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Affirmative Fixed Point Results for Four Weakly Compatible Self-maps in a Complete S_b-metric Space

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Abstract

In this article, an attempt is made to establish certain fixed point theorems for four pairwise self-maps that are weakly compatible in a complete S_b-metric space. Four self-maps in a complete S_b-metric space are considered, and a contractive condition is applied together with the weak compatibility of pairs of mappings. Some pre-existing results that hold in ordinary metric spaces are examined and validated in the setting of a complete S_b-metric space. By applying a contractive condition along with the weak compatibility of pairs of mappings, the existence of a fixed point is established, and its uniqueness is subsequently proved. Two fixed point theorems corresponding to two different contractive conditions are proved. In each case, a corollary is obtained by restricting the results to two self-maps.

Keywords: Coincidence point; Common fixed point; S_b-Metric space; Weak compatibility.

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1. Introduction

The fixed point theorem in an n-dimensional Euclidean space was first developed by Brouwer. Subsequently, existence theorems in the theory of differential equations were established in 1922 by Birkhoff and Kellogg using Brouwer's fixed point theorem. Later, Brouwer's theorem was extended by Schauder to the case where E is a compact convex subset of a normed space. The well-known fixed point theorem for contraction mappings was subsequently established by Banach; its proof is relatively straightforward and does not require extensive topological knowledge. Thereafter, fixed point results were extended by several researchers to *b-*, *S-*, and S_b-metric spaces.

The concept of an S_b-metric space was due to Souayah *et al.* [1] and Radenović *et al.* [2]. It was obtained by combining the notions of *S*-metric space [3] and *b*-metric space [4,5]. Following this development, various fixed point theorems in S_b-metric spaces were established by several authors [6–11].

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Inspired by the work done by several authors, in this paper, the existence of a fixed point is established and its uniqueness is then demonstrated by using a contractive condition in addition with the weak compatibility of pairs of mappings. We illustrate two fixed point theorems that correspond to two different contractive conditions. Limiting the results to two self-maps in each case yields a corollary. Our findings generalize and extend certain previous findings in the literature.

The definition and certain topological properties of S_b-metric spaces are now presented.

Definition 1.1. [2] Let X be a non-empty set and $b \geq 1$ be a real number. An S_b-metric on X is a function S: $X^3 \to [0,\infty)$ that satisfy the following conditions, for each $u, v, w, a \in X$,

(S_b1) $S(u,v,w) = 0$ if and only if $u = v = w$,

(S_b2) $S(u,v,w) \leq b[S(u,u,a) + S(v,v,a) + S(w,w,a)]$.

In this case, the pair *(X, S)* is called an S_b-metric space.

Since every *S*-metric is an S_b-metric with $b = 1$, it is clear that S_b-metric spaces are the generalisations of *S*-metric spaces.

Example 1.2. [2] Let $X = R$ and let the function $S_b: X^3 \to [0,\infty)$ be defined as $S_b(u,v,w) = \{|u-w| + |v-w|\}^2$ is an S_b-metric on X with $b = 4$.

Definition 1.3. [2] Let *(X, S)* be an S_b-metric space. A sequence $\{u_n\}$ in X

a. is said to converge to some $u \in X$, if to each $\varepsilon > 0$there exists a number $n_0 \in N$such that $S(u_n, u_n, u) < \in$ whenever $n > n_0$, that is, $S(u_n, u_n, u) \to 0$ as $n \to \infty$.

Here, we write $u_n \to u$as $n \to \infty$ or $\lim_{n\to\infty} u_n = u$.

b. is said to be a Cauchy sequence, if to each $\in > 0$ there exists a number $n_0 \in N$ such that $S(u_n, u_n, u_m) < \in$ whenever $m, n > n_0$, that is, $S(u_n, u_n, u_m) \to 0$ as $m, n \to \infty$.

The S_b-metric space X is said to be complete if every Cauchy sequence is convergent in *X.*

Lemma 1.4. [2] Let *(X,S)* is an S_b-metric space. Then for every $u, v, w \in X$,

(1) $S(u,u,v) \leq bS(v,v,u)$

(2) $S(u,u,v) \leq 2bS(u,u,w) + bS(v,v,w) \leq 2bS(u,u,w) + b^2 S(w,w,v)$.

Jungck [12] first proposed the idea of compatibility in 1986 as a generalisation of commutative property. Later, the concept of weak compatibility of mappings was put forward by Jungck and Rhoades [13]. Additionally, it was demonstrated that two mappings that are compatible are always weakly compatible, but the converse does not hold.

Definition 1.5. Let *(X, S)* be an S_b-metric space and *F, G* be two self-maps of *X.* Then the pair *(F, G)* is

(i) said to be compatible [12] if $\lim_{n\to\infty} S(FGu_n, FGu_n, GFu_n) = 0$ for every sequence $\{u_n\}$ in X such that $\lim_{n\to\infty} Fu_n = \lim_{n\to\infty} Gu_n = t \in X$

(ii) said to be weakly compatible [13] if $FGu = GFu$ for every $u \in X$ such that $Fu = Gu$.

Definition 1.6. Let X be a non-empty set and *F, G* be two self-maps of *X.* Then a point $u \in X$ is said to be a coincidence point of *F* and *G,* if $Fu = Gu$. We denote $C(F,G) = \{u \in X: Fu = Gu\}$

Some fixed point theorems for four pairwise weakly compatible self-maps in a complete S_b-metric space are established in this study.

2. Main Results

Theorem 2.1. Let *(X,S)* be a complete *S*$_b$-metric space. Suppose that the mappings $F,G,A,B\ :X\to X$ satisfy the condition

$S(Fu,Fu,Gv)\le\lambda S(Au,Au,Bv)$, where $\lambda\in\left(0,\frac{1}{b^4}\right)$ is a constant. (2.1)

If $F(X)\subseteq B(X)$ and $G(X)\subseteq A(X)$, and $A(X),B(X)$ are closed subspaces of *X,* then $C(A,F)$and $C(B,G)$ are nonempty. Moreover, if the pairs *(F,A)* and *(G,B*) are weakly compatible, then *F, G, A* and *B* have a unique common fixed point.

Proof. Suppose that u_0 be an arbitrary point of *X.* Since $F(X)\subseteq B(X)$, there exists $u_1\in X$ such that $Fu_0=Bu_1$.

Since $G(X)\subseteq A(X)$, there must be some $u_2\in X$ such that $Gu_1=Au_2$.

Proceeding like this, a sequence $\{v_n\}$ in *X* is obtained such that

$$\left.\begin{array}{l} v_{2n}=Fu_{2n}=Bu_{2n+1,}\\ v_{2n+1}=Gu_{2n+1}=Au_{2n+2}\end{array}\right\}\text{for all } n=0,1,2,\ldots$$

From equation (2.1), It follows that

$$\begin{aligned} S(v_{2n+1},v_{2n+1},v_{2n+2}) &= S(Gu_{2n+1},Gu_{2n+1,}Fu_{2n+2})\\ &\le bS(Fu_{2n+2},Fu_{2n+2},Gu_{2n+1})\\ &\le b\lambda S(Au_{2n+2},Au_{2n+2},Bu_{2n+1})\\ &= b\lambda S(v_{2n+1},v_{2n+1},v_{2n})\\ &\le b^2\lambda S(v_{2n},v_{2n},v_{2n+1}). \qquad (2.2)\end{aligned}$$

Also, $S(v_{2n+2},v_{2n+2},v_{2n+3})=S(Fu_{2n+2},Fu_{2n+2},Gu_{2n+3})$

$$\begin{aligned} &\le \lambda S(Au_{2n+2},Au_{2n+2},Bu_{2n+3})\\ &= \lambda S(v_{2n+1},v_{2n+1},v_{2n+2})\\ &\le b^2\lambda S(v_{2n+1},v_{2n+1},v_{2n+2}),\ \text{Since } b^2\ge 1 \qquad (2.3)\end{aligned}$$

From (2.2) and (2.3),

$$S(v_{n+1},v_{n+1},v_{n+2})\le b^2\lambda S(v_n,v_n,v_{n+1}) \text{ for } n=0,1,2\ldots..$$

Therefore, for every $n=0,1,2\ldots$,

$$\begin{aligned} S(v_{n+1},v_{n+1},v_{n+2}) &\le b^2\lambda S(v_n,v_n,v_{n+1})\\ &\le b^4\lambda^2 S(v_{n-1},v_{n-1},v_n)\\ &\text{----------------------------------}\\ &\text{----------------------------------}\\ &\le b^{2n+2}\lambda^{n+1}S(v_0,v_0,v_1)\ldots\ldots. \qquad (2.4)\end{aligned}$$

Now, for $m>n\ge 1$,

$$\begin{aligned} S(v_n,v_n,v_m) &\le 2bS(v_n,v_n,v_{n+1})+b^2S(v_{n+1},v_{n+1},v_m)\\ &\le 2bS(v_n,v_n,v_{n+1})+b^2\{2bS(v_{n+1},v_{n+1},v_{n+2})+b^2S(v_{n+2},v_{n+2},v_m)\}\\ &= 2bS(v_n,v_n,v_{n+1})+2b^3S(v_{n+1},v_{n+1},v_{n+2})+b^4S(v_{n+2},v_{n+2},v_m)\\ &\le 2bS(v_n,v_n,v_{n+1})+2b^3S(v_{n+1},v_{n+1},v_{n+2})+\\ &\quad 2b^5S(v_{n+2},v_{n+2},v_{n+3})+\ldots\ldots\\ &\quad \ldots..+b^{2(m-n-1)}S(v_{m-1},v_{m-1},v_m).\\ &\le 2bb^{2n}\lambda^nS(v_0,v_0,v_1)+2b^3b^{2(n+1)}\lambda^{n+1}S(v_0,v_0,v_1)+\\ &\quad 2b^5b^{2(n+2)}\lambda^{n+2}S(v_0,v_0,v_1)+\ldots\ldots.+b^{2(m-n-1)}.b^{2(m-1)}\lambda^{m-1}S(v_0,v_0,v_1)\\ &\le 2b^{2n+1}\lambda^n\{1+b^4\lambda+b^8\lambda^2+\ldots\ldots.\}S(v_0,v_0,v_1)\end{aligned}$$

$$= \frac{2b(b^2\lambda)^n}{1-b^4\lambda} S(v_0, v_0, v_1) \to 0 \text{ as } n \to \infty, \text{ since } b^2\lambda \le b^4\lambda < 1.$$

Thus $S(v_n, v_n, v_m) \to 0$ as $n, m \to \infty$.

This proves $\{v_n\}$ is a Cauchy sequence.

Since X is complete, there exists $q \in X$ such that $v_n \to q$.

Therefore $v_{2n} = Bu_{2n+1} \to q$ and $v_{2n+1} = Au_{2n+2} \to q$.

Since *A(X)* and *B(X)* are closed.

$q = Ar = Bs$ for some $r, s \in X$.

Now, It will be shown that $q = Ar = Fr = Bs = Gs$.

$$\begin{aligned} S(Ar, Ar, Fr) &\le 2bS(Ar, Ar, Au_{2n+2}) + bS(Fr, Fr, Au_{2n+2}) \\ &= 2bS(q, q, Au_{2n+2}) + bS(Fr, Fr, Gu_{2n+1}) \\ &\le 2bS(q, q, Au_{2n+2}) + b\lambda S(Ar, Ar, Bu_{2n+1}) \\ &= 2bS(q, q, Au_{2n+2}) + b\lambda S(q, q, Bu_{2n+1}) \to 0 \text{ as } n \to \infty \end{aligned}$$

This implies $Ar = Fr$. (2.5)

$$\begin{aligned} S(Bs, Bs, Gs) &\le 2bS(Bs, Bs, Bu_{2n+1}) + bS(Gs, Gs, Bu_{2n+1}) \\ &\le 2bS(q, q, Bu_{2n+1}) + b^2S(Bu_{2n+1}, Bu_{2n+1}, Gs) \\ &= 2bS(q, q, Bu_{2n+1}) + b^2S(Fu_{2n}, Fu_{2n}, Gs) \\ &\le 2bS(q, q, Bu_{2n+1}) + b^2\lambda S(Au_{2n}, Au_{2n}, Bs) \\ &= 2bS(q, q, Bu_{2n+1}) + b^2\lambda S(Au_{2n}, Au_{2n}, q) \to 0 \text{ as } n \to \infty. \end{aligned}$$

Hence $Bs = Gs$. (2.6)

Then $q = Ar = Fr = Bs = Gs$ from (2.5) and (2.6)

From the weak compatibility of the pairs *(F,A)* and *(G,B)*, it follows that

$$FAr = AFr \text{ and } GBs = BGs.$$

Then $Fq = Aq$ and $Gq = Bq$ (2.7)

Now from (2.1), $S(Fq, Fq, q) = S(Fq, Fq, Gs)$

$$\le \lambda S(Aq, Aq, Bs)$$
$$= \lambda S(Fq, Fq, q).$$

Since $\lambda < \frac{1}{b^4} \le 1$, there is a contradiction if $Fq \ne q$

Therefore, it must be that $Fq = q$ (2.8)

Also $S(q, q, Gq) = S(Fr, Fr, Gq)$

$$\le \lambda S(Ar, Ar, Bq)$$
$$= \lambda S(q, q, Gq)$$

Since $\lambda < 1$, this is a contradiction if $Gq \ne q$

Therefore $Gq = q$. (2.9)

From (2.7), (2.8) and (2.9), $q = Fq = Aq = Gq = Bq$.

To prove the uniqueness of q, let q^* be another common fixed point of *A,B,F,G*

Then $q^* = Fq^* = Aq^* = Gq^* = Bq^*$.

From (2.1), $S(q, q, q^*) = S(Fq, Fq, Gq^*)$

$$\le \lambda S(Aq, Aq, Bq^*)$$
$$= \lambda S(q, q, q^*).$$

Since $\lambda < \frac{1}{b^4} \le 1$, this is a contradiction if $q \ne q^*$.

Therefore $q = q^*$.

Corollary 2.2. Let (X, S) be a complete S_b −metric space. Suppose that the mappings

$G, A: X \to X$ satisfy the condition $S(Gu, Gu, Gv) \leq \lambda S(Au, Au, Av)$ where $\lambda \in \left(0, \frac{1}{b^4}\right)$ is a constant. If $G(X) \subseteq A(X)$, and $A(X)$ is closed, then $C(G, A) \neq \phi$.
Moreover, if the pair *(G,A)* is weakly compatible, then the maps *G* and *A* have unique common fixed point.
Proof. Proof is obtained by taking *F* = *G* and *B* = *A* in Theorem 2.1.

Theorem 2.3. Let *(X,S)* be a complete *S*$_b$-metric space. Suppose that the mappings *F, G, A* and *B: X*⟶*X* satisfy the condition

$$S(Fu, Fu, Gv) \leq \lambda[S(Fu, Fu, Au) + S(Gv, Gv, Bv)], \tag{2.10}$$

where $\lambda \in \left(0, \frac{1}{b^2(b^2+1)}\right)$ is a constant.
If $F(X) \subseteq B(X)$ and $G(X) \subseteq A(X)$, and $A(X), B(X)$ are closed subspaces of *X*, then $C(A, F)$ and $C(B, G)$ are nonempty.
Moreover, if the pairs *(F,A)* and *(G,B)* are weakly compatible, then *F, G, A* and *B* have a unique common fixed point.
Proof. Suppose $u_0 \in X$ be an arbitrary point of *X*
Since $F(X) \subseteq B(X)$, there exists $u_1 \in X$ such that $Fu_0 = Bu_1$
Since $G(X) \subseteq A(X)$, there exists $u_2 \in X$ such that $Gu_1 = Au_2$
Continuing this process, there exists a sequence $\{v_n\}$ in *X* such that

$$\left.\begin{aligned} v_{2n} &= Fu_{2n} = Bu_{2n+1} \\ v_{2n+1} &= Gu_{2n+1} = Au_{2n+2} \end{aligned}\right\}, n = 0,1,2.\ldots.$$

From equation (2.10), it follows that

$$\begin{aligned} S(v_{2n+1}, v_{2n+1}, v_{2n+2}) &= S(Gu_{2n+1}, Gu_{2n+1}, Fu_{2n+2}) \\ &\leq bS(Fu_{2n+2}, Fu_{2n+2}, Gu_{2n+1}) \\ &\leq b\lambda[S(Fu_{2n+2}, Fu_{2n+2}, Au_{2n+2}) + S(Gu_{2n+1}, Gu_{2n+1}, Bu_{2n+1})] \\ &= b\lambda[S(v_{2n+2}, v_{2n+2}, v_{2n+1}) + S(v_{2n+1}, v_{2n+1}, v_{2n})] \\ &\leq b^2\lambda S(v_{2n+1}, v_{2n+1}, v_{2n+2}) + b^2\lambda S(v_{2n}, v_{2n}, v_{2n+1}) \end{aligned}$$

$$S(v_{2n+1}, v_{2n+1}, v_{2n+2}) \leq \frac{b^2\lambda}{1-b^2\lambda} S(v_{2n}, v_{2n}, v_{2n+1}). \tag{2.11}$$

Also,

$$\begin{aligned} S(v_{2n+2}, v_{2n+2}, v_{2n+3}) &= S(Fu_{2n+2}, Fu_{2n+2}, Gu_{2n+3}) \\ &\leq \lambda[S(Fu_{2n+2}, Fu_{2n+2}, Au_{2n+2}) + S(Gu_{2n+3}, Gu_{2n+3}, Bu_{2n+3})] \\ &= \lambda[S(v_{2n+2}, v_{2n+2}, v_{2n+1}) + S(v_{2n+3}, v_{2n+3}, v_{2n+2})] \\ &\leq \lambda bS(v_{2n+1}, v_{2n+1}, v_{2n+2}) + \lambda bS(v_{2n+2}, v_{2n+2}, v_{2n+3}) \end{aligned}$$

$$S(v_{2n+2}, v_{2n+2}, v_{2n+3}) \leq \frac{\lambda b}{1-\lambda b} S(v_{2n+1}, v_{2n+1}, v_{2n+2}) \tag{2.12}$$

From (2.11) and (2.12),
$S(v_{n+1}, v_{n+1}, v_{n+2}) \leq \mu S(v_n, v_n, v_{n+1})$ for $n = 0,1,2,\ldots.$,
where $\mu = max\left\{\frac{b\lambda}{1-b\lambda}, \frac{b^2\lambda}{1-b^2\lambda}\right\} = \frac{b^2\lambda}{1-b^2\lambda} < \frac{1}{b^2} \leq 1$, since $0 < \lambda < \frac{1}{b^2(b^2+1)}$.
Therefore for $n = 0,1,2,\ldots..$

$$\begin{aligned} S(v_{n+1}, v_{n+1}, v_{n+2}) &\leq \mu S(v_n, v_n, v_{n+1}) \\ &\leq \mu^2 S(v_{n-1}, v_{n-1}, v_n) \\ &\leq \mu^3 S(v_{n-2}, v_{n-2}, v_{n-1}) \\ &\text{-------------} \\ &\text{-------------} \\ &\leq \mu^{n+1} S(v_0, v_0, v_1). \end{aligned} \tag{2.13}$$

Now, for $m > n \geq 1$,

$$S(v_n, v_n, v_m) \leq 2bS(v_n, v_n, v_{n+1}) + b^2 S(v_{n+1}, v_{n+1}, v_m)$$

$$\leq 2bS(v_n, v_n, v_{n+1}) + b^2\{2bS(v_{n+1}, v_{n+1}, v_{n+2}) + b^2S(v_{n+2}, v_{n+2}, v_m)\}$$
$$= 2bS(v_n, v_n, v_{n+1}) + 2b^3S(v_{n+1}, v_{n+1}, v_{n+2}) + b^4S(v_{n+2}, v_{n+2}, v_m)\}$$
$$-----------------$$

$$-------------------$$
$$\leq 2bS(v_n, v_n, v_{n+1}) + 2b^3S(v_{n+1}, v_{n+1}, v_{n+2}) + 2b^5S(v_{n+2}, v_{n+2}, v_{n+3}) + ----+b^{2(m-n-1)}S(v_{m-1}, v_{m-1}, v_m).$$
$$\leq 2b\mu^nS(v_0, v_0, v_1) + 2b^3\mu^{n+1}S(v_0, v_0, v_1) + 2b^5\mu^{n+2}S(v_0, v_0, v_1) + ----+b^{2(m-n-1)}\mu^{m-1}S(v_0, v_0, v_1)$$
$$\leq 2b\mu^n\{1 + b^2\mu + b^4\mu^2 + ...\}S(v_0, v_0, v_1)$$
$$= \frac{2b\mu^n}{1-b^2\mu}S(v_0, v_0, v_1) \to 0 \text{ as } m, n \to \infty, \text{ since } b^2\mu < 1.$$

Hence $\{v_n\}$ is a Cauchy sequence in *X*.

Since *X* is complete, there exists some $q \in X$ such that $v_n \to q$ as $n \to \infty$.

Therefore $v_{2n} = Bu_{2n+1} \to q$ and $v_{2n+1} = Au_{2n+2} \to q$

Since *A(X)* and *B(X)* are closed $q = Ar = Bs$ for same $r, s \in X$ (2.14)

It shall be shown that $q = Ar = Fr = Bs = Gs$.

$$S(Ar, Ar, Fr) \leq 2bS(Ar, Ar, Au_{2n+2}) + bS(Fr, Fr, Au_{2n+2})$$
$$= 2bS(q, q, Au_{2n+2}) + bS(Fr, Fr, Gu_{2n+1})$$
$$\leq 2bS(q, q, Au_{2n+2}) + b\lambda[S(Fr, Fr, Ar) + S(Gu_{2n+1}, Gu_{2n+1}, Bu_{2n+1})$$
$$= 2bS(q, q, v_{2n+1}) + b\lambda S(Fr, Fr, Ar) + b\lambda S(v_{2n+1}, v_{2n+1}, v_{2n})$$
$$(1 - b\lambda)S(Ar, Ar, Fr) \leq 2bS(q, q, v_{2n+1}) + b\lambda S(v_{2n+1}, v_{2n+1}, v_{2n}) \to 0 \text{ as } n \to \infty$$

Hence $S(Ar, Ar, Fr) = 0$, Since $b\lambda < 1$.

This implies $Ar = Fr$. (2.15)

Similarly, It can be shown that $Bs = Gs$. (2.16)

From (2.14), (2.15) and (2.16), $q = Ar = Fr = Bs = Gs$.

Since the pairs *(F,A)* and *(G,B)* are weakly compatible

$$AFr = FAr \text{ and } BGs = GBs.$$

This implies $Aq = Fq$ and $Bq = Gq$.

From (2.10), $S(Fq, Fq, q) = S(Fq, Fq, Gs)$
$$\leq \lambda[S(Fq, Fq, Aq) + S(Gs, Gs, Bs)]$$
$$= \lambda S(Fq, Fq, Aq) + \lambda S(q, q, q)$$
$$= 0.$$

Hence $Fq = q$. (2.17)

From (2.10), $S(q, q, Gq) = S(Fu, Fu, Gq)$
$$\leq \lambda[S(Ar, Ar, Fr) + S(Gq, Gq, Bq)]$$
$$= 0, \text{since } Ar = Fr \text{ and } Bq = Gq.$$

Hence $Gq = q$. (2.18)

From (2.17) and (2.18),

$q = Fq = Aq = Bq = Gq$ showing that q is a common fixed point of *A, B, F* and *G*.

To prove unique of q, let $q*$ be another common fixed point of *A, B, F* and *G*.

Therefore $q^* = Fq^* = Aq^* = Bq^* = Gq^*$.

From (2.10), $S(q, q, q^*) = S(Fq, Fq, Gq^*)$
$$\leq \lambda[S(Fq, Fq, Aq) + S(Gq^*, Gq^*, Bq^*)$$
$$= 0, \text{ since } Aq = Fq \text{ and } Bq^* = Gq^*.$$

This implies $q = q^*$.

Corollary 2.4. Let *(X,S)* be a complete S_b-metric space. Suppose that the mappings *G, A: X* $\rightarrow X$ satisfy the condition

$S(Gu, Gu, Gv) \leq \lambda[S(Gu, Gu, Au) + S(Gv, Gv, Av)]$,

where $\lambda \in \left(0, \frac{1}{b^2(b^2+1)}\right)$ is a constant. If $G(X) \subseteq A(X)$ and $A(X)$ is closed subspace of *X*, then $C(A, G)$ is nonempty.

Moreover, if the pair *(G,A)* is weakly compatible, then *G* and *A* have a unique common fixed point.

Proof. Proof is obtained by taking *F* = *G* and *B* = *A* in Theorem 2.3.

4. Conclusion

Common fixed point theorems for four weakly compatible self-maps in a complete S_b-metric space under suitable contractive conditions are established in this investigation. Without assuming continuity or commutativity of the mappings, the existence and uniqueness of a common fixed point are demonstrated. Corresponding results are presented together with corollaries where the result is limited to two self-maps, taking into account two distinct contractive conditions. These results add to the on-going advancement of fixed point theory in extended metric settings by extending a number of well-known fixed point theorems from metric and related generalized metric spaces to the broader framework of S_b-metric spaces.

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