delta SCl(1_\lambda - \alpha) = 1_\lambda - Nf_\delta SInt(\alpha), FNano\delta Scl(\alpha \vee \beta) \leq Nf_\delta SCl(\alpha) \vee Nf_\delta SCl(\beta)$.

Proof: Obvious.

Lemma 3.18 The following hold for a fuzzy subset α in a Nano fuzzy topological space $\left(X, \tau_{(R)}(\lambda)\right)$.

(i) $Nf_\delta pCl(\alpha) = \alpha \wedge NfCl\left(Nf_\delta Int(\alpha)\right) and\ Nf_\delta pInt(\alpha) = \alpha \vee NfInt\left(Nf_\delta Cl(\alpha)\right)$,

(ii) $Nf_\delta SInt(\alpha) = \alpha \wedge NfCl(Nf_\delta Int(\alpha))\ and\ Nf_\delta SCl(\alpha) = \alpha \vee NfInt(Nf_\delta Cl(\alpha))$.

Proof: Obvious.

Theorem 3.19 Let $(X, \tau_{(R)}(\lambda))$ be a Nano fuzzy topological space and $\alpha \leq \beta \leq X$. Then α is a Nano fuzzy Z - open set iff $\alpha = Nf_\delta SInt(\alpha) \vee Nf_\delta pInt(\alpha)$.

Proof: Let α be a Nano fuzzy Z-open set.

Then $\alpha \leq NfCl(Nf_\delta Int(\alpha)) \vee NfInt(NfCl(\alpha))$

and hence by Proposition 3.17 and Lemma3.18, $Nf_\delta SInt(\alpha) \vee NfpInt(\alpha) = (\beta \wedge NfCl(Nf_\delta Int(\alpha)) \vee (\beta \wedge NfInt(NfCl\ (\alpha))) =$
$\alpha \wedge (NfCl(Nf_\delta Int(\alpha))\ \vee NfInt(NfCl(\alpha)))\ = \alpha$.

It follows from Proposition 2.17 and Lemma 2.18 that the converse is true.

Theorem 3.20 Assume that α is a fuzzy subset of the space $(X, \tau_{(R)}(\lambda))$. Then

(i) $Nf_Z Cl(\alpha) = Nf_\delta SInt(\alpha) \wedge NfpCl(\alpha)$,

(ii) $Nf_Z Int(\alpha) = Nf_\delta SInt(\alpha) \vee NfpInt(\alpha)$.

Proof: (i) It is easy to see that $Nf_Z Cl(\alpha) \leq Nf_\delta SCl(\alpha) \wedge NfpCl(\alpha)$. Also, $Nf_\delta SCl(\alpha) \wedge NfpCl(\alpha) =$
$(\alpha \vee NfInt(\ Nf_\delta Cl(\alpha))) \wedge (\alpha \vee NfCl(NfInt(\alpha))\ = \alpha \vee (NfInt(\ Nf_\delta Cl(\alpha)) \wedge NfCl(NfInt(\alpha)))$.

Since $Nf_Z Cl(\alpha)$is Nano fuzzy Z- closed, then
$Nf_Z Cl(\alpha) \leq\ NfInt(Nf_\delta Cl(\ Nf_Z Cl(\alpha))) \wedge NfCl(NfInt(\ Nf_Z Cl(\alpha))) \geq NfInt(Nf_\delta Cl(\alpha)) \wedge\ NfCl(NfInt(\alpha))$.

Thus,
$\alpha \vee \left(NfInt\left(Nf_\delta Cl(\alpha)\right) \wedge NfCl\left(NfInt(\alpha)\right)\right) \leq \alpha \vee\ Nf_Z Cl(\alpha) =\ Nf_Z Cl(\alpha)$ and hence, $Nf_\delta SCl(\alpha) \wedge NfpCl(\alpha)\ \leq Nf_Z Cl(\alpha)$. So, $Nf_Z Cl(\alpha) = Nf_\delta SCl(\alpha) \wedge NfpCl(\alpha)$.

(ii) As a result of (i).

Numerical Example: In recent scenarios, diagnosing any disease and its cure appropriately is the biggest challenge in the medical field. As new diseases are diagnosed, their cure accordingly is very important. So combinations of different drugs according to the medical case is known are to play a crucial role in this field. Such data is available to clinicians, which includes vulnerabilities. This section will demonstrate the applicability and usefulness of the approach mentioned above.

Step-1: Problem field selection – Consider the following problem framed in Table –1 which provides information about five patients $P1$, $P2$, $P3$, $P4$, $P5$ who have consulted a physician and were prescribed Cephalexin (Ce), Clopidogrel (Cl), Levothyroxine (Le), Rifapentin (Ri), Aspirin (As). We need to find the patient and to find the disease Dropsy, Peripheral Artery Disease, Underactive Thyroid, Tuberculosis, and Arthritis of the patient based on drugs prescribed to them. Tables 1 and 2 describe the membership, indeterminacy, and non-membership functions of the patients and diseases, respectively.

Table 1. Fuzzy values for patients.

Drugs/Patients	P 1	P 2	P 3	P 4	P 5
Cephalexin	0.3	0.9	0.7	0.3	0.4
Clopidogrel	0.2	0.6	0.5	0.4	0.0
Levothyroxine	0.9	0.0	0.7	0.3	0.2
Rifapentin	0.5	0.4	0.9	0.9	0.6
Aspirin	0.6	0.2	0.3	0.4	0.3

Step-2: Construct the in-discernibility relation for the correlation between the symptoms given as

$X/R = \{\{Ce\}, \{Cl\}, \{Le\}, \{Ri\}, \{As\}\}$

Step-3: Form Nano fuzzy topologies (τ_i) and (ν_j) :

(i) $\tau_1 = \{0_\lambda, 1_\lambda, 0.3, 0.2, 0.9, 0.5, 0.6\}$

(ii) $\tau_2 = \{0_\lambda, 1_\lambda, 0.9, 0.6, 0.4, 0.2\}$

(iii) $\tau_3 = \{0_\lambda, 1_\lambda, 0.7, 0.5, 0.9, 0.3\}$

(iv) $\tau_4 = \{0_\lambda, 1_\lambda, 0.3, 0.9, 0.4\}$

(v) $\tau_5 = \{0_\lambda, 1_\lambda, 0.3, 0.6, 0.4, 0.2\}$

(i) $\nu_1 = \{0_\lambda, 1_\lambda, 0.7, 0.2, 0.4\}$

(ii) $\nu_2 = \{0_\lambda, 1_\lambda, 0.4, 0.2, 0.6\}$

(iii) $\nu_3 = \{0_\lambda, 1_\lambda, 0.4, 0.2, 0.2\}$

(iv) $\nu_4 = \{0_\lambda, 1_\lambda, 0.8, 0.9, 0.6\}$

(v) $\nu_5 = \{0_\lambda, 1_\lambda, 0.9, 0.3, 0.6\}$

Step-4: Find Nano fuzzy score functions: $(i)\ NFSF(\tau_1) = 0.4166$, $(ii)\ NFSF(\tau_2) = 0.525$, $(iii)\ NFSF(\tau_3) = 0.60$, $(iv)\ NFSF(\tau_4) = 0.533$, $(v)\ NFSF(\tau_5) = 0.375$

$(i)\ NFSF(\nu_1) = 0.325$, $(ii)\ NFSF(\nu_2) = 0.40$, $(iii)\ NFSF(\nu_3) = 0.262$,

$(iv)\ NFSF(\nu_4) = 0.575$, $(v)\ NFSF(\nu_5) = 0.60$

Step-5: Final decision: Arrange the Nano fuzzy score values for the alternatives $\tau_1, \tau_2, \tau_3, \tau_4, \tau_5$ and the attributes $\nu_1, \nu_2, \nu_3, \nu_4, \nu_5$ in ascending order. We get the following sequences $\tau_5 \leq \tau_1 \leq \tau_2 \leq \tau_4 \leq \tau_3$ and $\nu_3 \leq \nu_1 \leq \nu_2 \leq \nu_4 \leq \nu_5$. Thus, patient **P 5** was given Levothyroxine, but he was not suffering from Dropsy, and **P 4** was given Rifapentin as he was suffering from Tuberculosis. Rest all the patients have been diagnosed and prescribed appropriately except patient **P 5.**

Table 2. Fuzzy values for drugs.

Drugs/Patients	Dropsy	PAD	UT	TB	Arthritis
Cephalexin	0.9	0.0	0.4	0.2	0.2
Clopidogrel	0.2	0.9	0.5	0.0	0.0
Levothyroxine	0.0	0.0	0.2	0.3	0.0
Rifapentin	0.1	0.1	0.1	0.9	0.1
Aspirin	0.1	0.2	0.3	0.4	0.9

The results are presented in Fig. 1.

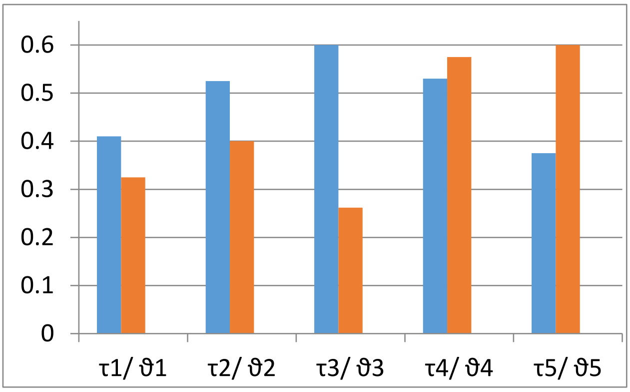


Fig. 1. Fuzzy score values for drugs and patients.

4. Conclusion

The characteristics of a recently defined set category known as Nano fuzzy Z-open sets in Nano fuzzy topological space have been introduced and explored in this article. Additionally, the properties of Nano fuzzy *Z*-interior, Nano fuzzy *Z*-closure along with their connection to the established Nano fuzzy sets (Nano fuzzy Z-open sets), were introduced and analyzed. Nano fuzzy δ open sets, Nano fuzzy δ semi-open sets, Nano fuzzy δ semi-open sets, and Nano fuzzy δ pre-open sets were defined in this article. In our numerical illustration, the Nano fuzzy score functions for both the topologies were computed. The fuzzy relationship between drug combinations and patients is represented by the first topology, while the second one pertains to drug combinations and the diseases they are effective in curing. After performing the calculations and organizing the data, we determined and conlcluded that patient **P 5** received Levothyroxine even though he did not have Dropsy, while patient **P 4** was prescribed Rifapentin due to his Tuberculosis condition. All the patients, except for **P 5,** received accurate diagnosis and appropriate prescriptions. The right drug combination can be selected to contribute accurate diagnosis and improved treatment.

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