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Some Features of α–Regular Spaces in Supra Fuzzy Topology

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Abstract

We introduce and study supra fuzzy α–regular spaces and we establish some relationships among them in this paper. We also study some other properties of these concepts and obtain their several features.

Keywords: Fuzzy regular spaces; Supra fuzzy regular spaces.

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1. Introduction

The fundamental concept of a fuzzy set was introduced by Zadeh [1] in 1965 to provide a foundation for the development of many areas of knowledge. Chang [2] in 1968 and Lowen [3] in 1976 developed the theory of fuzzy topological spaces using fuzzy sets. In 1983, Mashhour [4] introduced supra topological spaces and studied s-continuous functions and s^*-continuous functions. In 1987, Abd EL-Monsef [5] introduced the fuzzy supra topological spaces and studied fuzzy supra continuous functions and characterized a number of basic concepts. A note on fuzzy regularity concepts was given by Ali [6] in 1990. In this paper, we study some features of regular spaces and obtain their certain characterizations in supra fuzzy topological spaces. As usual $I = [0, 1]$ and $I_1 = [0, 1)$.

Definition 1.1 [1]: For a set X, a function $u : X \to [0,1]$ is called a fuzzy set in X. For every $x \in X$, $u(x)$ represents the grade of membership of x in the fuzzy set u. Some authors say that u is a fuzzy subset of X. Thus a usual subset of X, is a special type of a fuzzy set in which the range of the function is $\{0, 1\}$.

Definition 1.2 [1]: Let X be a nonempty set and A be a subset of X. The function

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$$1_A : X \to [0, 1] \text{ defined by } 1_A(x) = \begin{cases} 1 & if \quad x \in A \\ 0 & if \quad x \notin A \end{cases}$$

is called the characteristic function of A. We also write 1_x for the characteristic function of $\{x\}$. The characteristic functions of subsets of a set X are referred to as the crisp sets in X.

Definition 1.3 [2]: Let X be a non empty set and t be the collection of fuzzy sets in I^X. Then t is called a fuzzy topology on X if it satisfies the following conditions:

(i) $1, 0 \in t$,
(ii) If $u_i \in t$ for each $i \in \Lambda$, then $\cup_{i \in \Lambda} u_i \in t$.
(iii) If $u_1, u_2 \in t$ then $u_1 \cap u_2 \in t$.

If t is a fuzzy topology on X, then the pair (X, t) is called a fuzzy topological space (fts, in short) and members of t are called t- open (or simply open) fuzzy sets. If u is open fuzzy set, then the fuzzy sets of the form $1- u$ are called t- closed (or simply closed) fuzzy sets.

Definition 1.4 [3]: Let X be a nonempty set and t be a collection of fuzzy sets in I^X such that

(i) $1, 0 \in t$,
(ii) If $u_i \in t$ for each $i \in \Lambda$, then $\cup_{i \in \Lambda} u_i \in t$.
(iii) If $u_1, u_2 \in t$ then $u_1 \cap u_2 \in t$.
(iv) all constant fuzzy sets in X belong to t.

Then t is called a fuzzy topology on X.

Definition 1.5 [4]: Let X be a nonempty set. A subfamily t^* of I^X is said to be a supra topology on X if and only if

(i) $1, 0 \in t^*$,
(ii) If $u_i \in t^*$ for each $i \in \Lambda$, then $\cup_{i \in \Lambda} u_i \in t^*$.

Then the pair (X, t^*) is called a supra fuzzy topological spaces. The elements of t^* are called supra open sets in (X, t^*) and complement of supra open set is called supra closed set.

Example 1.6 Let $X = \{x, y\}$ and $u, v \in I^X$ are defined by $u(x) = 0.8$, $u(y) = 0.6$ and $v(x) = 0.6$, $v(y) = 0.8$. Then we have $w(x) = (u \cup v)(x) = 0.8$, $w(y) = (u \cup v)(y) = 0.8$ and $k(x) = (u \cap v)(x) = 0.6$, $k(y) = (u \cap v)(y) = 0.6$. If we consider t^* on X generated by $\{0, u, v, w, 1\}$, then t^* is supra fuzzy topology on X but t^* is not fuzzy topology. Thus we see that every fuzzy topology is supra fuzzy topology but the converse is not always true.

Definition 1.7 [4]: Let (X, t) and (X, s) be two topological spaces. Let t^* and s^* are associated supra topologies with t and s respectively and $f : (X, t^*) \rightarrow (Y, s^*)$ be a function. Then the function f is a supra fuzzy continuous if the inverse image of each i.e., if for any $v \in s^*$, $f^{-1}(v) \in t^*$. The function f is called supra fuzzy homeomorphic if and only if f is supra bijective and both f and f^{-1} are supra fuzzy continuous.

Definition 1.8 [4]: Let (X, t^*) and (Y, s^*) be two supra topological spaces. If u_1 and u_2 are two supra fuzzy subsets of X and Y respectively, then the Cartesian product $u_1 \times u_2$ is a supra fuzzy subset of $X \times Y$ defined by $(u_1 \times u_2)(x, y) = \min [u_1(x), u_2(y)]$, for each pair $(x, y) \in X \times Y$.

Definition 1.9[10]: Suppose $\{X_i, i \in \Lambda\}$,be any collection of sets and X denoted the Cartesian product of these sets, i.e., $X = \Pi_{i\in\Lambda} X_i$. Here X consists of all points $p = < a_i, i \in \Lambda >$, where $a_i \in X_i$. For each $j_o \in \Lambda$, we define the projection $\pi_{jo} : X \rightarrow X_{jo}$ by $\pi_{jo}(< a_i: i\in\Lambda >)=a_{jo}$. These projections are used to define the product supra topology.

Definition 1.10 [10]: Let $\{X_\alpha\}_{\alpha \in \Lambda}$ be a family of nonempty sets. Let $X = \Pi_{\alpha \in\Lambda} X_\alpha$ be the usual product of X_α 's and let $\pi_\alpha : X \rightarrow X_\alpha$ be the projection. Further, assume that each X_α is a supra fuzzy topological space with supra fuzzy topology t^*_α. Now the supra fuzzy topology generated by $\{ \pi_\alpha^{-1}(b_\alpha) : b_\alpha \in t^*_\alpha, \alpha \in \Lambda \}$ as a sub basis, is called the product supra fuzzy topology on X. Thus if w is a basis element in the product, then there exist $\alpha_1, \alpha_2, \ldots. \alpha_n \in \Lambda$ such that $w(x) = \min \{ b_\alpha(x_\alpha) : \alpha = 1, 2, 3, \ldots\ldots n \}$, where $x = (x_\alpha)_{\alpha \in \Lambda} \in X$.

Definition 1.11 [5]: Let (X, T) be a topological space and T^* be associated supra topology with T. Then a function $f : X \rightarrow R$ is lower semi continuous if and only if $\{x \in X : f(x) > \alpha\}$ is open for all $\alpha \in R$. The lower semi continuous topology on X associated with T^* is $\omega(T^*) = \{\mu: X \rightarrow [0,1], \mu \textit{ is supra lsc}\}$. If $\omega(T^*) : (X, T^*) \rightarrow [0,1]$ be the set of all lower semi continuous (lsc) functions, then we can easily show that $\omega(T^*)$ is a supra fuzzy topology on X. Let P be the property of a supra topological space (X, T^*) and FP be its supra fuzzy topological analogue. Then FP is called a 'good extension' of P " if and only if the statement (X, T^*) has P if and only if $(X, \omega(T^*))$ has FP " holds good for every supra topological space (X, T^*).

2. α–Regular Spaces in Supra Fuzzy Topology

Definition 2.1: Let (X, t) be a fuzzy topological space and t^* be associated supra topology with t and $\alpha \in I_1$. Then

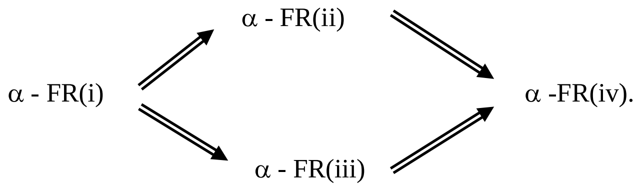
(a) (X, t^*) is an α - SFR (i) space if and only if for all $w \in (t^*)^c$ with $w(x) < 1$, $\forall x \in X$, there exist u, $v \in t^*$ such that $u(x) = 1$, $v(y) = 1$, $y \in w^{-1}\{1\}$ and $u \cap v \le \alpha$.

(b) (X, t^*) is an α -SFR(ii) space if and only if for all $w \in (t^*)^c$ with $w(x) < 1$, $\forall x \in X$, there exist u, $v \in t^*$ such that $u(x) > \alpha$, $v(y) = 1$, $y \in w^{-1}\{1\}$ and $u \cap v \le \alpha$.

(c) (X, t^*) is an α -SFR(iii) space if and only if for all $w \in (t^*)^c$ with $w(x) = 0, \forall x \in X$, there exist $u, v \in t^*$ such that $u(x) = 1$, $v(y) = 1$, $y \in w^{-1}\{1\}$ and $u \cap v \leq \alpha$.

(d) (X, t^*) is an α -SFR(iv) space if and only if for all $w \in (t^*)^c$ with $w(x) = 0, \forall x \in X$, there exist $u, v \in t^*$ such that $u(x) > \alpha$, $v(y) = 1$, $y \in w^{-1}\{1\}$, $u \cap v \leq \alpha$.

Theorem 2.2: Let (X, t) be a fuzzy topological space and t^* be associated supra topology with t. Then the following implications are true:
(a) (X, t^*) is α - SFR(i) implies (X, t^*) is α - SFR(ii) implies (X, t^*) is α - SFR(iv).
(b) (X, t^*) is α - SFR(i) implies (X, t^*) is α - SFR(iii) implies (X, t^*) is α - SFR(iv).
Also, these can be shown in a diagram as follows:



Proof. First, suppose that (X, t^*) is α - SFR (i). We have to prove that (X, t^*) is α - SFR(ii). Let $w \in (t^*)^c$ with $w(x)<1$, $x \in X$. Since (X, t^*) is α -SFR(i), for $\alpha \in I_1$, there exist $u, v \in t^*$ such that $u(x) = 1$, $v(y) = 1$, $y \in w^{-1}\{1\}$ and $u \cap v \leq \alpha$. Now, we see that $u(x) > \alpha$, $v(y) = 1$, $y \in w^{-1}\{1\}$ and $u \cap v \leq \alpha$. Hence by definition (X, t^*) is α -SFR (ii).
Next, Suppose that (X, t^*) is α - SFR (ii). Let $w \in (t^*)^c$ with $w(x)<1$, $x \in X$, then for $\alpha \in I_1$, there exist $u, v \in t^*$ such that $u(x)>1$, $v(y) = 1$, $y \in w^{-1}\{1\}$ and $u \cap v \leq \alpha$. Now, we see that $u(x)> \alpha$, $v(y) = 1$, $y \in w^{-1}\{1\}$ and $u \cap v \leq \alpha$. Hence by definition (X, t^*) is α -SFR (iv).
In the same way, we can prove that

(X, t^*) is α - SFR(i) $\Rightarrow$ (X, t^*) is α - SFR(iii) .
(X, t^*) is α - SFR(iii) $\Rightarrow$ (X, t^*) is α - SFR(iv) .

Now, we give some examples to show the non implication among α- SFR (i), α - SFR (ii), α- SFR (iii) and α - SFR (iv).

Example 2.3: Let $X = \{x , y \}$ and $u, v \in I^X$ are defined by $u(x) = 0.9$, $u(y) = 0$, $v(x) = 0.5$ and $v(y)=1$. Consider the supra fuzzy topology t^* on X generated by {*0*, *u*, *v*, *1*, constants}. Then for $w = 1 - u$ and $\alpha = 0.7$, we see that (X, t^*) is α -SFR(ii) but (X, t^*) is not α-SFR(i).

Example 2.4: Let X = $\{x, y\}$ and $u, v \in I^X$ are defined by $u(x)=0.2$, $u(y) = 0.3$, $v(x)= 0.3$, $v(y)=0.2$. Consider the supra fuzzy topology t^* on X generated by {*0*, *u*, *v*, *1*, Constants}. Then for $w = 1 - u$ and $\alpha=0.5$, we see that (X, t^*) is α - SFR(iii) and (X, t^*) is α -SFR(iv), but (X, t^*) is not α - SFR(ii) as they do not exist any $u, v \in t^*$ such that $u(x)>\alpha$, $v(y)= 1$, $y \in w^{-1}\{1\}$ and $u \cap v \leq \alpha$.

Example 2.5: Let $X = \{ x , y \}$ and $u , v , w \in I^X$ are defined by $u(x) = 0.9$, $u(y) = 0$, $v(x)=0.5$, $v(y)= 1$, $w(x)= 1$, $w(y)= 0$. Consider the supra fuzzy topology t^* on X generated by {0 , u , v, w, 1 , Constants}. Then for $\alpha = 0.6$ and $p = 1 - w$, it is seen that (X, t^*) is α - SFR(iv) but (X, t^*) is not α - SFR(iii).

This completes the proof.

Theorem 2.6: If $\alpha, \beta \in t^*$ with $0 \le \alpha \le \beta < 1$, then

(a) (X, t^*) is α -SFR(i) implies (X, t^*) is β - SFR(i) .
(b) (X, t^*) is α - SFR(iii) implies (X, t^*) is β - SFR(iii).

Proof. Suppose that (X, t^*) is α - SFR (i). We have to prove that (X, t^*) is β - SFR (i). Let $w \in (t^*)^c$ and $x \in X$ with $w(x) < 1$. Since (X, t^*) is α - SFR (i), for $\alpha \in I_1$, there exist $u, v \in t^*$ such that $u(x) = 1$, $v(y) = 1$, $y \in w^{-1}\{1\}$ and $u \cap v \le \alpha$. Since $\alpha \le \beta$, then $u \cap v \le \beta$. So it is observed that (X, t^*) is β - SFR (i).

Example 2.7: Let $X = \{ x , y \}$ and $u, v \in I^X$ are defined by $u(x) = 1$, $u(y) = 0$, $v(x) = 0.7$, $v(y)=1$. Consider the supra fuzzy topology t^* on X generated by {$0, u, v, 1$, constants }. Then for $w=1-u$, $\alpha=0.8$, $\beta=0.6$. We see that (X, t^*) is β-SFR (i) but (X, t^*) is not α-SFR(i). In the same way, we can prove that

(X, t^*) is α - SFR (iii) implies (X, t^*) is β - SFR (iii).

Theorem 2.8: Let (X, t^*) be a supra fuzzy topological space and $I_\alpha(t^*) = \{u^{-1}(\alpha, 1] : u \in t^*\}$, then (X, t^*) is 0 – SFR (i) implies $(X, I_0(t^*))$ is Regular.

Proof. Suppose (X, t^*) be a 0 –SFR (i). We have to prove that $(X, I_0(t^*))$ is Regular. Let V be a closed set in $I_0(t^*)$ and $x \in X$ such that $x \notin V$. Then $V^c \in I_0(t^*)$ and $x \in V^c$. So, by the definition of $I_0(t^*)$, there exists an $u \in t^*$ such that $V^c = u^{-1}(0, 1]$, i.e., $u(x) > 0$. Since $u \in t^*$, then u^c is closed supra fuzzy set in t^* and $u^c(x) < 1$. Since (X, t^*) is 0 –SFR(i), there exist $v, w \in t^*$ such that $v(x) = 1$, $w \ge 1_{(u^c)^{-1}\{1\}}$, $v \cap w = 0$.

(a) Since $v, w \in t^*$ then $v^{-1}(0, 1]$, $w^{-1}(0, 1] \in I_0(t^*)$ and $x \in v^{-1}(0, 1]$
(b) Since $w \ge 1_{(u^c)^{-1}\{1\}}$ then $w^{-1}(0, 1] \supseteq \left(1_{(u^c)^{-1}\{1\}}\right)^{-1}(0, 1]$.
(c) And $v \cap w = 0$, mean $(v \cap w)^{-1}(0, 1] = v^{-1}(0, 1] \cap w^{-1}(0, 1] = \phi$.

Now, we have

$$
\begin{aligned}
\left(1_{(u^c)^{-1}\{1\}}\right)^{-1}(0, 1] &= \{ x : 1_{(u^c)^{-1}\{1\}}(x) \in (0, 1] \} \\
&= \{ x : 1_{(u^c)^{-1}\{1\}}(x) = 1 \} \\
&= \{ x : x \in (u^c)^{-1}\{1\} \} \\
&= \{ x : u^c(x) = 1 \} \\
&= \{ x : u(x) = 0 \} \\
&= \{ x : x \notin V^c \} \\
&= \{ x : x \in V \} \\
&= V.
\end{aligned}
$$

Put $W = v^{-1}(0, 1]$ and $W^* = w^{-1}(0, 1]$, then $x \in W$, $W^* \supseteq V$ and $W \cap W^* = \phi$. Hence it is clear that $(X, I_0(t^*))$ is Regular.

Example 2.9: Let $X = \{x, y\}$, $u, v \in I^X$, where u, v are defined by $u(x)=0.8$, $u(y)=0$, $v(x)=0$ and $v(y)=1$, consider the supra fuzzy topological space t^* on X generated by $\{u, v\} \cup \{\text{constants}\}$. For $w = 1 - u$, we see that (X, t^*) is not 0-SFT(i). Now $I_0(t^*) = \{X, \phi, \{x\}, \{y\}\}$, here it is clear that $I_0(t^*)$ is a supra fuzzy topology on X and hence $(X, I_0(t^*))$ is a supra regular space.

Theorem 2.10: Let (X, t^*) be a supra fuzzy topological space $A \subseteq X$, and $t^*_A = \{u/A : u \in t^*\}$, then $1_{((u/A)^c)^{-1}\{1\}}(x) = (1_{(u^c)^{-1}\{1\}}/A)(x)$.

Proof. Let w be a closed supra fuzzy set in t^*_A, i.e., $w \in {t^*_A}^c$, then $u/A = w^c$, where $u \in t^*$.

Now, we have

$$1_{((u/A)^c)^{-1}\{1\}}(x) = \begin{cases} 0 & if \quad x \notin ((u/A)^c)^{-1}\{1\} \\ 1 & if \quad x \in ((u/A)^c)^{-1}\{1\} \end{cases}$$

$$= \begin{cases} 0 & if \quad x \notin \{y : (u/A)^c(y) = 1\} \\ 1 & if \quad x \in \{y : (u/A)^c(y) = 1\} \end{cases}$$

$$= \begin{cases} 0 & if \quad (u/A)^c(x) < 1 \\ 1 & if \quad (u/A)^c(x) = 1 \end{cases}$$

$$= \begin{cases} 0 & if \quad w(x) < 1 \\ 1 & if \quad w(x) = 1 \end{cases}$$

Again, $$1_{(u^c)^{-1}\{1\}}(x) = \begin{cases} 0 & if \quad x \notin (u^c)^{-1}\{1\} \\ 1 & if \quad x \in (u^c)^{-1}\{1\} \end{cases}$$

$$= \begin{cases} 0 & if \quad x \notin \{y : u^c(y) = 1\} \\ 1 & if \quad x \in \{y : u^c(y) = 1\} \end{cases}$$

$$= \begin{cases} 0 & if \quad u^c(x) < 1 \\ 1 & if \quad u^c(x) = 1 \end{cases}$$

Now, $$(1_{(u^c)^{-1}\{1\}}/A)(x) = \begin{cases} 0 & if \quad (u^c/A)(x) < 1 \\ 1 & if \quad (u^c/A)(x) = 1 \end{cases}$$

$$= \begin{cases} 0 & if \quad (u/A)^c(x) < 1 \\ 1 & if \quad (u/A)^c(x) = 1 \end{cases}$$

$$= \begin{cases} 0 & if \quad w(x) < 1 \\ 1 & if \quad w(x) = 1 \end{cases}$$

Therefore, we have $1_{((u/A)^c)^{-1}\{1\}}(x) = (1_{(u^c)^{-1}\{1\}} / A)(x)$.

Theorem 2.11: Let (X, t^*) be a supra fuzzy topological space and $A \subseteq X$ and $t^*_A = \{ u/A : u \in t^* \}$, then

(a) (X, t^*) is α -SFR (i) implies (A, t^*_A) is α - SFR (i).

(b) (X, t^*) is α -SFR (ii) implies (A, t^*_A) is α - SFR (ii).

(c) (X, t^*) is α -SFR (iii) implies (A, t^*_A) is α - SFR (iii).

(d) (X, t^*) is α -SFR(iv) implies (A, t^*_A) is α - SFR(iv) .

Proof. Let (X, t^*) be α-SFR(i). We have to prove that (A, t^*_A) is α-SFR(i). Let w be a closed fuzzy set in t^*_A, and $x^* \in A$ such that $w(x^*) < 1$. This implies that $w^c \in t^*_A$ and $w^c(x^*) > 0$. So there exists a $u \in t^*$ such that $u/A = w^c$ and clearly u^c is closed in t^* and $u^c(x^*) = (u/A)^c(x^*) = w(x^*) < 1$, i.e., $u^c(x^*) < 1$. Since (X, t^*) is α -SFR (i), for $\alpha \in I_1$, there exist $v, v^* \in t^*$ such that $v(x^*) = 1$, $v^* \geq 1_{(u^c)^{-1}\{1\}}$ and $v \cap v^* \leq \alpha$. Since $v, v^* \in t^*$, then v/A, $v^*/A \in t^*_A$ and $v/A(x^*) = 1$, $v^*/A \geq (1_{(u^c)^{-1}\{1\}}/A)$ and $v/A \cap v^*/A = (v \cap v^*)/A \leq \alpha$.

But $1_{(u^c)^{-1}\{1\}}/A = 1_{((u/A)^c)^{-1}\{1\}} = 1_{w^{-1}\{1\}}$, then $v^*/A \geq 1_{w^{-1}\{1\}}$. Hence it is clear that (A, t^*_A) is α - SFR (i).

The proofs of (b), (c) and (d) are similar.

Theorem 2.12: Let (X, T) be a topological space and T^* be associated supra topology with T. Consider the following statements:

1) (X, T^*) is a Regular space.
2) $(X, \omega(T^*))$ is α -SFR (i).
3) $(X, \omega(T^*))$ is α -SFR (ii).
4) $(X, \omega(T^*))$ is α -SFR (iii).
5) $(X, \omega(T^*))$ is α -SFR (iv).

Then the following statements are true:

(a) (1) implies (2) implies (3) implies (5) implies (1),

(b) (1) implies (2) implies (4) implies (5) implies (1).

Proof. First, suppose that (X, T^*) be regular space. We shall prove that $(X, \omega(T^*))$ is α -SFR (i). Let w be a closed supra fuzzy set in $\omega(T^*)$ and $x \in X$ such that $w(x) < 1$, then $w^c \in \omega(T^*)$ and $w^c(x) > 0$. Now we have $(w^c)^{-1}(0,1] \in T^*$, $x \in (w^c)^{-1}(0,1]$. Also it is clear that $[(w^c)^{-1}(0,1]]^c = w^{-1}\{1\}$ be a closed in T^* and $x \notin w^{-1}\{1\}$. Since (X, T^*) is Regular, then there exist $V, V^* \in T$ such that $x \in V$, $V^* \supseteq w^{-1}\{1\}$ and $V \cap V^* = \phi$. But by the definition of lower semi continuous functions $1_V, 1_{V^*} \in \omega(T^*)$ and $1_V(x) = 1$, $1_{V^*} \supseteq 1_{w^{-1}\{1\}}$, $1_V \cap 1_{V^*} = 1_{V \cap V^*} = 0$. Put $u = 1_V$ and $v = 1_{V^*}$, then, we have $u(x) = 1$, $v \supseteq 1_{w^{-1}\{1\}}$ and $u \cap v \le \alpha$. Hence $(X, \omega(T^*))$ is α - SFR (i).

We can easily show that (2) implies (3), (3) implies (5), (2) implies (4), (4) implies (5). We therefore prove that (5) implies (1).

Let $(X, \omega(T^*))$ be α - SFR (iv). Let $x \in X$ and V be a closed set in T^*, such that $x \notin V$. This implies that $V^c \in T^*$ and $x \in V^c$. But from the definition of $\omega(T^*)$, $1_{V^c} \in \omega(T^*)$, and $(1_{V^c})^c = 1_V$ closed in $\omega(T^*)$ and $1_V(x) = 0$. Since $(X, \omega(T^*))$ is α - SFR (iv), for $\alpha \in I_1$, there exist $u, v \in \omega(T^*)$ such that $u(x) > \alpha$, $v \ge 1_{(1_V)^{-1}\{1\}} = 1_V$ and $u \cap v \le \alpha$. Since $u, v \in \omega(T^*)$, then $u^{-1}(\alpha, 1]$, $v^{-1}(\alpha, 1] \in T^*$ and $x \in u^{-1}(\alpha, 1]$. Since $v \ge 1_V$, then $v^{-1}(\alpha, 1] \supseteq (1_V)^{-1}(\alpha, 1] = V$, and $u \cap v \le \alpha$ implies $(u \cap v)^{-1}(\alpha, 1] = u^{-1}(\alpha, 1] \cap v^{-1}(\alpha, 1] = \phi$. Then by definition, (X, T^*) is Regular space.

Thus it is seen that α - SFR (p) is a good extension of its topological counter part (p = i , ii , iii, iv).

Theorem 2.13: Let (X, t^*) and (Y, s^*) be two supra fuzzy topological spaces and $f : X \to Y$ be continuous , one-one , onto and open map, then

(a) (X, t^*) is α - SFR (i) implies (Y, s^*) is α - SFR (i).
(b) (X, t^*) is α - SFR (ii) implies (Y, s^*) is α - SFR (ii).
(c) (X, t^*) is α - SFR (iii) implies (Y, s^*) is α - SFR (iii).
(d) (X, t^*) is α - SFR (iv) implies (Y, s^*) is α - SFR (iv).

Proof. Suppose (X, t^*) be α - SFR (i). For $w \in (s^*)^c$ and $p \in Y$ such that $w(p) < 1$, $f^{-1}(w) \in (t^*)^c$ as f is continuous and $x \in X$ such that $f(x) = p$ as f is one-one and onto. Hence $f^{-1}(w)(x) = w(f(x)) = w(p) < 1$. Since (X, t^*) is α - SFR (i) , for $\alpha \in I_1$, then there exist $u, v \in t^*$ such that $u(x) = 1, v(y) = 1, y \in \{f^{-1}(w)\}^{-1}\{1\}$ and $u \cap v \le \alpha$. This implies that

$f(u)(p) = \{\text{Sup}\, u(x) \ : f(x) = p\} = 1$,

and $f(v) f(y) = \{\text{Sup}\, v(y)\} = 1$ as $f(f^{-1}(w)) \subseteq w \Rightarrow f(y) \in w^{-1}\{1\}$

and $f(u \cap v) \le \alpha$ as $u \cap v \le \alpha \Rightarrow f(u) \cap f(v) \le \alpha$.

Now, it is clear that for every $f(u)$, $f(v) \in s^*$ such that $f(u)(x) = 1$, $f(v)(f(y)) = 1$, $f(y) \in w^{-1}\{1\}$ and $f(u) \cap f(v) \le \alpha$. Hence (Y, s^*) is α - SFR (i).

Similarly (b), (c) and (d) can be proved.

Remark: Every homeomorphic image of α–regular space is also α–regular

Theorem 2.14: Let (X, t^*) and (Y, s^*) be two supra fuzzy topological spaces and $f : X \to Y$ be a continuous, one-one, onto and closed map then,

(a) (Y, s^*) is α - SFR (i) implies (X, t^*) is α - SFR (i).

(b) (Y, s^*) is α - SFR (ii) implies (X, t^*) is α - SFR (ii).

(c) (Y, s^*) is α - SFR (iii) implies (X, t^*) is α - SFR (iii).

(d) (Y, s^*) is α - SFR (iv) implies (X, t^*) is α - SFR (iv).

Proof. Suppose (Y, s^*) be α - SFR (i). For $w \in (t^*)^c$ and $x \in X$ with $w(x) < 1$, then $f(w) \in (s^*)^c$ as f is closed and we find $p \in Y$ such that $f(x) = p$ as f is one-one. Now we have $f(w)(p) = \{\text{Sup } w(x) \ : f(x) = p\} < 1$. Since (Y, s^*) is α - SFR (i), for $\alpha \in I_1$, then there exist $u, v \in s^*$ such that $u(f(x)) = 1$, $v(y) = 1$, $y \in (f(w))^{-1}\{1\}$ and $u \cap v \le \alpha$. This implies that $f^{-1}(u), f^{-1}(v) \in t^*$ as f is continuous and $u, v \in s^*$. Now $f^{-1}(u)(x) = u(f(x)) = u(p) = 1$ and $f^{-1}(v)(q) = v(f(q)) = v(y) = 1$ as $f(q) = y$, $y \in (f(w))^{-1}\{1\}$, i.e., $f(p) \in (f(w))^{-1}\{1\}$ $\Rightarrow q \in w^{-1}\{1\}$ and $f^{-1}(u) \cap f^{-1}(v) = f^{-1}(u \cap v) \le \alpha$ as $u \cap v \le \alpha$. Now we observe that there exist $f^{-1}(u), f^{-1}(v) \in t^*$ such that $f^{-1}(u)(x) = 1$, $f^{-1}(v)(q) = 1$, $q \in w^{-1}\{1\}$ and $f^{-1}(u) \cap f^{-1}(v) \le \alpha$. Hence (X, t^*) is α -SFR (i).

Similarly, (b), (c) and (d) can be proved.

Remark: Every inverse homeomorphic image of α–regular space is also α–regular.

Now we recall the following diagrams from refs. [11], [12] and [13], respectively:

(a)

(X, t) is α - T_0 (i)

(X, t) is α - T_0 (ii)

$\Rightarrow$ (X, t) is α - T_0 (iii) $\Rightarrow$ (X, t) is T_0 (iv) .

(b)

(X, t^*) is α -T_1(i).

(X, t^*) is α -T_1(ii)

$\Rightarrow$ (X, t^*) is α - T_1(iii) $\Rightarrow$ (X, t^*) is T_1(iv) .

(c)

(X, t) is α - T_2(i)

(X, t) is α - T_2(ii)

$\Rightarrow$ (X, t) is α - T_2(iii).

Theorem 2.15: The following are true:

(i) (X, t^*) is an α - SFR (iv) + α -T_0(i) $\Rightarrow$ α -T_2(i) $\Rightarrow$ α - T_1(i).

(ii) (X, t^*) is an α - SFR (iv) + α -T_0(ii) $\Rightarrow$ α -T_2(ii) $\Rightarrow$ α - T_1(ii).

Proof: The proof is easy.

However the arrows are in (i) and (ii) are not reversible

The following examples will serve the purpose.

Example 2.16: Let $X = \{x , y\}$ and u, v be fuzzy sets in X , where $u(x)$=1, $u(y)$ =0.5, $v(x)$ = 0.6, $v(y)$ =1. Consider the fuzzy supra topology t^* on X generated by$\{u, v\}\cup\{$Constants$\}$. For w = 1-u, it is clear that (X , t^*) is α-T_2(i) but it is not □- SFR (iv).

Similarly the non reverse civility of (ii) can be shown.
The proof is now complete.

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