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A Many-Component Cosmic Fluid with a Dynamic Dark Energy Term for the Accelerating Universe

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Abstract

Earlier studies have shown that in a two-component model of the universe with matter and the running vacuum energy, either eternal deceleration or acceleration is produced in the absence of a bare constant in the density of the running vacuum. Here, it is analytically solved for the Hubble parameter, in a spatially flat Friedmann universe with dark energy and matter as components, and the solution traces the evolutionary path from the prior decelerated to the late accelerated epoch. But along with the additive constant, equivalent to a cosmological constant, the model predicts a late time exponential acceleration in the expansion of the universe, and in the far future of the evolution it tends to de Sitter universe with deceleration parameter approaches -1. On contrasting the model with the cosmological data, it is favoured to the low value of present Hubble constant as measured from the CMB data.

Keywords: Friedmann equation; Dark energy; Cosmic acceleration; Flat geometry.

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1. Introduction

The accelerated expansion of the current universe is a confirmed phenomenon [1,2] as per the present observations. The observational data from the supernova type Ia (SNe Ia) [3,4], cosmic microwave background [5], large scale structure (LSS) [6], baryon acoustic oscillations [7] and weak lensing [8] are the strong supporting evidences for this. The reason for this accelerated expansion is argued to be due to the presence of gravitationally repulsive energy component known as dark energy (DE). The nature and evolution of DE is still not clear in spite of the numerous speculations existing in the current literature.

The simplest model for DE is by taking it as the vacuum energy or cosmological constant, leads to the standard ΛCDM model of the universe. This approach faces coincidence problem and fine-tuning problem [9]. The coincidence problems means that, even though the cosmic evolution of both dark matter and dark energy is different, their densities are of the same order in the present universe, a thing which is not accounted by the standard models of the universe. The fine-tuning problem is that, the theoretically

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predicted value of the cosmological constant is several orders higher than the observed value. These problems motivated the consideration of various dynamical DE models like quintessence [10,11], kessence [12], the Chaplygin gas model [13] and emerging DE models [14,15], in which the DE density is evolving with time. Another approach in explaining the recent acceleration of the universe, is by modifying the left-hand side of the Einstein equation, i.e., the geometry of the space time, which are generally called as modified gravity theories [16-18]. A class of models which were proposed to alleviate the problems of the ΛCDM model are the dynamical vacuum energy models. Recently much attention has been paid in these models in which DE treated as a time varying vacuum in which the density is varying as the universe expands, but equation of state stays constant around -1, and is often termed as running vacuum energy [19-23]. The main motivation for this approach is arising from vacuum energy, predicted by quantum field theory in curved space-time derived from the renormalization group. The vacuum energy predicted from such theories, possess constant equation of state but their density is varying during the evolution. The evolution of such a running vacuum energy is considered in literature [24], where the authors have shown that, the recent acceleration of the universe can be explained with varying density contain an extra constant term, and corresponding equation of state stands constant around -1. The model is free from future singularity under certain conditions on the model parameters.

In the curvature driven DE model [17], instead of postulating an additional DE term in the total energy density of the cosmic fluid, the authors propose a curvature driven DE to explain the present dynamics of the universe. However, the mechanism of energy exchange between matter and curvature is highly speculative with observational limitations. Gueorguiev *et al.* [18] postulated Einstein's cosmological constant as a source term for DE as $\Lambda = \Lambda_E \lambda^2$, where λ is fixed by scale invariant vacuum and independent of matter distribution, a property of space-time fabric. This complex scenario is simplified in this present work, where DE is described as a property of space which evolves with the expansion of the universe. The negative pressure due to warm dark matter particles forming reduced relativistic gas [25] is another attempt to explain the late accelerating phase, but clearly lacks any observational evidence for such a dominant particle creation in the late universe. The coasting evolution [26] is devoid of cosmic acceleration at any stage of evolution and hence can be considered only as a limiting case of a more general model. In the present work it is considered that our universe is approximated by a homogeneous and isotropic fluid which has many components: radiation, matter and a time varying dark component. They evolve differently with expansion as required by the conservation law and this simple model is free from age problem, coincidence problem, cosmological constant problem and also explains the recent observation of accelerated expansion. If we append a bare cosmological term to the dynamic term to represent vacuum energy, then the expansion eventually leads to de Sitter type model. The paper is organized as follows. Section 2 represents the dynamics of the universe in the new model. In section 3 we summarize the results and section 4 contains the conclusion.

2. New Model

Since the universe is flat [27] and Ω_m, the ratio of the baryonic plus dark matter density to the critical density is significantly less than unity and to make universe flat, there is a missing energy and it should possess negative pressure with equation of state parameter $\omega = \frac{p}{\rho c^2}$, with $-1 < \omega < -\frac{1}{3}$ [28-30]. It seems ideal to propose a model in which the energy density in the missing form or dark is comparable to the radiation density (to within a few orders of magnitude) at the end of inflation. One would want that the energy density of this dark component somehow tracks below the background density for most of the past history of the universe, and then, only recently grows to dominate the energy density and drive it into a period of accelerated expansion. Zlatev *et al.* proposed [31] a time varying component for the DE which decays as a constant power of the scale factor as $a^{-3(1+\omega_d)}$ with equation of state parameter for the unknown component related to that of backgound through $\omega_d = \frac{\frac{\alpha}{2}\omega_b - 1}{1+\frac{\alpha}{2}}$. If we choose the parameter $\alpha \approx 1$, then ω_d is negative always and turn to $-\frac{2}{3}$ when universe enters to matter domination and so the unknown dark component of energy density decays as a^{-1} from energy conservation law. As the universe evolves, dark component of energy density dominates in Ω and expansion of the universe shifts to an accelerating phase.

In the present work a dynamical dark energy with negative pressure is considered. It is an intrinsic property like surface tension or surface energy (proportional to area) of the physical universe at each epoch which increases as the universe expands due to negative pressure. So, the corresponding energy density varies as $\frac{a^2}{a^3}$. This dynamic missing energy of the cosmic fluid, can be thought of as a "mass" of empty space. As the density of other forms of matter decreases, the DE term will eventually dominate the universe's energy density. A component in the energy density varying as a^{-1} corresponds to a term with equation of state parameter $\omega = -\frac{2}{3}$, which is in agreement with recent observations [28]. Since the universe is spatially homogeneous and isotropic on very large scales, the space-time can be described by the spatially symmetric Friedmann-Robertson-Wlaker (FRW) metric: $ds^2 = c^2dt^2 - a(t)^2[\frac{dr^2}{1-kr^2} + r^2(d\vartheta^2 + sin^2\vartheta d\varphi^2)]$. Here k is the curvature parameter and is zero for flat universe. Incorporating a DE term, the field equation is,

$$R_{ik} - \frac{1}{2}g_{ik}R = \frac{8\pi G}{c^4}[T_{ik}^{(M)} + T_{ik}^{(d)}] \quad (1)$$

Last two terms on the r.h.s. represent energy-momentum tensor due to matter (both relativistic and non-relativistic) and dark component respectively. Here $T_{ik}^{(M)}$ or $T_{ik}^{(d)}$ are not separately conserved. Only their sum is conserved. Since the total energy of the comprehensive cosmic fluid is conserved and if it is considered as a perfect fluid, its energy-momentum tensor will be $T_{ik} = diagonal(\varepsilon, -p, -p, -p)$. The energy density ε and pressure p of the fluid comprises that of matter (relativistic and non-relativistic) and a dynamic DE. The Friedmann-Robertson-Walker metric with the assumption of spatially homogeneous and isotropic universe leads to Friedmann equations:

$$\frac{\dot{a}^2}{a^2} + \frac{kc^2}{a^2} = \frac{8\pi G}{3}[\rho_M + \rho_d] = \frac{8\pi G}{3}\rho \quad (2)$$

And

$$2\frac{\ddot{a}}{a}+\frac{\dot{a}^2}{a^2}+\frac{kc^2}{a^2}=-\frac{8\pi G}{c^2}[p_M+p_d]=-\frac{8\pi G}{c^2}p \tag{3}$$

Here ρ is the total energy density of the cosmic fluid. Corresponding pressure is p.
The acceleration equation is,

$$\frac{\ddot{a}}{a}=-\frac{4\pi G}{3c^2}[\rho c^2+3p] \tag{4}$$

The two Friedmann equations can be combined further to represent energy conservation law for an adiabatic evolution of the whole universe as

$$\frac{d}{da}(\rho a^3)+3\frac{a^2}{c^2}p=0 \tag{5}$$

The equation of state of the cosmic fluid is $p=\omega\rho c^2$, with constant ω, above equation is integrable to get the evolution of density as a function of scale factor. The comprehensive cosmic fluid is not uni-component, so a single ω is meaningless. However, it is reasonable to split the past epochs of universe into various eras (especially after matter domination) dominated by a single component in the fluid. This is justifiable, since each component in the cosmic fluid evolves differently without interaction and energy conservation can be applied individually.

The expansion rate of the universe is usually expressed as Hubble parameter $\frac{\dot{a}}{a}=H(t)$, whose present value is determined using the redshift-apparent magnitude data and is nearly $67kms^{-1}Mpc^{-1}$ [32-34]. With the density parameter $\Omega=\frac{\rho}{\rho_c}$. Friedmann equation is,

$$\frac{kc^2}{a^2}=[\Omega-1]H^2 \tag{6}$$

Where $\Omega=\frac{\rho_M+\rho_d}{\rho}$. In the relativistic and non relativistic matter era, ρ_M dominate and in dark era, ρ_d dominates in the total energy density. In the non-relativistic matter era, the dominant component is matter with equation of state parameter is approximately 0 and $\rho\sim\rho_M\propto a^{-3}$. If the present universe is matter dominated with density ρ_p, then $\frac{\rho}{\rho_p}=[\frac{a}{a_p}]^{-3}$.
Eq. (2) can be rewritten as

$$\dot{a}=H_p a_p[\Omega_p\frac{a_p}{a}-(\Omega_p-1)]^{\frac{1}{2}} \tag{7}$$

Since the universe is expanding and is expressed through the physically measurable quantity called redshift given by

$$1+z(t)=\frac{a_p}{a(t)} \tag{8}$$

The life span of non-relativistic matter era can be determined from

$$t=\int_{a_0}^{a}\frac{da}{\dot{a}} \tag{9}$$

Here a_0 is the scale factor of the early universe which is much smaller than present scale factor. With a substitution

$$x=\frac{a}{a_p}=(1+z)^{-1} \tag{10}$$

Life span of non-relativistic matter dominated era evaluated from Friedmann equations is

$$t_p=\int_0^{(1+z)^{-1}}\frac{dx}{\sqrt{1-\Omega_p+\Omega_p x^{-1}}} \tag{11}$$

If $\Omega=1$, as suggested by recent observations [27], then

$$t_p=\frac{2}{3H_p}\frac{1}{(1+z)^{\frac{3}{2}}} \tag{12}$$

Using similar arguments, for a relativistic matter dominated universe, expansion age will be

$$t_p = \frac{1}{2H_p}\frac{1}{(1+z)^2}. \tag{13}$$

If universe is matter dominated throughout its evolution after inflation, then $z = 0$ of the present epoch leads to $H_p t_p = \frac{2}{3}$. This is known as age problem in cosmology. Also, with non-relativistic matter domination in energy density, expansion rate of the universe will decrease with time. To overcome these difficulties, it is proposed that the universe has entered into a DE dominated era, around 9 billion years after the big bang which is supported by recent observations based on Type Ia supernovae data. To explain the accelerating phase of cosmic evolution, a dynamical DE with negative pressure, varying as a^{-1} is introduced. Such a proposition has great advantage over many recently proposed cosmological models in the literature, though the smooth time variation of DE term is not well defined to support all recent observations.

When universe enters to such a dark component dominated era, density becomes $\rho \sim \rho_d$. Also

$$\rho = \rho_p [\frac{a}{a_p}]^{-1} \tag{14}$$

Eq. (2) after a few simplifications becomes

$$\dot{a} = H_p a_p \sqrt{\Omega_p \frac{a}{a_p} - (\Omega_p - 1)} \tag{15}$$

If the universe is dominated with a dark component as above, the expansion age will be

$$t = \int_0^a \frac{da}{\dot{a}} = H_p^{-1} \int_0^{(1+z)^{-1}} \frac{dx}{\sqrt{\Omega_p x - (\Omega_p - 1)}} \tag{16}$$

Where $x = \frac{a}{a_p}$. For a flat geometry,

$$t_p = \frac{2}{H_p}\frac{1}{\sqrt{1+z}} \tag{17}$$

Using $z = 0$, gives $H_p t_p = 2$. This is for the dynamic DE dominated period with an accelerating expansion. Overall $H_p t_p$ will be less than 2 and age problem doesn't arise in this scenario.

3. Results

The radiation-dominated era lasted from about 10^{-43} seconds after the Big Bang to around 10,000 years. During this time, the energy density of electromagnetic radiation was greater than the energy density of matter. The radiation era transitioned to the matter era and this transition occurred thousands of years after the Big Bang. With the present estimate for Hubble parameter [34] as $H_p = 67 kms^{-1}MPc^{-1}$, the expansion age of the universe before entering into non-relativistic matter era is 7310 years if we use $z = 1000$ in Eq. (13). From Eq. (12), if we use $z = 0.1$, then $t_p = 8.45$ billion years as the expansion age of the non-relativistic matter dominated era. In the DE-dominated era, the universe's expansion accelerates. This era is thought to have begun when the universe was about 9 billion years old, after the matter-dominated era.

So, Eq. (17) can be redefined by performing the integration from $(1 + z_d)^{-1}$ to $(1 + z_p)^{-1}$.

Which corresponds DE dominated era to present epoch. This is reasonable if the missing component in the energy density varies as a^{-1}. This is the most important point in this phenomenological model. Eq. (17) becomes

$$t_d = \frac{2}{H_p}[(1+z_p)^{-\frac{1}{2}} - (1+z_d)^{-\frac{1}{2}}] \tag{18}$$

To get a best fit value and supported by Type Ia supernovae data, which predicts an accelerated expansion of the universe for the last 5 billion years and with $z_p = 0$, it follows that $z_d = 0.47$. If this redshift corresponds to the beginning of dynamical DE domination in total energy density, then non-relativistic matter dominated in the energy density only for epochs for which z is at least greater than 0.5 and Eq. (12) provides the life span for matter dominated universe as 5 billion years after big bang. A table showing the expansion age for matter dominated and DE dominated era for different redshifts and Hubble constant is illustrated below. Table 1 is for $H_0 = 67 km/s/Mpc$, and Table 2 is for $H_0 = 73\ km/Mpc$.

Table 1. Expansion age vs redshift with H_0 = 67 km/s/Mpc.

	Redshift (z)	Time elapsed (t) in billion years
Matter era	0.10	8.45
	0.47	5.5
Dark era	0.10	1.36
	0.47	5.12

Table 2. Expansion age vs redshift with H_0 = 73 km/s/Mpc.

	Redshift (z)	Time elapsed (t) in billion years
Matter era	0.10	7.75
	0.47	5.04
Dark era	0.10	1.25
	0.47	4.70

The deceleration parameter, which quantifies how the expansion rate of the universe changes over time will change from positive to a negative value as the universe enters to an accelerating phase. The deceleration parameter (q) is defined through $q = -\frac{1}{H^2}\frac{\ddot{a}}{a}$. In the accelerating phase q should be negative. Using Friedmann's acceleration equation,

$$q = \frac{8\pi G}{3H^2}\frac{1}{2}[\rho_M + \rho_d + \frac{3p_M}{c^2} + \frac{3p_d}{c^2}] \tag{19}$$

Redshift, $1 + z = \frac{a_p}{a(t)}$ and incorporating density parameter,

$$q = \frac{H_p^2}{H^2}\frac{1}{2}[\Omega_{Mp}(1+z)^3 - \Omega_{dp}(1+z)^1] \tag{20}$$

Subscript p stands for present epoch. If the present universe has only matter, then $q = +\frac{1}{2}$ and if it has only the dark component, then $q = -\frac{1}{2}$. If there is a bare cosmological constant in the field equation in addition to a dynamic epoch term, then q eventually approaches -1 (de Sitter universe). To get the evolution of q in the recent past, it is assumed that the universe entered into an accelerating phase at least 4 to 5 billion years ago and $H(t) \propto \frac{1}{t}$, so

$$q(z) \approx \frac{1}{2}[\Omega_{Mp}(1+z)^3 - \Omega_{dp}(1+z)^1] \tag{21}$$

$q(z)$ is plotted versus z in the recent past for two sets: one for $(\Omega_{Mp}, \Omega_{dp}) = (0.3, 0.7)$ and the second one for $(\Omega_{Mp}, \Omega_{dp}) = (0.2, 0.8)$ (Figs. 1 and 2). First set is more reasonable to explain the transition from decelerating phase to an accelerating phase around $z = 0.5$ and the value of deceleration parameter is nearly -0.2 for the present epoch. If the universe is approximated by a two-component cosmic fluid as in the present case, then q approaches $-\frac{1}{2}$ as $\Omega_d \to 1$. It is within the limits of recent observations [35].

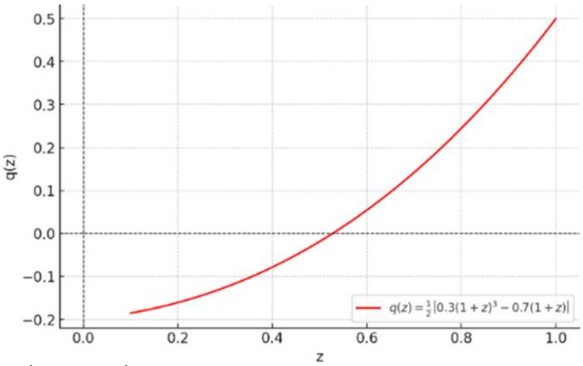

Fig. 1. $q(z)$ vs z for $(\Omega_{Mp}, \Omega_{dp}) = (0.3, 0.7)$.

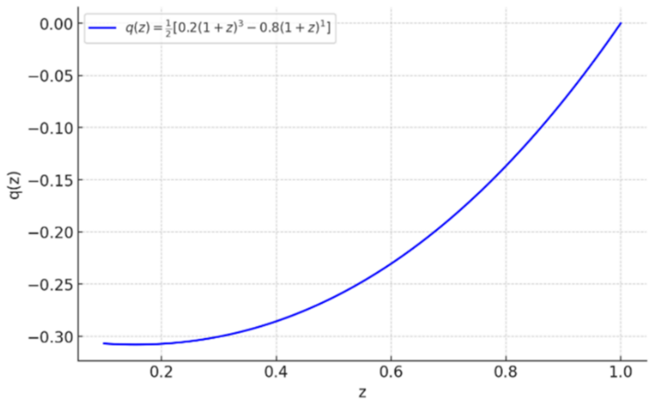

Fig. 2. $q(z)$ vs z for $(\Omega_{Mp}, \Omega_{dp}) = (0.2, 0.8)$.

A switch over from positive to negative value of q marks the transition point from a decelerating phase to an accelerating phase of cosmic expansion and varying q is the characteristic feature of a dynamic DE model as in the present case.

The apparent magnitude-redshift data of Type Ia supernovae call for modifications in the standard model energy densities. Under the circumstance that this modification cannot be limited to the addition of a mere cosmological constant, a serious situation has emerged in cosmology in which the energy densities in the universe have become largely speculative.

Homogeneous and isotropic cosmological models based on a time-varying cosmological term $\Lambda(t) \propto a^{-2}$, with $a(t)$ being the scale factor of the universe, do not suffer from the notorious problems of the standard hot big-bang cosmology such as the initial singularity, horizon, entropy, monopole, and cosmological constant problem. The coasting models, especially non-empty Milne cosmological models, also free from above problems of standard model. However, such models describe only uniform expansion rate ($q = 0$) throughout the history of the universe. These models lack consistency with recent observations of accelerated expansion. A model is presented in which a many-component homogeneous isotropic fluid represents universe. They are relativistic matter, non-relativistic matter and a time varying DE with total energy is conserved. Since their time evolution is different, dynamics of the universe in various epochs determined solely by respective components in the cosmic fluid. The model is free from major cosmological problems like cosmological constant problem, age problem etc. and is believed to be preferable than other DE models in the literature due to its novelty and simplicity.

The proposition of a time varying positive DE with $\rho_d = \rho_p \frac{a_p}{a}$ and a negative pressure can account for the recent accelerated expansion of the universe. This phenomenological model is also devoid of cosmological constant problem, because a decaying component as the missing energy is proposed. It has shown that universe enters into an accelerated expansion 4 to 5 billion years ago which corresponds to an epoch with redshift $z \approx 0.47$. The best fit value is obtained for the Hubble constant $H_0 = 67\ kms^{-1}Mpc^{-1}$ against $H_0 = 73\ kms^{-1}Mpc^{-1}$ (Hubble tension) in the present model as shown in the tables 1 and 2. The deceleration parameter $q_0 \sim -0.2$ is obtained for $(\Omega_{Mp}, \Omega_{dp}) = (0.3, 0.7)$ and can be fine-tuned with more accurate values of present cosmological parameters.

4. Conclusion

A phenomenological cosmological model is proposed in which the evolutionary history of the universe involves an exponential inflation from $10^{-36}s$ to $10^{-43}s$ after big bang, then a decelerated expansion upto 5 billion years. This may be followed by a coasting evolution of uniform rate and finally an accelerated expansion 5 billion years ago as supported by recent observations. If a bare cosmological constant is added to the total energy density, then the evolution of the universe finally enters to exponential nature (de Sitter universe with $q = -1$). The expression of deceleration parameter shows that the switch from the decelerating to accelerating phase can be readily obtained which reflects that the present model is physically sensible. In this report, one of the pioneering decaying DE models is studied and suggested an alternative scenario which is conceptually sounder. Though the resulting model faces some problems when concrete theoretical predictions, either on nucleosynthesis or on the relative abundance of the density parameters Ω_M and Ω_d are compared with observations, it has several positive features and raises certain fundamental issues which invite serious consideration. Although the model is phenomenological in nature, it gives very sensible conclusions about the universe and have the potential to become a precursor to a more concrete theory that might be developed in the future.

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