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Fuzzy Game with Qualitative Payoffs in Linguistic Terms

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Abstract

Game strategies are vital in Economics, Business Administration, Sociology, Social sciences, Military operations, etc. The fuzzy set theory offers a background and a better foundation for studying game theory problems in which fuzzy numbers represent the payoffs to address the qualitative data. This paper deals with game theory problems with qualitative payoffs in linguistic terms. The fuzzy numbers represent the qualitative data irrespective of their linear or non-linear membership functions. The mean value of the horizontal points on the left-right membership functions of the fuzzy numbers justifies their order. Ordering the qualitative payoffs provides scope for solving the game theory problems using the saddle point method. Finally, the technique is illustrated numerically.

Keywords: Game; Qualitative payoffs; Linguistic terms; Mean value of the horizontal points.

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1. Introduction

Fuzzy numbers and fuzzy logic are often used to quantify linguistic terms. They are widely used in real-life decision-making situations, artificial intelligence, machine learning social-life negotiations, electoral-voting optimization techniques, etc. However, the information available to choose an optimal strategy is not always quantitative and imprecise in real-life scenarios. Using fuzzy numbers and fuzzy logic, the qualitative payoffs in linguistic terms of a problem are quantified. Fuzzy game theory is a fascinating and important area of study that many researchers have explored over the years. Fuzzy game theory applies the principles of fuzzy logic to game theory to model situations where the payoffs, strategies, or information available to players are not precise but rather fuzzy or uncertain. This is particularly useful in real-world scenarios where information is often imprecise, vague, or incomplete. Many authors studied fuzzy game theory problems. Some of them use the ranking function to solve the game problem. In 1944, Newmann and Morgenstern [1] published "Theory of Games and Economic Behaviour," which formally formulated game theory. The best out of the worst concept, i.e. the idea of minimizing the maximum losses,

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served as the foundation for Von Newmann's strategy for solving the Game Theory problems. This idea can be applied to the majority of competitive game theory problems. However, the information in real-world scenarios is imprecise, and the system has some inherent vagueness or uncertainty. As a result, traditional mathematical methods might not be effective in formulating and resolving real-world issues. The fuzzy sets, Zadeh [2] introduced offer practical and efficient tools and approaches to deal with these problems. Numerous authors have studied fuzzy games, some with ranking functions to solve fuzzy game problems. Aristuidou and Sarangi [3] represented a non-cooperative model of a normal-form game using tools from fuzzy set theory. Gao [4] represented a strategic game with fuzzy payoffs. Medinechiene, *et al*. [5] described a model of dwelling selection using fuzzy games theory on buildings. Chakeri *et al*. [6] presented fuzzy Nash equilibrium in fuzzy games using ranking fuzzy numbers. Jawad [7] represented fuzzy sets and fuzzy processes with game theory to address the uncertainty in data for mobile phone companies. Kumar and Kumaraghura [8] presented a solution to the fuzzy game problems with triangular fuzzy numbers using a ranking function to compare the fuzzy numbers. Selvakumari and Lavanya [9] considered a two-person zero-sum game with imprecise (triangular or trapezoidal) fuzzy numbers using a ranking function as an approach to solving the problem. Kumar and Gnanaprakash [10] represented a (3×3) two-person zero-sum game with octagonal fuzzy payoffs using a ranking function to solve the fuzzy game. Khedekar, *et al.* [11] advocate an application of Fuzzy Game Theory to Industrial Decision Making. Krishnaven *et al.* [12] suggested a novel approach for fuzzy game theory problems. Thomas and Jose [13] presented a Pythagorean fuzzy approach to game theory problems. Soni *et al.* [14] presented a mathematical approach to fuzzy game. Hussein and Abood [15] used ranking functions to demonstrate fuzzy game problems. Gajalakshmi and Rabinson [16] solved game theory problems using reverse-order pentagonal fuzzy numbers. Mitlif [17] suggested a modified ranking function to compute fuzzy matrix games. Thomas and Jose [18] presented the Pythagorean fuzzy approach to the game.

This paper aims to suggest an algorithm for solving fuzzy game theory problems with qualitative payoffs. Trapezoidal fuzzy numbers represent the qualitative payoffs as quantitative payoffs. The horizontal mean value of points on the membership functions is used as a ranking tool to suggest the algorithm.

Apart from the above introduction, the rest of this paper comprises five sections. Section 2 presents preliminary definitions of the proposed approach. Section 3 reviews game theory. Section 4 presents the numerical illustration. Conclusions conclude the last section, 5.

2. Preliminaries

2.1. *Fuzzy number*

A fuzzy subset $\tilde{A}$ of the real line R is known as a fuzzy number if its membership function $f_{\tilde{A}}(x)$ which satisfies the following conditions for $a, b, c, d \in \tilde{A}$, $(a \leq b \leq c \leq d)$,

(i) $f_{\tilde{A}}(x)$ is a piece-wise continuous function of R to the closed interval $[0, 1]$,

(ii) $f_{\tilde{A}}(x)$ is strictly increasing on $[a, b]$,
(iii) $f_{\tilde{A}}(x) = 1$, for all $x \in [b, c]$,
(iv) $f_{\tilde{A}}(x)$ is strictly decreasing on $[c, d]$,
(v) $f_{\tilde{A}}(x) = 0$, for all $x \in]-\infty, a] \cup [d, \infty[$,

The fuzzy number in Def. 2.1 is conveniently represented as $A = (a, b, c, d)$, and its membership function $f_{\tilde{A}}(x)$ is expressed as

$$f_{\tilde{A}}(x) = \begin{cases} f_{\tilde{A}}^{L}(x)\,; & x \in [a, b] \\ 1; & x \in [b, c] \\ f_{\tilde{A}}^{R}(x)\,; & x \in [c, d] \\ 0; & Otherwise \end{cases}, \tag{1}$$

where $f_{\tilde{A}}^{L}(x): [a, b] \longrightarrow [0, 1]$ and $f_{\tilde{A}}^{R}(x): [c, d] \longrightarrow [0, 1]$ are known as the left and the right membership functions of the fuzzy number $\tilde{A}$, respectively. $f_{\tilde{A}}^{L}(x)$ is continuous and strictly increasing on $[a, b]$, whereas $f_{\tilde{A}}^{R}(x)$ is continuous and strictly decreasing on $[c, d]$.

2.2. *Mean value of the horizontal points as a ranking function of fuzzy numbers*

Let $P^L(x^L, y^L)$ and $P^R(x^R, y^R)$ are the points on the left and the right membership functions of a fuzzy number $\tilde{A} = (a, b, c, d)$, respectively. The visual depictions of these points are illustrated in Fig. 1.

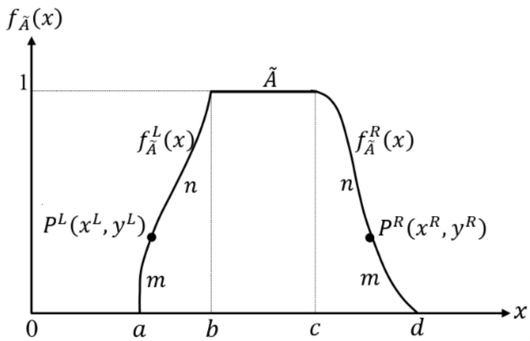

Fig. 1. Visual depictions of the horizontal points on the left-right membership functions.

The points divide the corresponding membership functions in the same ratio $m : n\ (m \neq n)$. The coordinates of the points $P^L(x^L, y^L)$ and $P^R(x^R, y^R)$ are obtained by solving the following equations.

$$n \int_{a}^{x^L} \sqrt{1 + \left(\frac{d\, f_{\tilde{A}}^{L}(x)}{dx}\right)^2}\, dx = m \int_{x^L}^{b} \sqrt{1 + \left(\frac{d\, f_{\tilde{A}}^{L}(x)}{dx}\right)^2}\, dx\,, \tag{2}$$

$$n \int_{0}^{y^L} \sqrt{1 + \left(\frac{d\, g_{\tilde{A}}^{L}(y)}{dy}\right)^2}\, dy = m \int_{y^L}^{1} \sqrt{1 + \left(\frac{d\, g_{\tilde{A}}^{L}(y)}{dy}\right)^2}\, dy\,, \tag{3}$$

$$m\int_{c}^{x^R}\sqrt{1+\left(\frac{d\,f_{\tilde{A}}^{R}(x)}{dx}\right)^2}\;dx \;=\; n\int_{x^R}^{d}\sqrt{1+\left(\frac{d\,f_{\tilde{A}}^{R}(x)}{dx}\right)^2}\;dx\,, \tag{4}$$

$$m\int_{1}^{y^R}\sqrt{1+\left(\frac{d\,g_{\tilde{A}}^{R}(y)}{dy}\right)^2}\;dy \;=\; n\int_{y^R}^{0}\sqrt{1+\left(\frac{d\,g_{\tilde{A}}^{R}(y)}{dy}\right)^2}\;dy, \tag{5}$$

where $g_{\tilde{A}}^{L}(y)$ and $g_{\tilde{A}}^{R}(y)$ are the inverse of the left and the right membership functions $f_{\tilde{A}}^{L}(x)$ and $f_{\tilde{A}}^{R}(x)$, respectively. The mean value of the two points $P^L(x^L, y^L)$ and $P^R(x^R, y^R)$ on the left and the right membership functions, respectively of the fuzzy number $\tilde{\mathrm{A}} = (a, \mathrm{b}, \mathrm{c}, \mathrm{d})$ is denoted by $\mathcal{M}(\tilde{\mathrm{A}})$ and defined as

$$\mathcal{M}(\tilde{\mathrm{A}}) = \frac{1}{2}(x^L + x^R). \tag{6}$$

2.3. *Ordering algorithm of fuzzy numbers*

Using the mean value of the horizontal points from Eq. (6), the ordering of the two fuzzy numbers $A_i = (a_i, b_i, c_i, d_i)$ and $A_j = (a_j, b_j, c_j, d_j)$; $i, j = 1, 2, \ldots n$ is defined as follows:

(i) If $\mathcal{M}(A_i) > \mathcal{M}(A_j)$, then $A_i \succ A_j$,
(ii) if $\mathcal{M}(A_i) < \mathcal{M}(A_j)$, then $A_i \prec A_j$, (7)
(iii) if $\mathcal{M}(A_i) = \mathcal{M}(A_j)$, then $A_i \sim A_j$.

2.4. *Trapezoidal fuzzy numbers*

A fuzzy number $\tilde{\mathrm{A}} = (a, \mathrm{b}, \mathrm{c}, \mathrm{d})$ is said to be a trapezoidal fuzzy number, if its membership function $f_{\tilde{A}}(x)$ is given by

$$f_{\tilde{A}}(x) = \begin{cases} f_{\tilde{A}}^{L}(x) = \frac{x-a}{b-a}\,; & x \in [a,b] \\ 1\,; & x \in [b,c] \\ f_{\tilde{A}}^{R}(x) = \frac{x-d}{c-d}\,; & x \in [c,d] \\ 0; & Otherwise \end{cases}, \tag{8}$$

For convenience, the trapezoidal fuzzy number is also denoted similarly as $\tilde{\mathrm{A}} = (a, \mathrm{b}, \mathrm{c}, \mathrm{d})$. In the case of trapezoidal fuzzy number $\tilde{\mathrm{A}} = (a, \mathrm{b}, \mathrm{c}, \mathrm{d})$, the points $P^L(x^L, y^L)$ and $P^R(x^R, y^R)$ on the respective left and right membership functions which divide them in the same ratio $m : n$ are obtained as follows:

$$x^L = \frac{mb+na}{m+n}\,;\; y^L = \frac{m}{m+n}\,, \tag{9}$$

$$x^R = \frac{mc+nd}{m+n}\,;\; y^R = \frac{m}{m+n}\,. \tag{10}$$

Hence, from Eq. (6), the mean value $\mathcal{M}(\tilde{\mathrm{A}})$ for the generalized trapezoidal fuzzy number $\tilde{\mathrm{A}} = (a, \mathrm{b}, \mathrm{c}, \mathrm{d})$ is given by

$$\mathcal{M}(\tilde{\mathrm{A}}) = \frac{1}{2}\left(\frac{\mathrm{mb}+na}{m+n} + \frac{mc+nd}{m+n}\right). \tag{11}$$

2.5. *Arithmetic operations of fuzzy number*

The arithmetic operations of any two fuzzy numbers $A_i = (a_i, b_i, c_i, d_i)$ and $A_j = (a_j, b_j, c_j, d_j)$, are defined as follows:

(i) Addition

$$A_i \oplus A_j = (a_i, b_i, c_i, d_i) \oplus (a_j, b_j, c_j, d_j), \\ = (a_i + a_j, b_i + b_j, c_i + c_j, d_i + d_j).$$

(ii) Subtraction

$$A_i \ominus A_j = (a_i, b_i, c_i, d_i) \ominus (a_j, b_j, c_j, d_j), \\ = (a_i - d_j, b_i - c_j, c_i - b_j, d_i - a_j).$$

(iii) Multiplication

$$A_i \otimes A_j = (a_i, b_i, c_i, d_i) \otimes (a_j, b_j, c_j, d_j), \\ = (a_i \times a_j, b_i \times b_j, c_i \times c_j, d_i \times d_j).$$

(iv) Division

$$A_i \oslash A_j = (a_i, b_i, c_i, d_i) \oslash (a_j, b_j, c_j, d_j), \\ = \left(\frac{a_i}{d_j}, \frac{b_i}{c_j}, \frac{c_i}{b_j}, \frac{d_i}{a_j}\right).$$

(v) Multiplication by a scalar 'k'

$$kA_i = \begin{cases} (ka_i, kb_i, kc_i, kd_i); & if\ k \geq 0 \\ (kd_i, kc_i, kb_i, ka_i); & if\ k < 0 \end{cases}.$$

3. Two Person Zero Sum Games with Fuzzy Payoffs

When Player I chooses pure strategy $i = 1, 2, \ldots m$ and Player II chooses pure strategy $j = 1, 2, \ldots n$, let $\tilde{a}_{ij}$ are the fuzzy payoffs for Player I and $-\tilde{a}_{ij}$ are the fuzzy payoffs for Player II, then the two-person zero-sum game with fuzzy payoffs can be represented as a fuzzy payoff matrix:

$$\begin{array}{cc} & \text{Player II} \\ & \begin{array}{cccc} 1 & 2 & - & n \end{array} \\ \text{Player I} \begin{array}{c} 1 \\ 2 \\ | \\ m \end{array} & \begin{bmatrix} \tilde{a}_{11} & \tilde{a}_{12} & - & \tilde{a}_{1n} \\ \tilde{a}_{21} & \tilde{a}_{22} & - & \tilde{a}_{2n} \\ | & | & | & | \\ \tilde{a}_{m1} & \tilde{a}_{m2} & - & \tilde{a}_{mn} \end{bmatrix} \end{array} \quad (12)$$

The game's outcome is zero-sum, so when one player receives a gain, the other suffers an equal loss. The matrix $(\tilde{a}_{ij})_{m \times n}$ in Eq. (12) is known as a fuzzy payoff matrix.

3.1. *Fuzzy saddle or fuzzy value of the game*

Let $\widetilde{M} \approx (\tilde{a}_{ij})_{m \times n}$ be a fuzzy payoff matrix, then the fuzzy game is said to have a fuzzy saddle point if

$$\max_{i=1,2,\ldots m} \min_{j=1,2,\ldots n} \tilde{a}_{ij} = \min_{j=1,2,\ldots m} \max_{i=1,2,\ldots n} \tilde{a}_{ji} \quad (13)$$

The common fuzzy value is taken to be the game's value. Otherwise, the game is said to have no saddle point.

4. Numerical Illustrations

In numerical examples, to compute the horizontal mean values of the fuzzy numbers, the numerical value of the ratio $m\!:\!n$ is taken as $2\!:3$ conveniently.

Example 1: Consider two major private limited companies of the same product regarded as players A and B competing for the business. Advertising of their product is the strategy (pure strategy) to optimize the profit/loss in the game for both players. Let the strategy of Player A be denoted by $A_1, A_2, and\ A_3$ whereas the strategy of Player B is denoted by $B_1, B_2, and\ B_3$. In a pure strategy, we assume that Player A wishes to maximize the minimum gain, whereas Player B wishes to minimize the maximum loss. Qualitative data represent the payouts, as displayed in the qualitative Payoff matrix below. The improved qualitative payoffs benefit Player A, whereas the worse one benefits Player B. Based on these conditions, the game is biased against Player B. Payments are to be made according to the choices made as shown in Table 1.

Table 1. Payoffs in linguistic terms of Ex. 1.

Choices	Payment
(A_1, B_1)	B pays extremely high to A
(A_1, B_2)	B pays high to A
(A_1, B_3)	B pays high to A
(A_2, B_1)	B pays low to A
(A_2, B_2)	B pays much high to A
(A_2, B_3)	B pays nil to A
(A_3, B_1)	B pays high to A
(A_3, B_2)	B pays extremely high to A
(A_3, B_3)	B pays low to A

The above payments can be easily arranged in the form of a matrix as follows

$$\text{Player B} \\ \text{Player } A \quad \begin{matrix} \\ A_1 \\ A_2 \\ A_3 \end{matrix} \begin{matrix} B_1 & B_2 & B_3 \\ \left[\begin{matrix} Extremely\ high & High & High \\ Low & Much\ High & nil \\ High & Extremely\ high & Low \end{matrix}\right] \end{matrix} \tag{14}$$

Solution: Using trapezoidal fuzzy numbers, qualitative payoffs were converted into quantitative payoffs, as displayed in Table 2.

Table 2. Qualitative data converted into quantitative data.

Payoffs' Qualitative Data	Payoffs' Quantitative Data (Trapezoidal number)
Extremely high	(1, 4, 6, 9)
Much High	(1, 3, 5, 7)
High	(1, 3, 3, 5)

Low	(1, 1, 1, 1)
nil	(0, 0, 0, 0)

Hence, the fuzzy payoff matrix (14) reduces to

$$\begin{array}{cc} & \text{Player B} \\ Player\ A & \begin{bmatrix} (1,4,6,9) & (1,3,5,7) & (1,3,5,7) \\ (1,1,1,1) & (1,3,3,5) & (0,0,0,0) \\ (1,3,5,7) & (1,4,6,9) & (1,1,1,1) \end{bmatrix} \end{array} \tag{15}$$

Using the formula for the horizontal mean value in Eq. (11), the ranking values of the trapezoidal fuzzy data are obtained as follows:

Table 3. Ranking values of the trapezoidal fuzzy data.

Payoffs' Quantitative Data (Trapezoidal number)	Horizontal Mean Value $\mathcal{M}(\tilde{A}) = \frac{1}{2}\left(\frac{2b+3a}{2+3} + \frac{2c+3d}{2+3}\right)$
(1, 4, 6, 9)	5
(1, 3, 5, 7)	4
(1, 3, 3, 5)	3
(1, 1, 1, 1)	1
(0, 0, 0, 0)	0

Using the ordering algorithm of fuzzy numbers in Eq. (7), maximin-minimax Eq. (13), and Table 3, we may obtain the saddle point of the fuzzy payoff matrix (15) as follows:

$$\underset{i=1,2,3}{max}\ \underset{j=1,2,3}{min}\ \tilde{a}_{ij} = (1,3,3,5) = \underset{j=1,2,3}{min}\ \underset{i=1,2,3}{max}\ \tilde{a}_{ji}$$

Hence, the value of the game is $high \approx (1,3,3,5)$.

Example 2: In a certain game, player A has three possible choices L, M, and N, while player B has two possible choices P and Q. Payments are to be made according to the choices made.

Table 4. Payoffs in linguistic terms of Ex. 2.

Choices	Payment
(L, P)	A pays much high to B
(L, Q)	B pays much high to A
(M, P)	A pays high to B
(M, Q)	B pays extremely high to A
(N, P)	B pays high to A
(N, Q)	B pays much high to A

The above payments can be easily arranged in the form of a matrix as follows:

$$\begin{array}{ccc} & & \text{Player B} \\ & & \begin{array}{cc} P & Q \end{array} \\ Player\ A & \begin{array}{c} L \\ M \\ N \end{array} & \begin{bmatrix} A \text{ pays much high to B} & B \text{ pays much high to } A \\ A \text{ pays high to } B & B \text{ pays extremely high to } A \\ B \text{ pays high to } A & B \text{ pays much high to } A \end{bmatrix} \end{array} \tag{16}$$

Solution: Using trapezoidal fuzzy numbers, qualitative payoffs in Table 4 were converted into quantitative payoffs, as displayed in Table 5.

Table 5. Qualitative payoffs converted into quantitative payoffs.

Choices	Payment (Qualitative)	Payment (Quantitative)
(L, P)	A pays much high to B	(-7, -5, -3, -1)
(L, Q)	B pays much high to A	(1, 3, 5, 7)
(M, P)	A pays high to B	(-5, -3, -3, -1)
(M, Q)	B pays extremely high to A	(1, 4, 6, 9)
(N, P)	B pays high to A	(1, 3, 3, 5)
(N, Q)	B pays much high to A	(1, 3, 5, 7)

Hence, the fuzzy payoff matrix is given by

$$\begin{array}{cc} & \text{Player B} \\ Player\ A & \begin{bmatrix} (-7,-5,-3,-1) & (1,3,5,7) \\ (-5,-3,-3,-1) & (1,4,6,9) \\ (1,3,3,5) & (1,3,5,7) \end{bmatrix} \end{array} \tag{17}$$

Using the formula for the horizontal mean value in Eq. (11), the ranking values of the trapezoidal fuzzy data are obtained as follows:

Table 6. Ranking values of the trapezoidal fuzzy data.

Payoffs' Quantitative Data (Trapezoidal number)	Horizontal Mean Value $\mathcal{M}(\tilde{A}) = \frac{1}{2}\left(\frac{2b+3a}{2+3} + \frac{2c+3d}{2+3}\right)$
(1, 4, 6, 9)	5
(1, 3, 5, 7)	4
(1, 3, 3, 5)	3
$(-7, -5, -3, -1)$	-4
$(-5, -3, -3, -1)$	-3

Using the ordering algorithm of fuzzy numbers in Eq. (7), maximin-minimax Eq. (13), and Table 6, we may obtain the saddle point of the fuzzy payoff matrix (17) as follows:

$$\underset{i=1,2,3}{max}\ \underset{j=1,2,3}{min}\ \tilde{a}_{ij} = (1,3,3,5) = \underset{j=1,2,3}{min}\ \underset{i=1,2,3}{max}\ \tilde{a}_{ji}$$

Hence, the value of the game is

$$B \text{ pays high to } A \approx (1,3,3,5).$$

5. Conclusion

Qualitative data refers to non-numeric information that describes qualities. This data type is often used in day-to-day life and is more subjective. Because qualitative data tends to capture personal experiences, opinions, or emotions very easily. It can vary from person to person frequently. Unlike quantitative data, qualitative data are more difficult to capture in standard metrics. They often require methods like fuzzy set representations to be evaluated. This paper suggests the solution of game theory problems with qualitative data in payoffs.

Such data are mostly used in our day-to-day life. Using the fuzzy number representation of the qualitative data and defining the ordering algorithm of fuzzy numbers, the problems are solved very significantly. The suggested technique can be important and valuable in Economics, Business Administration, Sociology, Political sciences, Military operations, etc.

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