JSR Publications

Available Online

JOURNAL OF SCIENTIFIC RESEARCH

J. Sci. Res. **17** (2), 519-529 (2025)

www.banglajol.info/index.php/JSR

A Fuzzy TOPSIS Framework for Assessment of Air Quality Index Values Across Different Stations- A Case Study on Kolkata in India

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Received 1 December 2024, accepted in final revised form 16 March 2025

Abstract

The Air Quality Index (AQI) is a critical measure for assessing air pollution levels, directly impacting public health and environmental sustainability. As urbanization and industrialization continue to expand, the need for advanced methodologies to evaluate and compare AQI across different locations has grown significantly. Traditional AQI computation methods often struggle to handle the uncertainties and complexities associated with multiple pollutants. This study applies a fuzzy Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) approach to enhance AQI evaluation and classification. Using data from the West Bengal Central Pollution Control Board (WBPCB) for the period January 2023 to December 2023, AQI values are computed based on 24-hour average pollutant concentrations. The fuzzy AQI (FAQI) values derived from the proposed method are compared with conventionally computed AQI values, demonstrating strong alignment while offering additional classification insights. The results indicate variations in air pollution levels across the seven stations analyzed, with Ultadanga emerging as the most polluted location and Tollygunge exhibiting the best air quality among them. In addition, the classification process categorizes the stations into different groups. Notably, none of the stations qualify as 'Good' in terms of air quality.

Keywords: AQI; FAQI; TOPSIS; Fuzzy preference degree.

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doi: https://dx.doi.org/10.3329/jsr.v17i2.78084 J. Sci. Res. **17** (2), 519-529 (2025)

1. Introduction

The air in this era is getting worse and has become a pressing global concern, significantly impacting human health, ecosystems, and overall environmental quality. The Air Quality Index (AQI) plays a significant role in assessing environmental conditions, providing a standardized measure of air pollution that helps the communities identify and respond to environmental concerns. Rapid urbanization and industrial expansion have led to a sharp increase in pollutant emissions, making air quality monitoring and assessment more critical

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than ever. Traditional AQI evaluation methods, though widely used, often fail to capture the inherent uncertainties associated with multiple pollutants and their varying impacts. This necessitates the development of more reliable and adaptable methodologies to accurately assess and compare air quality across different locations. In response to this challenge, this study introduces a fuzzy Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) approach to improve AQI assessment. It combines two powerful tools: fuzzy logic and TOPSIS (a technique for comparing options). This new method can handle the uncertainty in air quality data, providing a more accurate and flexible way to compare air quality in different regions. In addition, the methodology allows for the classification of different locations with respect to the pollution level in the air. The proposed approach is explained in detail, starting with the basics of fuzzy sets and TOPSIS to understand air quality data.

During the last few years, researchers have extensively studied and proposed various fuzzy based models for analyzing the pollution level of different countries. Fuzzy Inference systems can be used to determine the degree of pollution level accurately as it has the ability to model highly uncertain data [1-3]. The more detailed application of fuzzy inference systems in environmental sciences is discussed by Fisher [4] and Silvert [5]. According to them, Fuzzy logic is the most powerful tool by which one can quantitatively classify the environmental effects. Sowlat *et al*. [6] also introduced a fuzzy based air quality index for assessment and classification of air quality. Their result also showed that a fuzzy based air quality index is a most effective tool to produce accurate results in comparison to USEPA [7]. This study was carried out considering air quality data of five different stations of Tehran, Iran. Upadhyay *et al.* [8] proposed a fuzzy logic-based model to calculate the Air quality index in various countries. By applying the approach, they concluded that their model produced a satisfactory result than the usual linear conventional method. Fuzzy TOPSIS method was also used by Anwar et al. to predict the AQI level of Chennai during the festival [9].

In the work of Hernandez *et al.* two models were presented; one of which is a computational model while the other is a fuzzy inference system-based model [10] and a comparative analysis was studied among air quality indices developed by environmental agencies and other similar models. Later, Hamedian *et al*. [11] used a fuzzy inference system and fuzzy C-mean clustering method to calculate the degree of pollution level in Tehran, Iran. They showed that the major ambient pollutants of that city are not that much responsible for increasing the air pollution. Dass *et al*. [12] also discussed two types of model fuzzy type-1 and fuzzy type -2 to determine the level of Carbon Monoxide (CO) in a metropolitan city. They also provided a comparative analysis between the two methods which clearly reflected that type-2 method is more suitable than type-I method. Recently, a similar analysis of air quality index using type-I and type-II fuzzy systems was presented by Azad *et al.* [13]. Gorai *et al.* proposed a fuzzy synthetic evolution model to assess the air quality in four monitoring stations in Taaj area, Agra [14]. This model was based on the four major air pollutants (SPM, RSPM, SO_2 and NO_2) and their results indicated that the fuzzy air quality index exceeded the permissible level in each station. Hashimoto *et al*. [15]

the correlation between the pollutants PM and BC is analyzed during the Deepavali festival of Bengaluru which shows that PM levels have increased significantly more than BC levels. Numerous studies have explored the changes in air pollution situation across multiple cities of different countries during the lockdown period [16,17]

In addition to the above fuzzy based method, Marcovian fuzzy based system (MCFS), introduced by Alyousifi *et al.* [18], have also played a significant role in analyzing the air pollution index. Recently, many researchers also used an ensemble of deep learning frameworks that combine different machine learning algorithms to improve the prediction of AQI in Delhi [19,20]. The ensemble model achieved better results than the individual model, emphasizing the advantages of integrating different approaches.

The present study embraces the pollution level in Kolkata with respect to some specific locations from north to south. Using the Fuzzy TOPSIS method, the pollution level of some different stations in Kolkata has been analyzed and the results compared with those of the AQI calculated by the usual method [21]. This study is based on the data acquired from WBPCB (West Bengal Pollution Control Board, India) available for each month of seven selected stations from Kolkata for the period 2023, January to 2023, December [22].

The present work is covered in the following sections: Section 2 discusses the basics of fuzzy sets and TOPSIS, Section 3 explains the proposed methodology for comparing AQI using fuzzy TOPSIS, Section 4 presents a case study to demonstrate the practical applicability of this approach, and Section 5 concludes the study with key findings and implications.

2. Preliminaries

2.1. *Fuzzy sets and triangular fuzzy number*

Fuzzy logic, introduced by Zadeh [23], marked a significant departure from traditional two-valued logic by allowing for degrees of truth rather than strict binary classifications. Initially developed for engineering applications, its acceptance across various disciplines was gradual, as it challenged conventional deterministic approaches. Fuzzy sets are defined by membership functions, which can be either discrete or continuous, providing a mathematical framework to handle imprecision and uncertainty. These functions quantify the degree to which an element belongs to a given set, making fuzzy logic particularly useful in complex decision-making scenarios where traditional crisp classifications fall short. Let X denote a set of elements, commonly known as the universe of discourse. A fuzzy set is the set of ordered pairs $\widetilde{A}=\{(x, \mu_{\tilde{A}}(x)) : x \in X\}$ where $\mu_{\tilde{A}}(x)$ is known as the membership function or grade of membership of *x* in A defined as: $\mu_{\tilde{A}}(x): X \to [0,1]$.

A triangular fuzzy number $\tilde{A} = (a, b, c)$ is described as any fuzzy subset of the real line R with the membership function $\mu_{\tilde{A}}(x)$ in $[0,1]$ expressed as

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x-a}{b-a}, & a \le x \le b \\ 1, & x = b \\ \dfrac{c-x}{c-b}, & b \le x \le c \end{cases}$$

Here the two TFNs $\widetilde{A} = (a_1, a_2, a_3)$ and $\widetilde{B} = (b_1, b_2, b_3)$ are considered. The Fuzzy preference degree between two triangular fuzzy numbers is evaluated by using the formula [24].

$$P(\widetilde{A}, \widetilde{B}) = \frac{\sqrt{(y-a_1)^2+(y-a_2)^2+(y-a_3)^2}-\sqrt{(y-b_1)^2+(y-b_2)^2+(y-b_3)^2}}{3(y-x)} \tag{1}$$

where $x \le Min\{a_1, b_1\}$ *and* $y \ge Max\{a_3, b_3\}$.

Some basic properties of $P(\widetilde{A}, \widetilde{B})$ are:

i) If $P(\widetilde{A}, \widetilde{B}) = 0$ *then* $(\widetilde{A} = \widetilde{B})$

ii) $-1 \le P(\widetilde{A}, \widetilde{B}) \le 1$

iii) *If* $P(\widetilde{A}, \widetilde{B}) \le P(\widetilde{A}, \tilde{C})$, *then* $\widetilde{B} \le \widetilde{C}$

2.2. *Technique for order preference by similarity to ideal solution (TOPSIS)*

TOPSIS is a multi-criteria decision-making approach that evaluates and ranks alternatives by measuring their relative closeness to an ideal solution. It evaluates the alternatives by comparing their distance to the ideal solutions (the best possible values for all criteria) and the negative ideal solution (the worst possible values).

The process involves the following steps sequentially: Normalization, Weighting, Finding Positive and Negative Ideal Solutions, Evaluating Distances and Ranking. TOPSIS is widely used for multi-criteria decision-making due to its simplicity and effectiveness in handling quantitative and qualitative data.

3. Methodology

In this section, a methodology to provide the Fuzzy AQI (FAQI) of different stations is demonstrated. In this section, the key task is to establish the ranking of the stations according to their Fuzzy Air Quality Index (FAQI). This ordering provides a comparative assessment of air pollution levels, helping to identify the most and least affected areas based on the fuzzy evaluation method. The methodology is described stepwise.

Let $T(X_{ijm}) = (a_{ijm}, b_{ijm}, c_{ijm})$, i.e., $T(X_{ijm})$ is the TFN of the linguistic variable X_{ijm}, where X_{ijm} is the Linguistic Input for the i^{th} Station with respect to the j^{th} attribute on the m^{th} date. Also let i varies from 1 to r, j varies from 1 to s and m varies from 1 to t.

Step 1:

Let $T(Y_{jk}) = (a_{jk}, b_{jk}, c_{jk})$, where Y_{jk} is the Linguistic Input of the for the j^{th} attribute given by the k^{th} Expert. Let k varies from 1 to p.

Now the weight of the j^{th} attribute A_j is denoted by $\omega(A_j)$ and is defined by

ω (A_j) = (a_j, b_j, c_j), where $a_j = \underset{k}{Min}(a_{jk})$, $b_j = \frac{1}{p}\sum_{k=1}^{p}(b_{jk})$, and $c_j = \underset{k}{Max}(c_{jk})$.

This aggregation technique is nothing but the Min-Avg-Max principle of combining a set of TFNs.

Step 2:

Now the Fuzzy decision matrix (FDM) is constructed whose general element $\tilde{X}_{ij}$ is defined by its TFN,

T($\tilde{X}_{ij}$) = (p_{ij}, q_{ij}, r_{ij}) where

$p_{ij} = \underset{m}{Min}(a_{ijm})$,

$q_{ij} = \frac{1}{t}\sum_{m=1}^{t}(b_{ijm})$, and

$r_{ij} = \underset{m}{Max}(c_{ijm})$.

Thus, FDM is a $r \times s$ matrix.

Step 3:

The elements of the FDM are normalized by the greatest element M= $\underset{i,j}{Max}\{p_{ij}, q_{ij}, r_{ij}\}$. The elements of the Normalized Fuzzy Decision Matrix (NFDM) typically fall within the range of 0 to 1.

Step 4:

In this step the Weighted Normalized Fuzzy Decision Matrix (WNFDM) is constructed by multiplying each element of the NFDM by its corresponding Attribute's weights. This new variable is denoted by Z_{ij} and consider T(Z_{ij}) = (l_{ij}, m_{ij}, n_{ij}).

Thus (l_{ij}, m_{ij}, n_{ij}) = $\left(\frac{p_{ij}}{M}, \frac{q_{ij}}{M}, \frac{r_{ij}}{M}\right) \times (a_j, b_j, c_j)$,

i.e., $l_{ij} = \frac{p_{ij}}{M} \times a_j$, $m_{ij} = \frac{q_{ij}}{M} \times b_j$ and $n_{ij} = \frac{r_{ij}}{M} \times c_j$.

Step 5:

A pseudo alternative Station is constructed on the hypothesis that it is the best among all Stations over all attributes. This is called as Positive Ideal Station (PIS). So, the PIS must be represented by a set of m TFNs. For the j^{th} attribute A_j, the TFN of the PIS is denoted by $(a_j^{PIS}, b_j^{PIS}, c_j^{PIS})$ and is defined by

$a_j^{PIS} = \underset{i}{Max}(l_{ij})$,

$b_j^{PIS} = \underset{i}{Max}(m_{ij})$, and

$c_j^{PIS} = \underset{i}{Max}(n_{ij})$.

Again, a pseudo alternative is constructed on the hypothesis that it is the negative best (worst) among all Stations over all attributes. So, this alternative is called as Negative Ideal Station (NIS). So, the NIS must also be represented by a set of m TFNs. For the j^{th} attribute A_j, the TFN of the NIS is denoted by $(a_j^{NIS}, b_j^{NIS}, c_j^{NIS})$ and is defined by

$a_j^{NIS} = \underset{i}{Min}(l_{ij})$,

$b_j^{NIS} = \underset{i}{Min}(m_{ij})$, and

$c_j^{NIS} = \underset{i}{Min}(n_{ij}).$

Step 6:

The Fuzzy preference degree between two TFNs (discussed in section 2) is now applied to compare each Station with the Positive Ideal Station. Next, the preference degree of each of the following pairs: $(T(Z_{1j}), T(PIS_j))$, $(T(Z_{2j}), T(PIS_j))$, …are calculated.

Now P_i^+, the mean of the total preference degrees of the set of TFNs of the PIS and the set of TFNs of the i^{th} Station S_i is defined as: $P_i^+ = \frac{1}{s}\sum_{j=1}^{s} P\{T(Z_{ij}), T(PIS_j)\}.$

Again, the preference degrees of each of the pairs $(T(NIS_j), T(W_{1j}))$, $(T(NIS_j), T(W_{2j}))$, … and so on, are computed. Now, similarly, P_i^-, the mean of the total preference degrees of the set of TFNs of the NIS and the set of TFNs of the i^{th} Station S_i is defined as: $P_i^- = \frac{1}{s}\sum_{j=1}^{s} P\{T(NIS_j), T(Z_{ij})\}.$

Step 7:

Finally, the closeness co-efficient of each Station S_i is defined by $CC(S_i) = \frac{P_i^-}{P_i^+ + P_i^-}$.

The Station with maximum closeness co-efficient value will be taken as the Station having least air pollution.

Finally, a methodology for classifying stations based on Air Quality Index (AQI) data is proposed. Typically, in multi-attribute decision-making problems, ranking the stations is the final step in the evaluation process. However, categorizing stations on a finite scale significantly helps in identifying locations that require attention for air quality management. In this study, a three-tier classification system is proposed: Below Average, Medium, and Good. This approach can be further extended to classify stations into additional groups.

To classify the stations, the maximum Closeness Coefficient (CC) is determined for each category. For the 'Below Average' category, a hypothetical station, denoted as S_{BA} is introduced. The rating vector from all decision-makers (DMs) for this station is assumed to be {MP, MP, …, MP}. This assumption is based on the hypothesis that a station with poor air quality will generally receive lower ratings. The closeness coefficient of S_{BA}, denoted as α_{BA}, is computed. For the 'Medium' category, a hypothetical station, denoted as S_{MS}, is introduced. The rating vector from all DMs for this station is assumed to be {MG, MG, …, MG}. The closeness coefficient of S_{MS}, denoted as α_M, is calculated. For the Good category, the maximum CC value is 1. Here MP, MG stand for Medium Poor and Medium Good respectively.

The station with the highest value of closeness coefficient is considered the station with the least air pollution. Based on the computed CC values, the stations are classified into the following categories. A station S_i belongs to the 'Below Average' category if $CC(S_i)\in[0,\alpha_{BA}]$. A station S_i belongs to the 'Medium' category if $CC(S_i)\in(\alpha_{BA},\alpha_M]$. A station S_i belongs to the 'Good' category if $CC(S_i)\in(\alpha_M,1]$.

The classification of stations is a dynamic process, as air quality can vary over time due to environmental and anthropogenic factors. This classification system plays a crucial role in monitoring and managing air quality effectively.

4. Case Study

A case study is undertaken to analyze and compare the FuzzyAir Quality Index (FAQI) of the given stations:

S_1: Behala; S_2: Gariahat; S_3: Moulali; S_4: Shyambazar; S_5: Tollugunge; S_6: Beliaghata; S_7: Ultadanga. All the seven Stations are situated in Kolkata, West Bengal, India. In this case study, four major factors causing the air pollution have been considered as Attributes. They are A_1: NO_2, A_2: PM_{10}, A_3: $PM_{2.5}$, A_4: SO_2. Based on the data of WBPCB (West Bengal Central Pollution Control Board) of the period from 2023 January to 2023 December, the case study has been executed.

Step 1:

At first, to identify the weights of the Attributes, three Experts (E-1, E-2 and E-3) have been appointed to input their decision on the rating of the attributes. The inputs are Linguistic terms from the scale {Very Low (VL), Low (L), Medium Low (ML), Medium (M), Medium High (MH), High (H), Very High (VH)}. The Linguistic terms along with their corresponding TFNs are provided in Table 1.

Table 1. Linguistic terms and their corresponding TFNs.

Linguistic term	TFN
VL	(0.1, 0.1, 0.2)
L	(0.2, 0.3, 0.4)
ML	(0.3, 0.4, 0.5)
M	(0.4, 0.5, 0.6)
MH	(0.6, 0.7, 0.8)
H	(0.7, 0.8, 0.9)
VH	(0.9, 1, 1)

The inputs provided by the Experts are displayed in Table 2.

Table 2. Decision inputs from the experts.

	NO_2	PM_{10}	$PM_{2.5}$	SO_2
E-1	H	MH	VH	MH
E-2	MH	H	VH	M
E-3	H	H	VH	MH

Thus, the weights of the attributes are given in the following table.

Table 3. Weights of the attributes.

A_1	A_2	A_3	A_4
(0.6, 0.766, 0.9)	(0.6, 0.766,0.9)	(0.9, 1, 1)	(0.4, 0.633, 0.8)

Step 2:

Now the Fuzzy Decision Matrix (FDM) is constructed using the aggregation technique as described in the methodology. Thus, the FDM obtained along with the maximum values derived from the raw data for the attributes are shown in Table 4.

Table 4. FDM along with the maximum values of the Attributes.

	A_1			A_2			A_3			A_4		
	Min	Average	Max	min	Average	Max	Min	Average	Max	Min	Average	Max
S_1	17	35.365385	62	34	99.20192	254	19	51.35577	127	2	6.230769	16
S_2	19	31.673077	60	27	88.15385	243	18	44.73077	124	2	4.519231	13
S_3	22	36.701923	70	29	107.2596	267	19	55.38462	135	2	6.855769	17
S_4	21	34.538462	61	31	97.42308	250	19	50.31731	130	2	6.038462	15
S_5	17	29.884615	50	29	86.25	179	18	43.29808	89	2	4.394231	10
S_6	19	31.903846	53	29	92.48077	221	18	46.82692	113	2	4.826923	12
S_7	21	34.778846	67	28	111.5577	1124	19	51.97115	131	2	6.278846	18
Max value		70			1124			135			18	

Step 3:

Now the Normalized FDM (NFDM) is constructed by dividing the entries of an Attribute in the FDM by the corresponding Maximum values (available in the last row of Table 4). The NFDM is shown in Table 5.

Table 5. NFDM with corresponding attribute weights.

	A_1	A_2	A_3	A_4
S_1	(0.243,0.505,0.886)	(0.03,0.088,0.226)	(0.141,0.38,0.941)	(0.111,0.346,0.889)
S_2	(0.271,0.452,0.857)	(0.024,0.078,0.216)	(0.133,0.331,0.919)	(0.111,0.251,0.722)
S_3	(0.314,0.524,1)	(0.026,0.095,0.238)	(0.141,0.41,1)	(0.111,0.381,0.944)
S_4	(0.3,0.493,0.871)	(0.026,0.087,0.222)	(0.141,0.373,0.963)	(0.111,0.335,0.833)
S_5	(0.243,0.427,0.714)	(0.026,0.077,0.159)	(0.133,0.321,0.659)	(0.111,0.244,0.556)
S_6	(0.271,0.456,0.757)	(0.026,0.082,0.197)	(0.133,0.347,0.837)	(0.111,0.268,0.667)
S_7	(0.3,0.497,0.957)	(0.025,0.099,1)	(0.141,0.385,0.97)	(0.111,0.349,1)
Wts.	(0.6,0.766,0.9)	(0.6,0.766,0.9)	(0.9,1,1)	(0.4,0.633,0.8)

Step 4:

After that the weighted NFDM is calculated by multiplying the NFDM with the corresponding weights of the Attributes and WNFDM is demonstrated in Table 6.

Table 6. Weighted NFDM with corresponding attribute weights.

	A_1	A_2	A_3	A_4
S_1	(0.1458,0.3868,0.7974)	(0.018,0.0674,0.2034)	(0.1269,0.38,0.941)	(0.0444,0.2190,0.7112)
S_2	(0.1626,0.3462,0.7713)	(0.0144,0.0597,0.1944)	(0.1197,0.331,0.919)	(0.0444,0.1589,0.5776)
S_3	(0.1884,0.4014,0.9)	(0.0156,0.0728,0.2142)	(0.1269,0.41,1)	(0.0444,0.2412,0.7552)
S_4	(0.18,0.3776,0.7839)	(0.0168,0.0666,0.1998)	(0.1269,0.373,0.963)	(0.0444,0.2121,0.6664)
S_5	(0.1458,0.3271,0.6426)	(0.0156,0.059,0.1431)	(0.1197,0.321,0.659)	(0.0444,0.1545,0.4448)
S_6	(0.1626,0.3493,0.6813)	(0.0156,0.0628,0.1773)	(0.1197,0.347,0.837)	(0.0444,0.1696,0.5336)
S_7	(0.18,0.3807,0.8613)	(0.015,0.0758,0.9)	(0.1269,0.385,0.97)	(0.0444,0.2209,0.8)

Step 5:

Here, the PIS and the NIS as described in the methodology are constructed. The decision vectors for NIS and PIS are shown in Tables 7 and 8 respectively.

Table 7. TFN for the PIS.

	A_1	A_2	A_3	A_4
PIS	(0.1884,0.4014,0.9)	(0.018,0.0758,0.9)	(0.1269,0.41,1)	(0.0444,0.2412,0.8)

Table 8: TFN for the NIS.

	A_1	A_2	A_3	A_4
NIS	(0.1458,0.3271,0.6426)	(0.0144,0.0590,0.1431)	(0.1197,0.321,0.569)	(0.0444,0.1545,0.4448)

Step 6:

Now, using the decision vectors of both PIS and NIS, the respective Fuzzy preference degree of each Station with respect to the PIS and NIS is calculated, as described in the methodology.

Table 8. Fuzzy preference degree of each Station with respect to the PIS.

	A_1	A_2	A_3	A_4	Average
S_1	0.0191	0.0730	0.0062	0.0103	0.02715
S_2	0.0244	0.0768	0.0183	0.0349	0.0386
S_3	0	0.0706	0	0.0027	0.018325
S_4	0.0128	0.0740	0.0073	0.0154	0.027375
S_5	0.0437	0.0856	0.0364	0.0517	0.05435
S_6	0.0313	0.0789	0.0181	0.0376	0.041475
S_7	0.0078	0.0007	0.0049	0.0042	0.0044

Table 9. Fuzzy preference degree of each Station with respect to the NIS.

	A_1	A_2	A_3	A_4	Average
S_1	0.0246	0.0128	0.0301	0.0413	0.0272
S_2	0.0193	0.009	0.0181	0.0167	0.015775
S_3	0.0437	0.0152	0.0364	0.0490	0.036075
S_4	0.0397	0.0118	0.0291	0.0363	0.027025
S_5	0	0.0002	0	0	0.00005
S_6	0.0124	0.0070	0.0183	0.0141	0.01295
S_7	0.0359	0.0851	0.0315	0.0475	0.05

Step 7:

Now, the values of closeness coefficients, i.e., the Fuzzy based Air Quality Index values of the Stations are calculated using the process described in the methodology and finally the values are as follows:

$CC(S_1) = 0.50046$, $CC(S_2) = 0.290115$, $CC(S_3) = 0.663143$, $CC(S_4) = 0.496783$, $CC(S_5) = 0.000919$, $CC(S_6) = 0.237942$, $CC(S_7) = 0.919118$.

Therefore, $S_5 < S_6 < S_2 < S_4 < S_1 < S_3 < S_7$, i.e., Tollygunge < Beliaghata < Gariahat < Shyambazar < Behala < Moulali < Ultadanga, which means that, "Ultadanga" is the most air polluted station and "Tollygunge" is the least air polluted station among the Stations considered in this case study.

Now, based on the data of WBPCB (West Bengal Central Pollution Control Board) of the period from January 2023 to December 2023, the AQI value of every station is calculated by taking the 24 h average value of each pollutant. The linear interpolation method is used to calculate the subindices of each pollutant for every station and the highest sub-indices gives the final AQI value of every station. Here, The FAQI value, obtained by Fuzzy TOPSIS method and the AQI value, obtained by the conventional method are provided in Table 10.

Table 10. Outputs of AQI and FAQI for the study period of different sampling stations.

Stations	AQI	FAQI	Ranking Based on AQI	Ranking Based on FAQI
Behala Chowrasta	99	0.50046	3	3
Gariahat	88	0.290115	6	5
Moulali	105	0.663143	2	2
Shyambazar	97	0.496783	4	4
Tollygunge	86	0.000919	7	7
Beliaghata	92	0.237942	5	6
Ultadanga	108	0.919118	1	1

Following the process of classifying the Stations, as illustrated in Section 3, the following values have been derived: $\alpha_{BA} = 0.29035$ and $\alpha_M = 0.93042$. So, a station S_i will belong to the 'Below Average' category if $CC(S_i) \in [0, 0.29035]$. A station S_i will be classified as 'Medium' if $CC(S_i) \in (0.29035, 0.93042]$. A station S_i will be categorized as 'Good' if $CC(S_i) \in (0.93042, 1]$. Hence, Tollygunge, Beliaghata, and Gariahat are categorized as 'Below Average' stations; Shyambazar, Behala Chowrasta, Moulali, Ultadanga are categorized as 'Medium' stations in terms of air quality. No station considered here can be classified as a 'Good' station.

5. Conclusion

This study has demonstrated the effectiveness of a fuzzy TOPSIS approach for comparing Air Quality Index (AQI) values across different monitoring stations. The approach not only ranks air quality levels based on real data but also classifies stations into different categories. By incorporating fuzzy logic into the TOPSIS method, the proposed model enhances credibility in air quality assessment. The results indicate that Ultadanga is the most polluted station, while Tollygunge is the least polluted among the locations considered. The classification of stations based on their Closeness Coefficient (CC) further refines the interpretation of air quality levels, facilitating more precise monitoring and targeted interventions. The comparison of AQI and FAQI rankings reveals consistency in identifying pollution-affected areas, demonstrating the reliability of this methodology. These findings highlight the applicability of fuzzy TOPSIS in air quality monitoring, which assists policymakers and environmental agencies in making informed decisions. Future research can explore its potential for classifying and evaluating other environmental indices where Multi-Attribute Decision Making (MADM) plays a crucial role in managing uncertainty.

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