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Bianchi Type-V Dark Energy Cosmological Model in $f(R,T)$ Theory of Gravitation

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Abstract

The spatially homogenous and anisotropic Bianchi type-V dark energy cosmological model has been examined in the context of $f(R,T)$ gravity with an appropriate choice of the function $f(R,T)$. To obtain a deterministic solution, the fact that shear scalar σ is proportional to the scalar expansion θ is taken into consideration, which results in a relationship between metric potentials $B = C^n$. The mathematical condition that EoS parameter ω is proportional to skewness parameter δ is also used. The derived cosmological is free from initial singularity i.e. at $t = 0$. The main objective behind this paper is to explore some physically significant discussions on the evolution of the universe. Some crucial cosmological parameters of this model are also discussed. The spatial volume increase with cosmic time *t*, confirming accelerated expansion of the universe. Positive value of Hubble parameter indicates that the universe is expanding gradually. EoS parameter has a negative sign, which is in agreement with recent observations. Positive value of deceleration parameter shows that our model decelerates in the standard way. Pressure of dark energy is negative as we desire and it vanishes in large time limit. The significance of statefinder and jerk parameters are discussed to differentiate our model from other dark energy models.

Keywords: Bianchi type-V; $f(R,T)$ theory; Dark energy.

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1. Introduction

The accelerated expansion of the universe has been confirmed by recent cosmological observations [1-7]. These findings also support the theory that this late-time acceleration is caused by an exotic energy having negative pressure known as dark energy (DE). Dark energy is widely regarded as a best candidate to explain cosmic acceleration. It is now thought that the universe's energy composition consists of 5 % ordinary matter, 27 % dark matter, and 68% dark energy [8]. Thus dark energy cosmological models become an interesting subject of investigation for several authors.

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Dark energy is typically defined by the equation of state (EoS) parameter, given by $\omega(t) = p / \rho$, which is not always constant where p is the fluid pressure and ρ is energy density. Dark energy models with variable EoS parameter have been investigated by Akarsu and Kilinc [9], Ray *et al.* [10], Caroll and Hoffman [11], Yadav and Yadav [12], Amirhashchi [13], and Pradhan *et al.* [14]. Shaikh and Wankhede [15] developed a dark energy cosmological model for hypersurface-homogeneous space-time filled with perfect fluid in $f(R, T)$ theory of gravity. Shekh [16] investigated models of holographic dark energy using FLRW cosmological model in the context of modified gravity.

Several modified theories of gravity have been developed in recent years to explain the presence of dark energy, dark matter, and the mechanism behind the universe's late-time acceleration. Harko *et al.* [17] have developed a new modified theory of gravity known as $f(R, T)$ gravity. Numerous researchers are interested in this modified the theory since it is believed to provide a natural gravitational alternative to dark energy. Adhav [18], Sharif and Zubair [19], and Mahanta [20] have examined the Bianchi type-I cosmological model in $f(R, T)$ gravitational theory. The Bianchi type-V cosmological model has been investigated in the framework of $f(R, T)$ gravity by Naidu *et al.* [21], Ahmed and Pradhan [22], and Pawar *et al.* [23]. Shaikh and Bhoyar [24] studied plane symmetric universe in $f(R, T)$ gravity. Mishra *et. al* [25] presented cosmological models with variable anisotropic parameter in $f(R, T)$ gravity. Brahma and Dewri [26] studied Bianchi type-V modified $f(R, T)$ gravity cosmological models in Lyra geometry. Singh and Devi [27] studied interacting anisotropic LRS Bianchi type-I dark energy cosmological model with hybrid expansion in $(R,)$ gravity. Mule *et al.* [28] studied the Bianchi type-III cosmological model in the presence of Holographic dark energy within $(R,)$ gravity. Ugale and Deshmukh [29] investigated the anisotropic Bianchi type VI_0 cosmological models with modified Holographic Ricci Dark Energy in $f(R, T)$ gravity. Bianchi type V DE cosmological model is studied with the electromagnetic field in Lyra geometry based on $f(R, T)$ gravity by Brahma and Dewri [30]. As a result of above studies, $f(R, T)$ theory seems to be more convenient to explain the accelerating phase of the universe.

In most of the cases, expansion of the universe is described using spatially homogeneous and isotropic FRW models, which are widely considered as a good approximation of the present and early stages of the universe. The recent observations from various experiments like Cosmic Microwave Background (CMB) temperature and polarization anisotropy fundamentals [31], Cosmic Background Explorers (COBE) [32], Wilkinson Microwave Anisotropy Probe [33,34] and Planks collaboration [35] indicate that universe might have been anisotropic in initial phase that approaches to an isotropic phase later on. This prediction motivates us to study anisotropic universe using Bianchi model instead of FRW model.

The study of Bianchi type-V cosmological models is significant in the understanding of the universe since these models have isotropic special instances and allow arbitrary small anisotropy levels at some point in time. Being the natural generalization of open universe in Friedmann-Robertson-Walker (FRW) models with negative curvature, study of Bianchi type-V universe is significant in the context of dark energy (DE). Pradhan *et al.* [36], Baillie and Madsen [37], Ram *et al.* [38], Lorenz [39], Venkateswarulu and Reddy [40], Ram and Singh [41], Bali and Singh [42], Beesham [43], Banerjee and Sanyal [44], Roy and Prasad [45], Camci *et al.* [46] and Singh [47] are some authors who have studied Bianchi type-V cosmological models. Trivedi and Bhabor [48] investigated five dimensional LRS Bianchi type-V string cosmological models in scalar-tensor theory of gravitation. Mahanta *et al.* [49] have investigated Bianchi type-V universe in $f(R,T)$ gravity with variable cosmological constant and a quadratic equation of state.

Inspired by above discussion and investigations, in this paper, we have examined spatially homogeneous and anisotropic Bianchi type-V dark energy cosmological model in the framework of $f(R,T)$ gravity. The main goal of this research is to explore this Bianchi type-V Dark Energy cosmological model in $f(R,T)$ gravity in the view of various issues concerning the late time cosmic acceleration and cosmic anisotropy.

2. Metric and Field Equation

The anisotropic and spatially homogenous Bianchi type-V space-time is given by

$$ds^2 = dt^2 - A^2 dx^2 - e^{-2mx}\left(B^2 dy^2 + C^2 dz^2\right), \tag{1}$$

where A, B, C are functions of cosmic time t only and m is a constant.
The energy momentum tensor for anisotropic dark energy is given by

$$T_\nu^\mu = diag\left[\rho, -p_x, -p_y, -p_z\right] = diag\left[1, -\omega_x, -\omega_y, -\omega_z\right]\rho \ , \tag{2}$$

where ρ is the fluid's energy density and, p_x, p_y, p_z are the pressures along x, y, and z axes respectively. Here ω is the EoS parameter and ω_x, ω_y, and ω_z are the EoS parameters in the directions of x, y and, z respectively. The energy momentum tensor can be parameterized as follows:

$$T_\nu^\mu = diag\left[1, -\omega, -(\omega+\gamma), -(\omega+\delta)\right]\rho \tag{3}$$

For the sake of simplicity, we choose $\omega_x = \omega$. Skewness parameters γ and δ represents the deviations from the ω on y and z axes, respectively. The field equations of $f(R,T)$ gravity are derived from variational principle. The action of $f(R,T)$ gravity is given by

$$S = \frac{1}{2k}\int f(R,T)\sqrt{-g}\ d^4x + \int L_m\sqrt{-g}\ d^4x \ , \tag{4}$$

which can be varied with respect to the metric tensor $g_{\mu\nu}$ to obtain the gravitational field equation for $f(R,T)$ gravity as

$$f_R(R,T)R_{\mu\nu} - \frac{1}{2}f(R,T)g_{\mu\nu} + (g_{\mu\nu}\nabla^\mu\nabla_\mu - \nabla_\nu\nabla_\nu)f_R(R,T) = kT_{\mu\nu} - f_T(R,T)\,T_{\mu\nu} - f_T(R,T)\theta_{\mu\nu} \ , \tag{5}$$

where $\theta_{\mu\nu} = g^{\alpha\beta}\dfrac{\partial T_{\alpha\beta}}{\partial g_{\mu\nu}}$ and $T_{\mu\nu}$ is energy momentum tensor.

Here $f_R = \dfrac{\partial f(R,T)}{\partial R}$, $f_T = \dfrac{\partial f(R,T)}{\partial T}$, ∇_μ is covariant derivative. $k = \dfrac{8\pi G}{c^4}$, where G and c are the Newtonian Gravitational constant and speed of light in vacuum respectively.
According to Harko *et al.* [17], three different of $f(R,T)$ gravity model are possible. In this paper, we considered the functional as,

$$f(R,T) = R + 2f(T)\,, \tag{6}$$

where $f(T)$ is an arbitrary function of the trace of the energy-momentum tensor.
Now, the corresponding field equations become,

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = kT_{\mu\nu} + 2f_T T_{\mu\nu} + \left[f(T) + 2p_{\wedge} f_T\right]g_{\mu\nu}\,, \tag{7}$$

where f_T denotes the partial derivative of f with respect to T.
With specific choice of function $f(T) = \lambda T$ as given by Harko *et al.* [17], where λ is arbitrary constant and assuming commoving coordinate system, the field equations (7) for the metric (1) using (2) and (3), results in following system of equations:

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} - \frac{m^2}{A^2} = \rho\left[(8\pi + 2\lambda)\omega - (1 - 3\omega - \gamma - \delta)\right] - 2\lambda p\,, \tag{8}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} - \frac{m^2}{A^2} = \rho\left[(8\pi + 2\lambda)(\omega + \gamma) - (1 - 3\omega - \gamma - \delta)\right] - 2\lambda p\,, \tag{9}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{m^2}{A^2} = \rho\left[(8\pi + 2\lambda)(\omega + \delta) - (1 - 3\omega - \gamma - \delta)\right] - 2\lambda p\,, \tag{10}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} - \frac{3m^2}{A^2} = -\rho\left[8\pi + 2\lambda + (1 - 3\omega - \gamma - \delta)\right] - 2\lambda p\,, \tag{11}$$

$$\frac{2\dot{A}}{A} - \frac{\dot{B}}{B} - \frac{\dot{C}}{C} = 0\,, \tag{12}$$

here an overhead dot indicates differentiation with respect to cosmic time t.

3. Solution of Field Equations

Solving (12) gives
$A^2 = \alpha BC$, where α is constant.
Without loss of generality, taking $\alpha = 1$ gives

$$A^2 = BC\,. \tag{13}$$

Subtracting (9) from (10) yeilds

$$\frac{\ddot{B}}{B} - \frac{\ddot{C}}{C} + \frac{\dot{A}}{A}\left(\frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right) = 0\,. \tag{14}$$

The field equations (8)-(12) consist of five independent equations with seven unknowns A, B, C, ρ, p, δ, and ω. As a result, two more conditions are necessary in order to obtain a deterministic solution. The following conditions are considered:

i) The physically feasible condition that the shear scalar σ is proportional to scalar expansion θ [50], which gives

$$B = C^n . \tag{15}$$

ii) The EoS parameter ω is proportional to skewness parameter δ (mathematical condition) [51] such that

$$\omega + \delta = 0 . \tag{16}$$

Using (12) in (14) yeilds

$$\frac{\ddot{B}}{B} - \frac{\ddot{C}}{C} + \frac{1}{2}\left(\frac{\dot{B}^2}{B^2} - \frac{\dot{C}^2}{C^2} \right) = 0 . \tag{17}$$

Solving (17) gives

$$\frac{\dot{B}}{B} - \frac{\dot{C}}{C} = \frac{K'}{(BC)^{3/2}} . \tag{18}$$

Solving (18) and using (15), we get

$$C = (k_1 t + k_2)^{\frac{2}{3(n+1)}} ,$$

(19)

$$B = (k_1 t + k_2)^{\frac{2n}{3(n+1)}} ,$$

(20) $A = (k_1 t + k_2)^{\frac{1}{3}}$,

(21)

where $k_1 = \frac{3}{2}(n+1)\frac{k'}{n-1}$ and $k_2 = \frac{3}{2}(n+1)k^*$ are constant of integration.

Now, the metric (1) becomes

$$ds^2 = dt^2 - (k_1 t + k_2)^{\frac{2}{3}} dx^2 - e^{-2mx}\left[(k_1 t + k_2)^{\frac{4n}{3(n+1)}} dy^2 + (k_1 t + k_2)^{\frac{4}{3(n+1)}} dz^2 \right]. \tag{22}$$

4. Physical Parameters of Model

- The volume V is obtained as $V = ABC = (k_1 t + k_2)$. (23)

This model's spatial volume vanishes at $t = -k_2 / k_1$ and becomes infinite as $t \to \infty$, which indicate that the universe begins with zero volume and expands uniformly.

- The average scale factor for metric (1), which represents the model's rate of expansion, is obtained as

$$a(t) = (ABC)^{\frac{1}{3}} = V^{\frac{1}{3}} = (k_1 t + k_2)^{\frac{1}{3}} . \tag{24}$$

- The directional Hubble parameters are

$$H_1 = \frac{\dot{A}}{A} = \frac{k_1}{3(k_1 t + k_2)}, \quad H_2 = \frac{\dot{B}}{B} = \frac{2nk_1}{3(n+1)(k_1 t + k_2)}, \quad H_3 = \frac{\dot{C}}{C} = \frac{2k_1}{3(n+1)(k_1 t + k_2)} \quad . \tag{25}$$

- The mean Hubble parameter H, which determines the universe's volumetric expansion rate, is given by

$$H = \frac{\dot{a}}{a} = \frac{1}{3}\frac{\dot{V}}{V} = \frac{1}{3}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) = \frac{k_1}{3(k_1 t + k_2)} \tag{26}$$

The Hubble parameter H decreases with time and vanishes for large values of t. A positive value for the Hubble parameter indicates that the universe is expanding.

- The scalar expansion θ, which deals with the universe's expansion, is represented in the form

$$\theta = \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) = 3H = \frac{k_1}{(k_1 t + k_2)} \quad . \tag{27}$$

In this case, scalar expansion has an infinite value $t \to 0$ and tends to zero as $t \to \infty$.

- The shear scalar σ obtained as

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{1}{3}\left[\left(\frac{\dot{A}}{A}\right)^2 + \left(\frac{\dot{B}}{B}\right)^2 + \left(\frac{\dot{C}}{C}\right)^2 - \frac{\dot{A}\dot{B}}{AB} - \frac{\dot{B}\dot{C}}{BC} - \frac{\dot{A}\dot{C}}{AC}\right]$$

$$= \frac{1}{3}\left[\left(\frac{\dot{A}}{A}\right)^2 + \left(\frac{\dot{B}}{B}\right)^2 + \left(\frac{\dot{C}}{C}\right)^2 - \frac{\dot{A}\dot{B}}{AB} - \frac{\dot{B}\dot{C}}{BC} - \frac{\dot{A}\dot{C}}{AC}\right]$$

$$= \frac{1}{9}\left[\frac{k_1^{\,2}(n-1)^2}{(n+1)^2 (k_1 t + k_2)^2}\right] . \tag{28}$$

The shear scalar becomes infinite in the early the universe i.e. as $t \to 0$, and decreases to zero as cosmic time $t \to \infty$. It observes that $\frac{\sigma}{\theta} =$ constant, implying that the model is anisotropic for $n \neq 1$.

- The anisotropy parameter A_m, is given by

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2 = \frac{2}{3}\left[\frac{(n-1)^2}{(n+1)^2}\right]. \tag{29}$$

The mean anisotropic parameter A_m is constant throughout the evolution of the universe and $A_m \neq 0$ for $n \neq 1$. Thus model is anisotropic except for $n = 1$.

- The deceleration parameter q is is given by

$$q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1 = 2 \quad (30)$$

It may be noted that Bianchi models represent cosmos in its early stage of evolution.

Here the deceleration parameter is positive which shows early deceleration. Since $q > 0$, this model decelerates in the standard way. Despite the fact that the universe decelerates in a standard way, it will accelerate in finite time due to cosmic re-collapse where the universe in turns inflates "decelerates and then accelerates" [52].

- The average density parameter defined as $\Omega = \frac{\rho}{3H^2}$ and given by

$$\Omega = \frac{1}{(8\pi + 2\lambda)}\left[\frac{-4(n^2 + 4n + 1)}{3(n+1)^2} + \frac{6m^2 (k_1 t + k_2)^{4/3}}{k_1{}^2}\right]. \quad (31)$$

- The energy density and pressure of the fluid is obtained by subtracting (11) from (10) and using (19-21), which is given by

$$\rho = \frac{1}{(8\pi + 2\lambda)}\left[\frac{-2k_1{}^2(2n^2 + 8n + 2)}{9(n+1)^2 (k_1 t + k_2)^2} + \frac{2m^2}{(k_1 t + k_2)^{2/3}}\right] = -p\,, \quad (32)$$

since in the case of accelerated expansion we have $\rho + p = 0$.

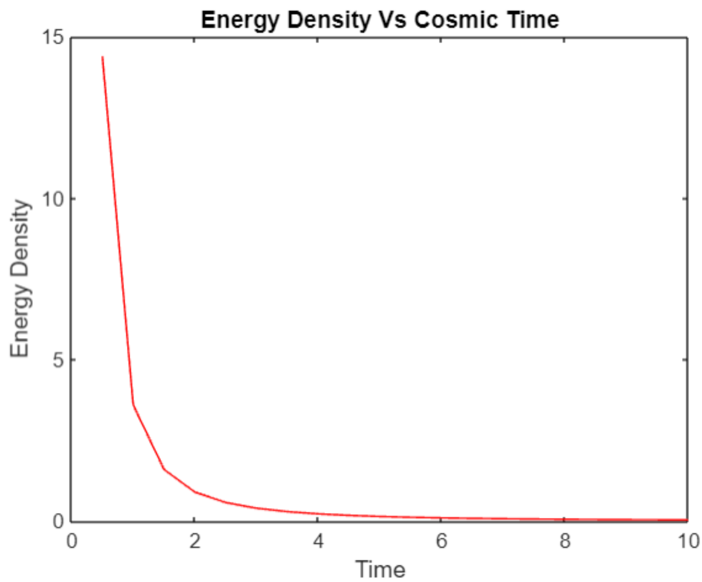

Fig. 1. Energy density versus cosmic time for $k_1 = 1$, $k_2 = 0$, $n = 1$, $m = 1$, $\lambda = 1$.

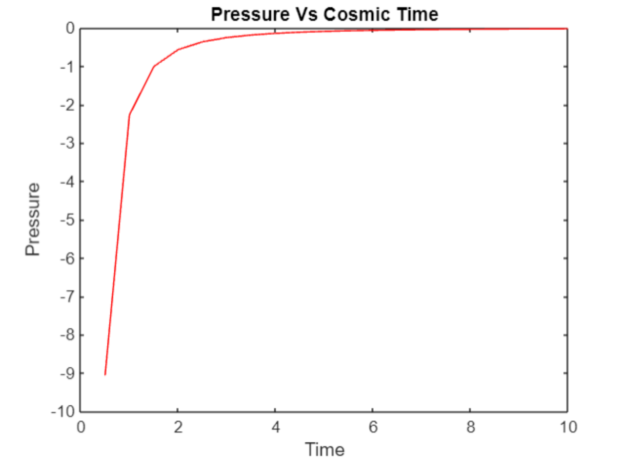

Fig. 2. Pressure versus cosmic time for $k_1 = 1,\ \ k_2 = 0,\ \ n = 1,\ \ m = 1,\ \ \lambda = 1$.

It is observed that energy density decreases as time increases and it vanishes as $t \to \infty$. The pressure of dark energy (fluid) is negative as desire and it also vanishes in large time limit.

The Equation of state (EoS) ω and Skewness parameter δ in the model are obtained by subtracting (11) from (8) and using (16), (19-21), which is given by

$$\omega = -1 + \frac{1}{\rho\left(8\pi + 2\lambda\right)}\left[\frac{-2k_1^{\ 2}\left(2n^2 + 8n + 2\right)}{9(n+1)^2(k_1 t + k_2)^2} + \frac{2m^2}{(k_1 t + k_2)^{2/3}}\right] = -\delta\ . \tag{33}$$

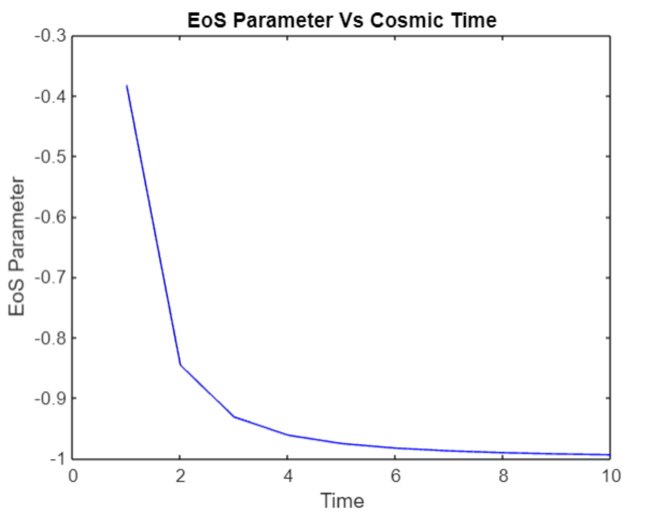

Fig. 3. Equation of State (EoS) versus cosmic time for $k_1 = 1,\ \ k_2 = 0,\ \ n = 1,\ \ m = 1,\ \ \lambda = 1,\ \rho = 0.5$.

- In our model, the EoS parameter is evolving with a negative sign, which can be explained by the current accelerated expansion of the universe. From Fig. 2, it is observed that initially universe expand with quintessence $\omega > -1$ region and at late time it approaches the cosmological constant $\omega = -1$ scenario. This is a situation in the early universe where the quintessence-dominated universe [53] may be playing an important role for the EoS parameter. It is important to note that negative value of EoS parameter is supported by SNe-Ia data.
- The skewness parameter γ is obtained by subtracting (8) from (9) and using (19-21) is $\gamma = 0$. (34)

5. Some Observational Parameters

- The redshift z is related to the scale factor $a(t)$ given by $z = \frac{a(t_0)}{a(t)} - 1$, where $a(t_0)$ is present value of scale factor. Using (24) in above equation yeilds

$$z = \frac{\left(k_1 t_0 + k_2\right)^{\frac{1}{3}}}{\left(k_1 t + k_2\right)^{\frac{1}{3}}} - 1 \ . \tag{35}$$

- The luminosity distance is described by the simple expression $d_L = r_1 a_0 (1+z)$, where $r_1 = \int_t^{t_0} \frac{1}{a(t)} dt$. Hence, the expression for luminosity distance is obtained as

$$d_L = \frac{3(k_1 t_0 + k_2)^{1/3}}{2k_1} \left[(k_1 t_0 + k_2)^{2/3} (k_1 t + k_2)^{-1/3} - (k_1 t + k_2)^{1/3} \right],$$

$$d_L = \frac{3}{2k_1} \left[(1+z)\ (k_1 t_0 + k_2)^{2/3} - (k_1 t_0 + k_2)^{1/3} (k_1 t + k_2)^{1/3} \right]. \tag{36}$$

- The distance modulas is given by the expression $D(Z) = 5 Log\ d_L + 25$, where d_L is the luminosity distance. Thus distance modulas is obtained as

$$D(Z) = 5 Log \left\{ \frac{3}{2k_1} \left[(1+z)\ (k_1 t_0 + k_2)^{2/3} - (k_1 t_0 + k_2)^{1/3} (k_1 t + k_2)^{1/3} \right] \right\} + 25 \ . \tag{37}$$

The distance modulas seems to be increasing function of redshift z.

- Jerk parameter is the third-order derivative of the scale factor. It is convenient to describe models close to ΛCDM. The transition from the decelerating to the accelerating phase of the phase the universe is thought to be caused by a cosmic cosmic jerk [54-56]. It is defined and obtained as

$$j = \frac{\dddot{a}}{H^3 a} = 10 \ . \tag{38}$$

Thus, in this model jerk parameter is positive throughout the evolution of the universe indicating accelerated expansion of the universe. This model deviates from ΛCDM since $j \neq 1$.

- The statefinder parameters $\{r, s\}$ proposed by Sahni *et al.* [57] are useful in identifying different forms of dark energy. In the sense that these two parameter solely depend on the scale factor, they are dimensionless and geometric. When $\{r, s\} = (1, 1)$, it implies cold dark matter (CDM) limit while $\{r, s\} = (1, 0)$ gives ΛCDM limit. Also, when $r < 1$ we have quintessence DE region and for $s > 0$ phantom DE regions.

  The pair of state finder diagnostic has defined and obtained as

$$r = \frac{\dddot{a}}{aH^3} = 10 \,, \; s = \frac{r-1}{3\left(q - \frac{1}{2}\right)} = 2 \,. \tag{39}$$

It is observes that the obtained model differs significantly from ΛCDM cosmology throughout the cosmic evolution since $\{r, s\} \neq (1, 0)$ and as $s > 0$, it shows phantom DE region.

6. Conclusion

In this paper, we have studied Bianchi Type-V Dark Energy Cosmological Model In $f(R,T)$ Theory of Gravitation. The model obtained (36) has no singularity for $n > 0$. The spatial volume of this model vanishes at $t = -k_2 / k_1$ and becomes infinite as $t \to \infty$, this indicates that the universe begins with zero volume and expands uniformly. The Hubble parameter H, the expansion scalar θ and shear scalar σ decrease as time increase. It is evident from the positive values of the expansion scalar and the Hubble parameter throughout the evolution that the universe is expanding gradually. Since $\sigma / \theta =$ constant, the model remains anisotropic except for $n = 1$. It is observes that anisotropic parameter $A_m \neq 0$ for $n \neq 1$. Thus model is anisotropic except for $n = 1$. Energy density and pressure increase with cosmic time and vanishes in large time limit. The experiments show that there is a certain amount of anisotropy in the universe and hence anisotropic space-times are important. Here $q > 0$, thus obtained model decelerates in the standard way. Despite the fact that the universe decelerates in a standard way, it will accelerate in finite time due to cosmic re-collapse where the universe in turns inflates “decelerates and then accelerates”.. It is also observed that initially universe expand with quintessence $\omega > -1$ region and at late time it approaches the cosmological constant $\omega = -1$ scenario. It is interesting to note that EoS parameter takes a negative value which is supported by SNe-Ia data. Energy density is found to be decreasing function of time and it vanishes as $t \to \infty$. It is noted that pressure of dark energy (fluid) is negative as we desire and it vanishes in large time limit. In this model jerk parameter is positive throughout the evolution of the universe indicating accelerated expansion of the universe. This model deviates from ΛCDM since $j \neq 1$ .The pair of state finder diagnostic is determine, which shows that this model does not approach to ΛCDM limit but since $s > 0$, it shows phantom DE region. Finally we can conclude that these cosmological models are new and different in some aspects from the various Bianchi type-V dark energy model presented by different authors.

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