Publications

Available Online

J. Sci. Res. **16** (1), 213-219 (2024)

JOURNAL OF SCIENTIFIC RESEARCH

www.banglajol.info/index.php/JSR

Fair Detour Domination in the Corona of Two Graphs: Optimizing CCTV Camera Installation for Efficient Surveillance

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Received 20 July 2023, accepted in final revised form 21 October 2023

Abstract

The objective of our research is to analyze the properties of FDD sets in the corona of two graphs and explore their practical application in CCTV camera installation. A set F$\subseteq V(G)$ is considered to be a Fair Detour Dominating set (FDD-set) if it is detour dominating and the number of neighbors within set F is the same for any pair of vertices outside of F. Among these FDD sets, the $f\gamma_d$-set refers to the FDD-set with the smallest number of vertices, and its order defines the Fair Detour Domination number ($f\gamma_d$ (G)). We have established that for any arbitrary graphs G_1 and G_2 , $f\gamma_d(G_1 \circ G_2) = |V(G_1)|$ iff fd$(G_2) = 1$ $and,$ we have determined the $f\gamma_d$ number of corona products of any connected graph G with the path graph as well as the cycle graph. We also characterized FDD sets in the corona product of two connected graphs and provided a thorough description of how FDD sets can be used in optimizing CCTV camera installation for efficient surveillance.

Keywords: Detour set; Fair detour domination; Application of FDD.

doi: http://dx.doi.org/10.3329/jsr.v16i1.67723 J. Sci. Res. **16** (1), 213-219 (2024)

1. Introduction

The term' dominating set' refers to a group of vertices in a graph that exerts control over the other vertices. The idea of domination first appeared in graph theory in the early 1960s. Over the years, domination has maintained its significance, attracting the attention of numerous researchers due to its wide range of practical applications. A few intriguing aspects of domination, like Total Equitable domination and Restrained edge domination, have been examined in references [1,2].

One noteworthy extension of dominating sets in graph theory is the concept of 'Fair domination,' which was introduced by Caro *et al.* [3]. This strategy seeks to identify a subset of nodes that act as dominating nodes while ensuring an equitable distribution of the domination responsibilities among the members of the graph. Since then, many studies have been conducted, resulting in new and improved approaches. Our current research introduces a new domination parameter named "fair detour domination." This parameter

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aims to determine a subset of nodes that engage in fair domination and are interconnected by detour paths within the graph.

In this study, we focus on investigating the properties and concepts related to graphs G = (V, E), which are simple connected graphs with finite order $p \geq 2$.

The detour distance, denoted as D(u, w), represents a graph's longest path length between vertices u and w [4]. We define a Detour set, denoted as $D \subseteq V$, as a set where every vertex in the graph is located on a detour connecting certain pairs of vertices from D [5]. A dominating set, on the other hand, refers to a set where each vertex in the graph is either adjacent to a vertex within the set or is itself a member of the set. The γ-set corresponds to the dominating set with the fewest number of vertices; its order is referred to as the domination number, γ [6].

Furthermore, we explore the notion of Detour Dominating sets, which are subsets of V(G) that simultaneously act as dominating sets and detour sets, with the size of the γ_d-set denoted as the detour domination number, γ_d [7]. The notation $N(w) = \{u \in V(G)/ uw \in E\}$ is used to represent all the neighbors of vertex w.

A γ-set $F \subseteq V(G)$, is considered a fair dominating set (FD-set) if, for any pair of vertices x and y outside of F, the number of neighbors in F is equal, that is, $|N(x) \cap F| = |N(y) \cap F|$, where $x, y \notin F$. FD sets are further classified as m-FD sets when a vertex $x \notin F$ has m neighbors in F, for any integer $m \geq 1$ [3, 8].

The Corona of two graphs G and H, represented as $G \circ H$, is created by taking a single copy of G and replicating H, |V(G)| times, with the i-th vertex of G connected to each vertex in the i-th replica of H. Previous studies of FD set in this binary operation can be seen in [9,10].

We introduce the concept of FDD [11] sets, denoted as $F \subseteq V(G)$, which are γ_d-sets exhibiting fairness, wherein any pair of vertices not belonging to F have an equal number of neighbors within F. Specifically, when a vertex $x \notin F$ has m neighbors in F, for any integer $m \geq 1$, we refer to F as m-FDD set. The $f\gamma_d$-set corresponds to the smallest set among all such FDD sets, and its cardinality represents the fair detour domination number, $f\gamma_d$

In graph theory, the FDD set is a relatively new idea that seeks to strike a balance between efficiency and equality in the detour domination problem. With the help of k-FDD, we can install ATM machines, cell phone towers, CCTV cameras, etc., for a range of k. This concept has several real-world applications, some of which are discussed in this work.

The following result is used to prove our main result:

Theorem 1.1: The fair domination number for the cycle graph C_p, $p \geq 3$ is given by

$$f\gamma_d\,(C_p)=\begin{cases} \left\lceil \frac{P}{3} \right\rceil \; if\ p \equiv 0, 1\ (mod\ 3) \\ \left\lceil \frac{P}{3} \right\rceil + 1\ if\ p \equiv 2\ (mod\ 3) \end{cases}$$

2. Main Results

***FDD in Corona of Two Graphs*:** Let G_2^x denote the copy of the graph G_2 corresponding to the vertex x of G_1 and $x + G_2^x$ be the join graph of x and G_2.

Theorem 2.1. $f\gamma_d(G_1 \circ G_2) = |V(G_1)|$ holds iff $fd(G_2) = 1$ for arbitrary graphs G_1 and G_2.

Proof: Suppose $f\gamma_d(G_1 \circ G_2) = |V(G_1)|$. Let $F = V(G_1)$. Then, the entire vertex set of $G_1 \circ G_2$ can not be covered by a detour connecting pairs of vertices of F. Hence, the required $f\gamma_d$-set of $G_1 \circ G_2$ of cardinality $|V(G_1)|$ must not have vertices of G_1. Then a vertex $u \in V(G_2^x)$ must exist corresponding to each vertex x of G_1 which fairly dominates $x + G_2^x$. This will happen only when $u \in V(G_2^x)$ is adjacent to every vertex of G_2^x. Then $fd(G_2^x) = 1$ and hence $fd(G_2) = 1$.

Conversely, Let $x_1 \in G_1$. By construction of $G_1 \circ G_2$, $u_1 \in V(G_2^{x_1})$ is adjacent to every other vertex of $x_1 + G_2^{x_1}$ and so $\{u_1\}$ forms a minimum fd-set of $(x_1 + G_2^{x_1})$. Similarly, for each $x_i \in G_1$, $\{u_i\}$ is the fd set of $(x_i + G_2^{x_i})$. Let $F = \{u_1, u_2, \ldots, u_{|V(G_1)|}\}$. Then, this F is a FD set of $G_1 \circ G_2$ and every vertex of $G_1 \circ G_2$ is located on a detour connecting pairs of vertices of F. Hence, F will form the minimum FDD set, and so $f\gamma_d(G_1 \circ G_2) = |V(G_1)|$.

Corollary 2.2. For a complete graph G_2, $f\gamma_d(G_1 \circ G_2) = |V(G_1)|$.

Results 2.3.

1. In the case where we have graphs G_1 and G_2, if $f\gamma_d(G_1 \circ G_2) = 2|V(G_1)|$ holds then $fd(G_2) = 2$
2. For the graphs G and the Path graph P_m,

$$f\gamma_d(G \circ P_m) = \begin{cases} |V(G)| \; if \; m = 2,3 \\ |V(G)| \, (fd(P_m) + 1) \; if \; m \geq 4 \end{cases}$$

Theorem 2.4. For any graph G and the Cycle graph C_p of order $p \geq 3$,

$$f\gamma_d \, (G \circ C_p) = \begin{cases} |V(G)| \text{ if m=3} \\ 2|V(G)| \text{ if m=4} \\ |V(G)|\left(fd\left(C_p\right)+1\right) \text{ if m} \geq 5 \end{cases}$$

Proof:

Case 1: $p = 3$. We have $fd(C_3) = 1$. Then, by theorem 2.1, $f\gamma_d(G \circ C_3) = |V(G)|$

Case 2: $p = 4$. By theorem 1.1, $fd(C_p) = 2$. Let $F = \{u, w\}$ be FD set of C_4^v. For each $v \in G$, we have F as the FD set of $v + C_4^v$. This gives an FD set of cardinalities $2|V(G)|$ and it will also be the detour-dominating set. Hence $f\gamma_d(G \circ C_p) = 2|V(G)|$.

Case 3: $p \geq 5$.

Sub Case(a): $p \equiv 0,1 \pmod 3$. Then, as theorem 1.1 states, $fd(C_p) = \left\lceil \frac{p}{3} \right\rceil$. Let $F = \{u_1, u_2, \ldots, u_{\lceil \frac{p}{3} \rceil}\}$ be the FD set of C_p^v. These $\left\lceil \frac{p}{3} \right\rceil$ vertices will not be enough to fairly dominate $v + C_p^v$ for each $v \in G$. These $\left\lceil \frac{p}{3} \right\rceil$ vertices along with v fairly dominate $v + C_p^v$.

Then the set $F' = \{v\} \cup \{u_1, u_2, \dots, u_{\lceil \frac{p}{3} \rceil}\}$ is an FD set of $v + C_p{}^v$, $v \in G$. Thus, we got an FD set F'' of $G \circ C_p$ of cardinality $(|V(G|)\left(\left\lceil \frac{p}{3} \right\rceil + 1\right)$. Also, each vertex $v_i \in V(G \circ C_p)$ is on the longest path connecting pairs of vertices that belong to F''. Hence $f\gamma_d(G \circ C_p) = |V(G)|(fd(C_p) + 1)$.

Sub Case(b): $p \equiv 2 \pmod 3$. By theorem 1.1, $fd(C_p) = \left\lceil \frac{p}{3} \right\rceil + 1$. By a similar argument, we get a FDD set of $G \circ C_p$ of cardinality $|V(G|\left(\left(\left\lceil \frac{p}{3} \right\rceil + 1\right) + 1\right)$. Hence $f\gamma_d(G \circ C_p) = |V(G)|(fd(C_p) + 1)$.

Theorem 2.5. For the graphs G_1 and G_2, a set $F \subseteq V(G_1 \circ G_2)$ is an FDD set of $G_1 \circ G_2$ if either of the subsequent conditions hold:

(i) $F = V(G_1) \cup (\cup F_x)$ where F_x is an FD set of $G_2{}^x$ and the union is taken over all $x \in G_1$.

(ii) $F = \cup F_x$ where F_x is an FD set of $G_2{}^x$ and the union is taken over all $x \in G_1$.

Proof:

Let $G = G_1 \circ G_2$ and F be a FDD-set of G.

Case 1: Suppose $V(G_1) \subset F$. Let $w \in F \setminus V(G_1)$ which implies that w belongs to $F \cap (\bigcup_{x \in V(G_1)} V(G_2{}^x))$.

Let $F_x = F \cap V(G_2{}^x)$.

This implies $w \in \bigcup_{x \in V(G_1)} F_x$. Therefore, $F \setminus V(G_1) \subseteq \bigcup_{x \in V(G_1)} F_x$.

Hence $F \subseteq V(G_1) \cup \left(\bigcup_{x \in V(G_1)} F_x\right)$.

Also, $\bigcup_{x \in V(G_1)} F_x \subseteq F$ and $V(G_1) \subset F$. Then $V(G_1) \cup \left(\bigcup_{x \in V(G_1)} F_x\right) \subseteq F$.

Hence $F = V(G_1) \cup \left(\bigcup_{x \in V(G_1)} F_x\right)$.

Suppose F_x is not an FD set of $G_2{}^x$. Then there exists $u, w \in V_{G_2{}^x} \setminus F_x$ such that $\left|N_{G_2{}^x}(u) \cap F_x\right| \neq \left|N_{G_2{}^x}(w) \cap F_x\right|$. Then $u, w \in V(G) \setminus F$ and $|N_G(u) \cap (x \cup F_x)| \neq |N_G(w) \cap (x \cup F_x)|$.

$\Longrightarrow \left|N_G(u) \cap \left(\bigcup_{x \in V(G_1)} \{x \cup F_x\}\right)\right| \neq \left|N_G(w) \cap \left(\bigcup_{x \in V(G_1)} \{x \cup F_x\}\right)\right|$

$\Longrightarrow \left|N_G(u) \cap \{V(G_1) \cup \left(\bigcup_{x \in V(G_1)} F_x\right)\}\right| \neq \left|N_G(w) \cap \{V(G_1) \cup \left(\bigcup_{x \in V(G_1)} F_x\right)\}\right|$

$\Longrightarrow |N_G(u) \cap F| \neq |N_G(w) \cap F|$ which contradicts F being an FDD set.

Thus $F = V(G_1) \cup \left(\bigcup_{x \in V(G_1)} F_x\right), F_x$ is an FD set of $G_2{}^x$.

Case 2: Suppose $V(G_1) \nsubseteq F$. Let $w \in F$. This implies $w \in F \cap (\bigcup_{x \in V(G_1)} V(G_2{}^x))$ and $w \notin V(G_1)$.

Let $F_x = F \cap V(G_2{}^x)$.

$\Longrightarrow w \in \bigcup_{x \in V(G_1)} F_x$

$\Longrightarrow F \subseteq \bigcup_{x \in V(G_1)} F_x$.

Also, since $F_x \subseteq F$ for all $x \in G_1$. This gives $\bigcup_{x \in V(G_1)} F_x \subseteq F$.

Hence $F = \bigcup_{x \in V(G_1)} F_x$. By a similar argument used in the above case, F_x is an FD set of $G_2{}^x$.

Conversely, let $F = V(G_1) \cup (\cup F_x)$ where F_x is an FD set of $G_2{}^x$. Since $V(G_1) \subset F$, $V(G_1)$ will be the dominating set. Since F_x is an FD set of $G_2{}^x$, we have $v, w \in V(G_2{}^x) \setminus F_x$ and $\left|N_{G_2{}^x}(v) \cap F_x\right| = \left|N_{G_2{}^x}(w) \cap F_x\right|$. Also, $v, w \in V(G) \setminus F$.

Consider $|N_G(v) \cap F|$ for any $v, w \in V(G) \setminus F$ then

$$
\begin{aligned}
|N_G(v) \cap F| &= \left|N_G(v) \cap V(G_1) \cup \left(\cup_{x \in V(G_1)} F_x\right)\right| \\
&= \left|N_G(v) \cap \left(\cup_{x \in V(G_1)} \{x \cup F_x\}\right)\right| \\
&= |N_G(v) \cap (x \cup F_x)| \\
&= \left|N_{G_2^x}(v) \cap F_x\right| \\
&= \left|N_{G_2^x}(w) \cap F_x\right| \\
&= |N_G(w) \cap (x \cup F_x)| \\
&= \left|N_G(w) \cap \left(\cup_{x \in V(G_1)} \{x \cup F_x\}\right)\right| \\
&= \left|N_G(w) \cap \{V(G_1) \cup \left(\cup_{x \in V(G_1)} F_x\right)\}\right| \\
&= |N_G(w) \cap F|
\end{aligned}
$$

This implies that F is an FD set. All the vertices of G lie on the longest paths of the same lengths, connecting pairs of vertices of F, and so F will be the FDD-set of G.

Suppose $F = \cup F_x$ where F_x is an FD set of ${G_2}^x$. F_x will be the γ-set of $x + {G_2}^x$. Thus, $\cup F_x$ will generate a γ set for G and all $x \in V(G)$ lies on the same length detours connecting pairs of vertices of F. By a similar argument used above, we get F as an FD set. Hence, F is an FDD set of G.

3. Application of FDD in Optimizing CCTV Camera Installation for Efficient Surveillance

CCTV surveillance is an important method for increasing security and reducing crime. However, placing CCTV cameras is difficult since it requires determining the optimal locations for cameras to maximize coverage while minimizing cost and privacy invasion. Fair detour domination is one solution to this problem. This method involves identifying a subset of CCTV camera locations covering all vertices in the network while guaranteeing a fair distribution of coverage. This strategy helps reduce concerns about discriminating or biased monitoring that may occur if cameras are installed randomly, recklessly, or in a discriminatory manner.

We intend to install CCTV cameras for the newly built gated residential community (Fig. 1). The first stage in applying fair detour domination to installing CCTV cameras is to create a graph representing the Residential area. The vertices in the graph represent intersections where at least two roads intersect. Cameras can be put at these intersections. The edges between the vertices represent the routes between those intersections, while the end vertices represent the entrances and exits. We obtain the graph shown in Fig. 2. Once the graph has been formed, fair detour domination can be used to select a set of vertices to place CCTV cameras.

Fig. 1. Residential area (figure created with Icograms).

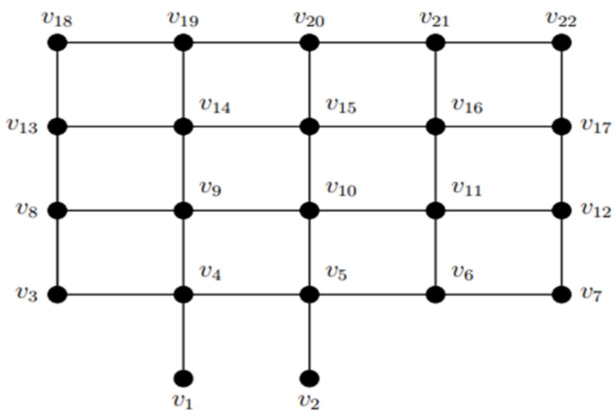

Fig. 2. Graph of the residential area.

In order to prevent biased installation, we have to place cameras in the selected subset of intersections so that every other intersection is within the field view of exactly one camera, provided all the entrances and exits must be under surveillance. To be specific, we have to find a 1-FDD set. One such required set is $F = \{v_1, v_2, v_8, v_{11}, v_{12}, v_{17}, v_{19}, v_{20}\}$. Install cameras in these intersections, which provides fair coverage and ensures that every intersection not in our set is within the coverage of exactly one camera. Also, since all these cameras lie in a detour path connecting a pair of camera locations, when there is a burglary, trespassing, or any other crime, the criminal must pass through at least one of these cameras' fields of view. Hence this footage can be used to identify and catch the criminal. This may be sufficient for places that require minimum security.

For solid security without blind spots, we will install cameras where each intersection of the houses lies within the range of two cameras. In other words, the problem is to find a subset with a minimum number of locations where cameras need to be installed such that every other location is within the range of two cameras, provided all the entrances and exits must be monitored by CCTV cameras. That is, we have to find a 2-FDD set for this graph. Then $F = \{v_1, v_2, v_4, v_7, v_8, v_{11}, v_{15}, v_{16}, v_{18}, v_{19}, v_{22}\}$ is one such required set. Installing cameras in these locations prevents and reduces blind spots. This can be used to install cameras in places like Banks, Prison, Country borders, etc., where maximum security is required.

4. Conclusion

Fair detour domination is an effective tool capable of tackling a wide range of challenges in various fields. This is useful in network design, facility location, traffic management, and other areas. The number of resources necessary to cover a network or a system can be reduced while ensuring that all essential vertices are covered fairly using fair detour domination. This can result in more efficient and effective resource allocation and management, which is vital for a wide range of real-world applications.

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