Journal of Statistical Research *https://doi.org/10.3329/jsr.v58i1.75424*
2024, Vol. 58, No. 1, pp. 181-196 *ISSN 0256 - 422 X*

DIAGNOSTICS FOR ONE-WAY RANDOM EFFECTS PANEL DATA MODELS

SIBNARAYAN GURIA

Department of Statistics, West Bengal State University, Barasat, India.
Email: sng_65@yahoo.co.in

SUGATA SEN ROY*

Department of Statistics, University of Calcutta, Calcutta, India.
Email: ssrstat@caluniv.ac.in

SUMMARY

In this paper we consider a random effects panel data model with one-way error components and derive methods for identifying outliers using the deletion technique. These outliers may be a single unit, a single time-point or a particular time-point in a particular unit. Expressions for the diagnostics for each of these three cases are derived and the technique illustrated through an example.

Keywords and phrases: Panel data, random effects, outlier detection, deletion technique.

AMS Classification: 62J20

1 Introduction

In most economic or biological studies data are not obtained either spatially or in the form of a time-series. Data more generally are in the form of a panel taken on a set of units (households, individuals, firms or countries) over several equidistant time-points. Some examples are the incomes of a cross-section of households observed over a number of years or the conditions of a group of patients studied over a number of days. The fundamental advantage of a panel or longitudinal data set over cross-sectional data is that it allows greater flexibility in modeling differences in behaviour across units. This is natural because the heterogeneity among individuals or units or that over time cannot be captured through either a cross-sectional or a time-series model.

Various models have been suggested over the years to study the effects of a set of regressors on the response obtained in the form of panels. The study and analysis related to these models have been the subject of one of the most active and innovative bodies of literature in both econometrics and biostatistics, partly because panel data provide a rich environment for the development of sophisticated inferential techniques. Comprehensive discussions on such models can be found in Diggle et.al. (1994), Baltagi (2008) and Greene (2005).

However, very few studies have been conducted on detection of outliers in panel data. Outliers often exert an inordinate influence on the estimates of the parameters and distorts the inferences and predictions based on

* Corresponding author

these estimates. It is thus necessary to detect these outliers and, if found present, take appropriate remedial measures so as to obtain a good fit. Although outlier detection studies have a long history in regression analysis, these studies have not been greatly extended to panel data models.

In regression models two techniques are generally employed to detect outliers - the deletion technique and the perturbation technique. The deletion technique, initially based on independent errors, was extensively studied by, among others, Belsley et. al. (1980), Cook (1986) and Chatterjee and Hadi (1988). Later the results were extended by Haslett and Hayes (1998), Haslett (1999) and Sen Roy and Guria (2004) to models with correlated errors as proposed by Beach and MacKinnon (1978). Zhu et. al. (2012) looked at the problem caused by different degrees of perturbation arising from the deletion of different numbers of observation and proposed a new measure to take account of it.

The perturbation technique too has mostly been applied to models with spherical error structures. Extensions to models with correlated error structures were made by Schall and Dunne (1991), Tsai and Wu (1992) and Kim amd Huggins (1998). Here, to check whether a particular observation is an outlier, a weight is attached to it, the other observations being unweighted. The differences in the estimators and fitted values obtained with and without the weight are then observed to identify whether the particular observation is an outlier or not. An alternative way is often to look at the impact of the weight on the log-likelihood function as observed through the likelihood displacement, the plot of which gives the influence graph. However, in this paper we only seek to extend the deletion diagnostics to panel data models.

In the early diagnostics studies of longitudinal models Banerjee and Frees (1997) used the partial influence technique to take account of the effects of subject-specific parameters and to measure the effect of a subject on the population parameters. Banerjee (1998) also noted that the effectiveness of Cook's distance as an influence measure in longitudinal data was limited. Ouwens et. al. (1999) demonstrated the necessity to use observation-oriented influence measures in addition to subject-oriented influence measures and showed that subject-oriented measures may fail to detect influential subjects, owing to the relative position of the observations within and across subjects. Tan et. al. (2001) proposed an alternative version of the Cook's distance by conditioning on the subjects in the sample. Yang and Chang (2006), considered a longitudinal model with mixed effect and taking cognisance of the high within subject dependence, proposed multiple quantitative indices and plots to check for outliers.

In the context of panel data, outliers can be of three different types - an unit may be an outlier, all observations at a particular time point may be outliers, or only a single observation corresponding to a particular unit at a particular time point may be an outlier. For example, the U.S. Gross Investment data which we use in Section 3 to illustrate our results, consist of observations over 20 years across 10 firms. Here a single firm can be an outlier because its performance is different from the remaining nine, or a particular year may be an outlier because there is an upturn or downturn in the economy, or a specific year of a particular firm may be an outlier, because of short-term changes in its internal structure or policies. Thus the diagnostics would relate to the effect produced on the regression coefficients or the predictors by the deletion respectively of either one unit or one time point or one single observation. In the first case, deletion generally leaves the structure of the dispersion matrix unaffected. However, this is not true in the latter two cases and the problem becomes more complex.

In Section 2 we describe our model and the diagnostic tools while Section 3 gives the main results. Section 4 indicates the changes in the results under an alternative error structure. A numerical example is given in Section 5 to illustrate our technique and some concluding remarks are made in Section 6.

2 The Model and the Problem

Suppose n individuals are observed over T time points. Let y_{it} and $\mathbf{x}_{it}$ be respectively the response and the p-dimensional vector of explanatory variables on the i^{th} individual at time t. We then write the model as

$$y_{it} = \mathbf{x}'_{it}\boldsymbol{\beta} + u_{it}, \quad \text{with} \tag{2.1}$$

$$u_{it} = \mu_i + \nu_{it}, \tag{2.2}$$

where $\boldsymbol{\beta}$ is the $p \times 1$ parameter vector. In (2.2) we have taken the usual one-way error component model, where μ_i is the effect due to the i^{th} individual and ν_{it} the random error. We assume that the μ_i'a are i.i.d. $N(0, \sigma^2_\mu)$ independently of the ν_{it}'s which are also i.i.d. $N(0, \sigma^2_\nu)$.

Let $\mathbf{y}_i = (y_{i1}, \ldots, y_{iT})'$, $\mathbf{X}_i = (\mathbf{x}_{i1}, \ldots, \mathbf{x}_{iT})'$ and $\mathbf{u}_i = (u_{i1}, \ldots, u_{iT})'$. Then writing $\mathbf{y} = (\mathbf{y}'_1, \ldots, \mathbf{y}'_n)'$, $\mathbf{X} = (\mathbf{X}'_1, \ldots, \mathbf{X}'_n)'$ and $\mathbf{u} = (\mathbf{u}'_1, \ldots, \mathbf{u}'_n)'$, (2.1) can be written as

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{u}, \tag{2.3}$$

where **y** and **u** are $nT \times 1$ vectors and **X** is a $nT \times p$ matrix. Since

$$cov(u_{it}, u_{js}) = \begin{cases} \sigma^2_\mu + \sigma^2_\nu & \text{for} \quad i = j, t = s; \\ \sigma^2_\mu & \text{for} \quad i = j, t \neq s; \\ 0 & \text{otherwise,} \end{cases}$$

the dispersion of **u** is given by $\Omega = \mathbf{I}_n \otimes \mathbf{V}$, where $\mathbf{I}_n$ is the identity matrix of order n and $\mathbf{V}$ is a $T \times T$ matrix of the form

$$\mathbf{V} = \sigma^2 \begin{pmatrix} 1 & \tau & \tau & \cdots & \tau \\ \tau & 1 & \tau & \cdots & \tau \\ & \vdots & \vdots & \ddots & v \\ \tau & \tau & \tau & \cdots & 1 \end{pmatrix},$$

with $\sigma^2 = (\sigma^2_\mu + \sigma^2_\nu)$ and $\tau = \sigma^2_\mu/(\sigma^2_\mu + \sigma^2_\nu)$. Using the generalized least-squares, the best linear unbiased estimator of $\boldsymbol{\beta}$ is given by

$$\begin{aligned} \hat{\boldsymbol{\beta}} &= (\mathbf{X}'\Omega^{-1}\mathbf{X})^{-1}\mathbf{X}'\Omega^{-1}\mathbf{y} \\ &= \Big(\sum_{i=1}^{n} \mathbf{X}'_i\mathbf{V}^{-1}\mathbf{X}_i\Big)^{-1} \sum_{i=1}^{n} \mathbf{X}'_i\mathbf{V}^{-1}\mathbf{y}_i, \end{aligned} \tag{2.4}$$

with the residual vector defined as $\mathbf{e} = \mathbf{y} - \mathbf{X}\hat{\boldsymbol{\beta}} = (\mathbf{e}'_1, \ldots, \mathbf{e}'_n)'$, where $\mathbf{e}_i = \mathbf{y}_i - \mathbf{X}_i\hat{\boldsymbol{\beta}}$ is the residual vector corresponding to the i^{th} individual $i = 1, \ldots, n$.

Let $\mathbf{H} = \mathbf{X}(\mathbf{X}'\Omega^{-1}\mathbf{X})^{-1}\mathbf{X}'$ with $\mathbf{H}_{ii} = \mathbf{X}_i(\mathbf{X}'\Omega^{-1}\mathbf{X})^{-1}\mathbf{X}'_i$, the block diagonal corresponding to the i^{th} individual. The diagonal elements of $\mathbf{H}$ give the leverages.

In practice, Equation (2.4) leads to a reasonably accurate estimate of $\boldsymbol{\beta}$ provided of course there are no outliers in the data. An intial idea of any such outlier is provided by the residuals or the leverages. Generally, the residuals identify outliers in the response while the leverages indicate outliers in the independent variable. However, to get a more comprehensive view of outliers, and particularly to identify influential observations, we need to look further.

3 The Main Results

Suppose for any m, the m observations $(y_{i_1t_1}, \mathbf{x}'_{i_1t_1}), \ldots, (y_{i_mt_m}, \mathbf{x}'_{i_mt_m})$ are deleted from the data set. Let $\mathbf{K} = \{(i_j, t_j) : 1 \le i_j \le n, 1 \le t_j \le T, j = 1 \ldots, m\}$ i.e. $\mathbf{K}$ is the set of indices corresponding to the deleted observations. Then let $(\mathbf{y}_{(\mathbf{K})}, \mathbf{X}_{(\mathbf{K})})$ be the $(nT - m)$ observations remaining after deleting the m observations from $(\mathbf{y}, \mathbf{X})$. The estimator of $\boldsymbol{\beta}$ based on these $(nT - m)$ observations is

$$\hat{\boldsymbol{\beta}}_{(\mathbf{K})} = (\mathbf{X}'_{(\mathbf{K})}\Omega^{-1}_{(\mathbf{K})}\mathbf{X}_{(\mathbf{K})})^{-1}\mathbf{X}'_{(\mathbf{K})}\Omega^{-1}_{(\mathbf{K})}\mathbf{y}_{(\mathbf{K})}, \tag{3.1}$$

with the corresponding residuals defined as $\mathbf{e}_{(\mathbf{K})} = \mathbf{y}_{(\mathbf{K})} - \mathbf{X}_{(\mathbf{K})}\hat{\boldsymbol{\beta}}_{(\mathbf{K})} = (\mathbf{e}'_{1(\mathbf{K})}, \ldots, \mathbf{e}'_{n(\mathbf{K})})'$.

The deletion technique for identification of influential observations then considers the difference in the estimators $\hat{\boldsymbol{\beta}}$ and $\hat{\boldsymbol{\beta}}_{(\mathbf{K})}$ given by $DFBETA_{\mathbf{K}} = \hat{\boldsymbol{\beta}} - \hat{\boldsymbol{\beta}}_{(\mathbf{K})}$. However, it is often more convenient to look at either the difference in the predicted values of y_K, $DFFIT_{\mathbf{K}} = \mathbf{X}_K(\hat{\boldsymbol{\beta}} - \hat{\boldsymbol{\beta}}_{(\mathbf{K})})$ or the Cook's distance, $CD_{\mathbf{K}} = (\hat{\boldsymbol{\beta}} - \hat{\boldsymbol{\beta}}_{(\mathbf{K})})'\mathbf{X}'\Omega^{-1}\mathbf{X}(\hat{\boldsymbol{\beta}} - \hat{\boldsymbol{\beta}}_{(\mathbf{K})})/p$.

As discussed earlier, in looking for outliers in panel data, three distinct cases may arise. The diagnostics in each of these cases would be different and hence each needs to be tried out while searching for outliers.

Case 1: A particular unit is an outlier

Very often a unit itself is an outlier. Hence all its observations over time would be different from those of the other units. To identify such an outlier, we need to delete the unit as a whole and observe its effect on the regression parameters. Deleting the j^{th} unit i.e. $\mathbf{K} = \{(j, 1), (j, 2), \ldots, (j, T)\}$, we have the following proposition

Proposition 3.1. For the j^{th} unit deleted, $j = 1, \ldots, n$,

$$\begin{aligned} DFBETA_j &= (\mathbf{X}'\Omega^{-1}\mathbf{X})^{-1}\mathbf{X}'_j[\mathbf{V} - \mathbf{H}_{jj}]^{-1}\mathbf{e}_j \\ DFFIT_j &= \mathbf{H}_{jj}[\mathbf{V} - \mathbf{H}_{jj}]^{-1}\mathbf{e}_j \\ CD_j &= p^{-1}\mathbf{e}'_j[\mathbf{V} - \mathbf{H}_{jj}]^{-1}\mathbf{H}_{jj}[\mathbf{V} - \mathbf{H}_{jj}]^{-1}\mathbf{e}_j. \end{aligned}$$

Proof. For any $j = 1, \dots, n$,

$$
\begin{aligned}
\hat{\beta}_j &= \Big(\sum_{i=1 \neq j}^{n} \mathbf{X}_i' \mathbf{V}^{-1} \mathbf{X}_i \Big)^{-1} \Big(\sum_{i=1 \neq j}^{n} \mathbf{X}_i' \mathbf{V}^{-1} \mathbf{y}_i \Big) \\
&= \Big(\sum_{i=1}^{n} \mathbf{X}_i' \mathbf{V}^{-1} \mathbf{X}_i - \mathbf{X}_j' \mathbf{V}^{-1} \mathbf{X}_j \Big)^{-1} \Big(\sum_{i=1}^{n} \mathbf{X}_i' \mathbf{V}^{-1} \mathbf{y}_i - \mathbf{X}_j' \mathbf{V}^{-1} \mathbf{y}_j \Big) \\
&= \Bigg[\Big(\sum_{i=1}^{n} \mathbf{X}_i' \mathbf{V}^{-1} \mathbf{X}_i \Big)^{-1} + \Big(\sum_{i=1}^{n} \mathbf{X}_i' \mathbf{V}^{-1} \mathbf{X}_i \Big)^{-1} \mathbf{X}_j' \Big\{ \mathbf{V} - \mathbf{X}_j \Big(\sum_{i=1}^{n} \mathbf{X}_i' \mathbf{V}^{-1} \mathbf{X}_i \Big)^{-1} \mathbf{X}_j' \Big\}^{-1} \\
&\qquad \mathbf{X}_j \Big(\sum_{i=1}^{n} \mathbf{X}_i' \mathbf{V}^{-1} \mathbf{X}_i \Big)^{-1} \Bigg] \Bigg[\sum_{i=1}^{n} \mathbf{X}_i' \mathbf{V}^{-1} \mathbf{y}_i - \mathbf{X}_j' \mathbf{V}^{-1} \mathbf{y}_j \Bigg] \\
&= \hat{\beta} + \Big(\sum_{i=1}^{n} \mathbf{X}_i' \mathbf{V}^{-1} \mathbf{X}_i \Big)^{-1} \mathbf{X}_j' \Big\{ \mathbf{V} - \mathbf{X}_j \Big(\sum_{i=1}^{n} \mathbf{X}_i' \mathbf{V}^{-1} \mathbf{X}_i \Big)^{-1} \mathbf{X}_j' \Big\}^{-1} \mathbf{X}_j \hat{\beta} \\
&\qquad - \Big(\sum_{i=1}^{n} \mathbf{X}_i' \mathbf{V}^{-1} \mathbf{X}_i \Big)^{-1} \mathbf{X}_j' \Big\{ \mathbf{V} - \mathbf{X}_j \Big(\sum_{i=1}^{n} \mathbf{X}_i' \mathbf{V}^{-1} \mathbf{X}_i \Big)^{-1} \mathbf{X}_j' \Big\}^{-1} \\
&\qquad \Bigg[\Big\{ \mathbf{V} - \mathbf{X}_j \Big(\sum_{i=1}^{n} \mathbf{X}_i' \mathbf{V}^{-1} \mathbf{X}_i \Big)^{-1} \mathbf{X}_j' \Big\} \mathbf{V}^{-1} \mathbf{y}_j + \mathbf{X}_j \Big(\sum_{i=1}^{n} \mathbf{X}_i' \mathbf{V}^{-1} \mathbf{X}_i \Big)^{-1} \mathbf{X}_j' \mathbf{V}^{-1} \mathbf{y}_j \Bigg] \\
&= \hat{\beta} - \Big(\sum_{i=1}^{n} \mathbf{X}_i' \mathbf{V}^{-1} \mathbf{X}_i \Big)^{-1} \mathbf{X}_j' \Bigg[\mathbf{V} - \mathbf{X}_j \Big(\sum_{i=1}^{n} \mathbf{X}_i' \mathbf{V}^{-1} \mathbf{X}_i \Big)^{-1} \mathbf{X}_j' \Bigg]^{-1} \Big(\mathbf{y}_j - \mathbf{X}_j \hat{\beta} \Big) \qquad (3.2)
\end{aligned}
$$

from which the expression of $DFBETA_j$ follows. $DFFIT_j$ is obtained by pre-multiplying (3.2) by X_j, while the use of the $DFBETA_j$ expression gives CD_j. □

Case 2: A particular time point is deleted

Sometimes a single time point may be an outlier. In such a case all the units will show an unusual value at this time-point. To detect such outliers we need to delete the time point from all the units and observe the effect on the regression coefiicients. If the n observations corresponding to the time point s are deleted i.e. if $\mathbf{K} = \{(1, s), (2, s), \dots, (n, s)\}$, then along with the s^{th} row in each of $(\mathbf{y}_i, \mathbf{X}_i), i = 1, \dots, n$, the s^{th} row and column of $\mathbf{V}$ will also need to be deleted. To observe the effect of this, let $a = \tau / \big(1 + (T-1)\tau\big)$ and define $\mathbf{a}_s$ as a T-component vector with s^{th} component $1 - a$ and all other components a. Also let $\mathbf{c}_s = (0, \dots, 1, \dots, 0)'$ be the s^{th} unit vector. Then with $\mathbf{A}_s = \mathbf{I}_n \otimes \mathbf{a}_s'$ and $\theta = \sigma^2(1-\tau)(1-a)$, we have the following proposition.

Proposition 3.2. If the s^{th} time point is deleted, $s = 2, \dots, T$,

$$
\begin{aligned}
DFBETA_s &= (\mathbf{X}'\Omega^{-1}\mathbf{X})^{-1}\mathbf{X}'\mathbf{A}_s'[\theta\mathbf{I}_n - \mathbf{A}_s\mathbf{H}\mathbf{A}_s']^{-1}\mathbf{A}_s\mathbf{e} \\
DFFIT_s &= (I_n \otimes \mathbf{c}_s')\mathbf{H}\mathbf{A}_s'[\theta\mathbf{I}_n - \mathbf{A}_s\mathbf{H}\mathbf{A}_s']^{-1}\mathbf{A}_s\mathbf{e} \\
CD_s &= p^{-1}\mathbf{e}'\mathbf{A}_s'[\theta\mathbf{I}_n - \mathbf{A}_s\mathbf{H}\mathbf{A}_s']^{-1}\mathbf{A}_s\mathbf{H}\mathbf{A}_s'[\theta\mathbf{I}_n - \mathbf{A}_s\mathbf{H}\mathbf{A}_s']^{-1}\mathbf{A}_s\mathbf{e}.
\end{aligned}
$$

Proof. For observations corresponding to i^{th} unit, the deletion of the s^{th} time point leads to

$$\begin{aligned}
\mathbf{X}_i'\mathbf{V}^{-1}\mathbf{X}_i &= \frac{1}{\sigma^2(1-\tau)}\{\mathbf{X}_i'\mathbf{X}_i - \frac{\tau}{1+(T-1)\tau}\mathbf{X}_i'\mathbf{1}\mathbf{1}'\mathbf{X}_i\} \\
&= \frac{1}{\sigma^2(1-\tau)}\{\mathbf{X}_{i(s)}'\mathbf{X}_{i(s)} - \frac{\tau}{1+(T-1)\tau}\mathbf{X}_{i(s)}'\mathbf{1}_s\mathbf{1}_s'\mathbf{X}_{i(s)}\} \\
&+ \frac{1}{\sigma^2(1-\tau)}\{\mathbf{x}_{is}\mathbf{x}_{is}' - \frac{\tau}{1+(T-1)\tau}(\mathbf{X}_{i(s)}'\mathbf{1}_s\mathbf{x}_{is}' + \mathbf{x}_{is}\mathbf{1}_s'\mathbf{X}_{i(s)} + \mathbf{x}_s\mathbf{x}_s')\} \\
&= \frac{1}{\sigma^2(1-\tau)}\{\mathbf{X}_{i(s)}'\mathbf{X}_{i(s)} - \frac{\tau}{1+(T-2)\tau}\mathbf{X}_{i(s)}'\mathbf{1}_s\mathbf{1}_s'\mathbf{X}_{i(s)}\} \\
&+ \frac{1}{\sigma^2(1-\tau)}\{\frac{\tau^2}{(1+(T-2)\tau)(1+(T-1)\tau)}\mathbf{X}_{i(s)}'\mathbf{1}_s\mathbf{1}_s'\mathbf{X}_{i(s)} \\
&- \frac{\tau}{1+(T-1)\tau}(\mathbf{X}_{i(s)}'\mathbf{1}_s\mathbf{x}_{is}' + \mathbf{x}_{is}\mathbf{1}_s'\mathbf{X}_{i(s)}) + \frac{1+(T-2)\tau}{1+(T-1)\tau}\mathbf{x}_{is}\mathbf{x}_{is}'\} \\
&= \mathbf{X}_{i(s)}'\mathbf{V}_s^{-1}\mathbf{X}_{i(s)} \\
&+ \frac{1}{\sigma^2(1-\tau)}\{(\frac{1+(T-1)\tau}{1+(T-2)\tau}\mathbf{x}_{is} - \frac{T\tau}{1+(T-2)\tau}\bar{\mathbf{X}}_i)(\mathbf{x}_{is} - \frac{T\tau}{1+(T-1)\tau}\bar{\mathbf{X}}_i)'\}
\end{aligned}$$

Define $\mathbf{w}_{is} = \mathbf{x}_{is} - Ta\bar{\mathbf{X}}_i = \mathbf{a}_s'\mathbf{X}_i$ and $z_{is} = (y_{is} - Ta\bar{y}_i) = \mathbf{a}_s'\mathbf{y}_i$. Then

$$\mathbf{X}_{i(s)}'\mathbf{V}_s^{-1}\mathbf{X}_{i(s)} = \mathbf{X}_i'\mathbf{V}^{-1}\mathbf{X}_i - \frac{1}{\theta}\mathbf{w}_{is}\mathbf{w}_{is}'.$$

Similarly

$$\mathbf{X}_{i(s)}'\mathbf{V}_s^{-1}\mathbf{y}_{i(s)} = \mathbf{X}_i'\mathbf{V}^{-1}\mathbf{y}_i - \frac{1}{\theta}\mathbf{w}_{is}z_{is}.$$

Next considering the s^{th} time points in all the n units and defining $\mathbf{z}_s = (z_{1s}, z_{2s}, \ldots, z_{ns})' = \mathbf{A}_s\mathbf{y}$, $\mathbf{W}_s = (\mathbf{w}_{1s}, \mathbf{w}_{2s}, \ldots, \mathbf{w}_{ns})' = \mathbf{A}_s\mathbf{X}$ and $\mathbf{e}_s = (e_{1s}, e_{2s}, \ldots, e_{ns})' = \mathbf{A}_s\mathbf{e}$, where

$$\begin{aligned}
e_{is} &= y_{is} - \mathbf{x}_{is}'\hat{\boldsymbol{\beta}} \\
\hat{\boldsymbol{\beta}}_s &= \hat{\boldsymbol{\beta}} - (\sum_{i=1}^{n}\mathbf{X}_i'\mathbf{V}^{-1}\mathbf{X}_i)^{-1}\mathbf{W}_s(\theta\mathbf{I}_n - \mathbf{W}_s'(\sum_{i=1}^{n}\mathbf{X}_i'\mathbf{V}^{-1}\mathbf{X}_i)^{-1}\mathbf{W}_s)^{-1}\mathbf{e}_s,
\end{aligned}$$

from which the results follow. □

Case 3: A particular time-point in a particular unit is deleted

In many situations a single observation in a particular unit may be unusual. To observe the impact of this particular observation, say the s^{th} observation of unit j, on the fitted model, we take $\mathbf{K} = \{(j, s)\}$. This requires deleting the s^{th} row of $(\mathbf{y}_j, \mathbf{X}_j)$ along with the deletion of s^{th} row and column of $\mathbf{V}$ for the j^{th} unit only. The resultant effect is given in the following proposition.

Proposition 3.3. For any $j = 1, \ldots, n$ and $s = 1, \ldots, T$,

$$\begin{aligned} DFBETA_{js} &= (\mathbf{X}'\Omega^{-1}\mathbf{X})^{-1}\mathbf{X}_j\mathbf{a}_s\mathbf{a}_s'\mathbf{e}_j/[\theta - \mathbf{a}_s'\mathbf{H}_{jj}\mathbf{a}_s] \\ DFFIT_{js} &= \mathbf{c}_s'\mathbf{H}_{jj}\mathbf{a}_s\mathbf{a}_s'\mathbf{e}_j/[\theta - \mathbf{a}_s'\mathbf{H}_{jj}\mathbf{a}_s] \\ CD_{js} &= p^{-1}[\theta - \mathbf{a}_s'\mathbf{H}_{jj}\mathbf{a}_s]^{-2}\mathbf{e}_j'\mathbf{a}_s\mathbf{a}_s'\mathbf{H}_{jj}\mathbf{a}_s\mathbf{a}_s'\mathbf{e}_j. \end{aligned}$$

Proof. For the j^{th} unit, deleting the s^{th} observation gives

$$\begin{aligned} \mathbf{X}_{j(s)}'\mathbf{V}_s^{-1}\mathbf{X}_{j(s)} &= \mathbf{X}_j'\mathbf{V}^{-1}\mathbf{X}_j - \frac{1}{\theta}\mathbf{w}_{js}\mathbf{w}_{js}', \\ \mathbf{X}_{j(s)}'\mathbf{V}_s^{-1}\mathbf{y}_{j(s)} &= \mathbf{X}_j'\mathbf{V}^{-1}\mathbf{y}_j - \frac{1}{\theta}\mathbf{w}_{js}z_{js}, \end{aligned}$$

with no change in the remaining units. Thus

$$\begin{aligned} \hat{\beta}_{js} &= (\sum_{i=1\neq j}^{n}\mathbf{X}_i'\mathbf{V}^{-1}\mathbf{X}_i + \mathbf{X}_{j(s)}'\mathbf{V}_s^{-1}\mathbf{X}_{j(s)})^{-1}(\sum_{i=1\neq j}^{n}\mathbf{X}_i'\mathbf{V}^{-1}\mathbf{y}_i + \mathbf{X}_{j(s)}'\mathbf{V}_s^{-1}\mathbf{y}_{j(s)}) \\ &= (\sum_{i=1}^{n}\mathbf{X}_i'\mathbf{V}^{-1}\mathbf{X}_i - \frac{1}{\theta}\mathbf{w}_{js}\mathbf{w}_{js}')^{-1}(\sum_{i=1}^{n}\mathbf{X}_i'\mathbf{V}^{-1}\mathbf{y}_i - \frac{1}{\theta}\mathbf{w}_{js}z_{js}) \end{aligned}$$

from which the results follow. □

Generally for comparisons the standardized versions of DFBETA and DFFIT are used. For simplicity these are very often normed by $Var(\hat{\boldsymbol{\beta}})$ and $Var(\mathbf{X}\hat{\boldsymbol{\beta}})$ respectively. However, it's always better to use instead $Var(DFBETA_{\mathbf{K}})$ and $Var(DFFIT_{\mathbf{K}})$. The following proposition gives these expressions for the three cases.

Proposition 3.4. (i) For the j^{th} unit deleted,

$$\begin{aligned} Var(DFBETA_j) &= (\mathbf{X}'\Omega^{-1}\mathbf{X})^{-1}\mathbf{X}_j'(\mathbf{V} - \mathbf{H}_{jj})^{-1}\mathbf{X}_j(\mathbf{X}'\Omega^{-1}\mathbf{X})^{-1} \\ Var(DFFIT_j) &= \mathbf{H}_{jj}(\mathbf{V} - \mathbf{H}_{jj})^{-1}\mathbf{H}_{jj} \end{aligned}$$

(ii) For the s^{th} time-point deleted,

$$\begin{aligned} Var(DFBETA_{\mathbf{K}}) &= (\sum_{i=1}^{n}\mathbf{X}_i'\mathbf{V}^{-1}\mathbf{X}_i)^{-1}\mathbf{X}'\mathbf{A}_s'(\theta\mathbf{I}_n - \mathbf{A}_s\mathbf{H}\mathbf{A}_s'))^{-1}\mathbf{A}_s\mathbf{X}(\sum_{i=1}^{n}\mathbf{X}_i'\mathbf{V}^{-1}\mathbf{X}_i)^{-1} \\ Var(DFFIT_{\mathbf{K}}) &= \mathbf{A}_s'\mathbf{H}\mathbf{A}_s'(\theta\mathbf{I}_n - \mathbf{A}_s\mathbf{H}\mathbf{A}_s')^{-1}\mathbf{A}_s\mathbf{H}\mathbf{A}_s' \end{aligned}$$

(iii) For the s^{th} time point of j^{th} unit deleted,

$$\begin{aligned} Var(DFBETA_{is}) &= (\mathbf{X}'\Omega^{-1}\mathbf{X})^{-1}\mathbf{X}_j'\mathbf{a}_s'(\theta - \mathbf{a}_s\mathbf{H}_{jj}\mathbf{a}_s')^{-1}\mathbf{a}_s\mathbf{X}_j(\mathbf{X}'\Omega^{-1}\mathbf{X})^{-1} \\ Var(DFFIT_{js}) &= \mathbf{a}_s'\mathbf{H}_{jj}\mathbf{a}_s(\theta - \mathbf{a}_s\mathbf{H}_{jj}\mathbf{a}_s')^{-1}\mathbf{a}_s'\mathbf{H}_{jj}\mathbf{a}_s \end{aligned}$$

Proof. Consider any set of m deleted observations as defined by $\mathbf{K}$.

$$
\begin{aligned}
Var(DFBETA_{\mathbf{K}}) &= Var(\hat{\beta}) + Var(\hat{\beta}_{\mathbf{K}}) - 2Cov(\hat{\beta}, \hat{\beta}_{\mathbf{K}}) \\
&= (\mathbf{X}'\Omega^{-1}\mathbf{X})^{-1} + (\mathbf{X}'_{(\mathbf{K})}\Omega^{-1}_{(\mathbf{K})}\mathbf{X}_{(\mathbf{K})})^{-1} \\
&\quad -2(\mathbf{X}'\Omega^{-1}\mathbf{X})^{-1}\mathbf{X}'\Omega^{-1}Cov(\mathbf{y}, \mathbf{y}'_{(\mathbf{K})})\Omega^{-1}_{(\mathbf{K})}\mathbf{X}_{(\mathbf{K})}(\mathbf{X}'_{(\mathbf{K})}\Omega^{-1}_{(\mathbf{K})}\mathbf{X}_{(\mathbf{K})})^{-1} \qquad (3.3)
\end{aligned}
$$

Let $\Omega^*_{\mathbf{K}}$ denote the $nT \times (nT - m)$ matrix obtained from Ω by deleting the m columns corresponding to $\mathbf{K}$ from Ω. Then $Cov(\mathbf{y}, \mathbf{y}'_{\mathbf{K}}) = \Omega^*_{\mathbf{K}}$. Writing $\mathbf{E_K}$ as a $nT \times (nT - m)$ matrix obtained from the nT^{th} order identity matrix by deleting the m columns corresponding to $\mathbf{K}$, $\mathbf{X}'\Omega^{-1}\Omega^*_{\mathbf{K}} = \mathbf{X}'\mathbf{E_K} = \mathbf{X}'_{(\mathbf{K})}$. Substituting this in (3.3) simplifies the 3^{rd} term on the right-hand-side to $2(\mathbf{X}'\Omega^{-1}\mathbf{X})^{-1}$. Hence
$Var(DFBETA_{\mathbf{K}}) = (\mathbf{X}'_{(\mathbf{K})}\Omega^{-1}_{(\mathbf{K})}\mathbf{X}_{(\mathbf{K})})^{-1} - (\mathbf{X}'\Omega^{-1}\mathbf{X})^{-1}$.
The results can then be obtained as special choices of $\mathbf{K}$. Pre- and post-multiplying with the corresponding $\mathbf{x}_{it}$'s give the respective variances of $DFFIT$s. ∇

Remark : Propositions 3.1 to 3.4, involve the unknown variances σ^2_μ and σ^2_ν, which thus need to be estimated and substituted before the $DFBETA$s can be computed. Averaging over the T observations on the i^{th} individual, model (2.1) can be written as

$$y_{i0} = \mathbf{x}'_{i0}\boldsymbol{\beta} + u_{i0}, \qquad (3.4)$$

where the suffix 0 indicates the average over the corresponding index. Following Baltagi (2008) and defining $\sigma^{*2} = T\sigma^2_\mu + \sigma^2_\nu$, the estimators can then be obtained as

$$
\begin{aligned}
\hat{\sigma}^{*2} &= \frac{T}{n}\sum_{i=1}^{n} u^2_{i0} \\
\text{and} \quad \hat{\sigma}^2_\nu &= \frac{1}{n(T-1)}\sum_{i=1}^{n}\sum_{t=1}^{T}(u_{it} - u_{i0})^2,
\end{aligned}
$$

with $\hat{\sigma}^2_\mu = T^{-1}(\hat{\sigma}^{*2} - \hat{\sigma}^2_\nu)$.

4 Model with AR(1) error

An alternative error structure that is often used instead of (2.2) in panel data modelling is the autoregressive errors of order one or AR(1),

$$u_{it} = \rho u_{i(t-1)} + \epsilon_{it}, \qquad (4.1)$$

where the ϵ_{it}'s are uncorrelated with $E(\epsilon_{it}) = 0$ and $E(\epsilon_{it}^2) = \sigma^2 \quad \forall \quad i$ and t and $|\rho| < 1$.

The dispersion of Ω is then of the form $\Omega = \mathbf{I}_n \otimes \mathbf{M}$, where

$$M = \frac{\sigma^2}{(1-\rho^2)} \begin{pmatrix} 1 & \rho & \rho^2 & \cdots & \rho^{T-1} \\ \rho & 1 & \rho & \cdots & \rho^{T-2} \\ & & \vdots & \ddots & \vdots \\ \rho^{T-1} & \rho^{T-2} & \rho^{T-3} & \cdots & 1 \end{pmatrix}.$$

Proposition 3.1 remains unchanged for this error structure except for V being replaced by M. As for the other results, let $\mathbf{a}_s^* = (1+\rho^2)^{-1/2}(\rho \mathbf{p}_{s+1} - \mathbf{p}_s)$, where $\mathbf{p}_s'$ is the k^{th} row of the $n \times n$ non-singular matrix P obtained from $M^{-1} = P'P$, M being positive definite. Then Propositions 3.2, 3.3 and 3.4 hold with A_s defined as $A_s = I_n \otimes \mathbf{a}_s^{*'}$ and θ replaced by one.

Here also the $DFBETA$s will depend on the unknown parameter ρ. A simple method to obtain an estimate of ρ would be to ignore both the $((i-1)T+t)^{th}$ and $((i-1)T+t+1)^{th}$ terms for all indices $(i,t) \in K$ and estimate ρ from the remaining observations. For a detailed discussion on this see Sen Roy and Guria (2004).

5 Numerical Illustrations

In this section we illustrate our results through the Gross Investment data as used by Grunfeld (1958) and Baltagi (2008). It is a 20 years data (1935-1954) on 10 U.S. firms, where annual real gross investment (y) is the response and real value of the firm (shares outstanding) (x_1) and real value of the capital stock (x_2) are explanatory variables. The model used is (2.1) with $p = 3$ (including the intercept). Initial simple regressions for each of the firms separately revealed that there is variation in the intercept term. Since there was no perceptible time effect, the one-way error structure (2.2) is assumed. All three types of influential observations are then searched for.

Table 1 shows the $DFBETA$ and $DFFIT$ values when the units are deleted one at a time. Although not so apparent from the $DFBETA$ values, the $DFFIT$s clearly indicate that the 2^{nd} and 3^{rd} firms are outliers.

Tables 2 and 3 give respectively the $DFBETA$ and $DFFIT$ values when each of the 20 time points are deleted one at a time. However, the results show that all the time points give quite small values of both and hence cannot be considered as outliers.

Table 4 shows the $DFFIT$ values when each observation corresponding to each firm and every time point is deleted (the corresponding $DFBETA$s are too numerous and hence are not shown). It is observed that most of the observations corresponding to Firms 2 and 3 are large. Also the time points 19 and 20 are large for most firms except 4, 6, 7 and 10, while the first firm has large values for time points 3, 4, 5, 6, 8 and 12.

Taking all of these into account it can be said that firms 2 and 3 are different from the other firms. The first firm experienced some fluctuations in the initial years which later stablized. Also there is an indication of change in the last two years, although this is not reflected in Tables 2 and 3 because it is yet to affect all the firms.

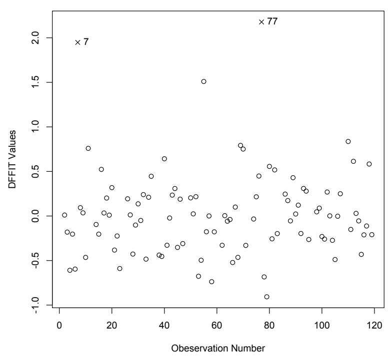

Figure 5. 1: $DFFITS$ when individual points 7^{th} and 77^{th} are outliers

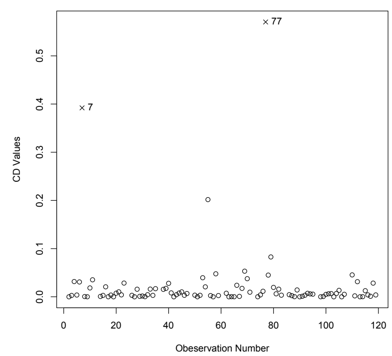

Figure 5. 2: Cook's-Distances when individual points 7^{th} and 77^{th} are outliers

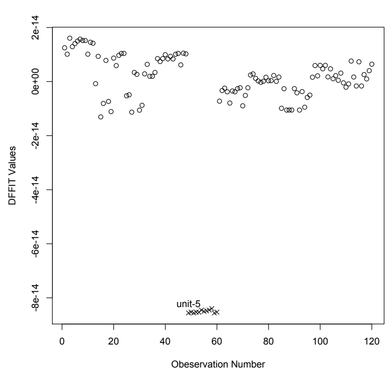

Figure 5. 3: $DFFITS$ when unit-5 is outlier

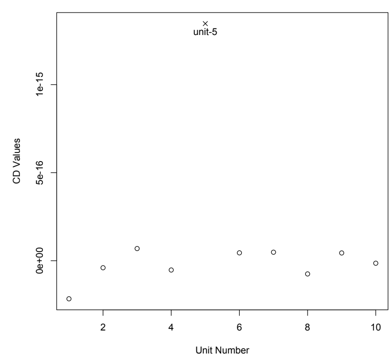

Figure 5. 4: Cook's Distance when unit-5 is outlier

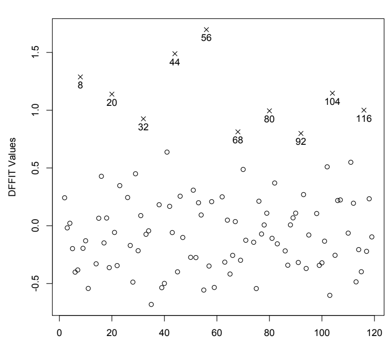

Figure 5. 5: $DFFITS$ when Time point-8 is outlier

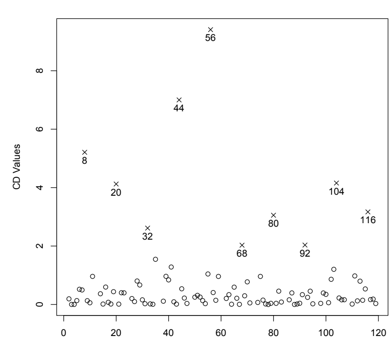

Figure 5. 6: Cook's Distance when Time point-8 is outlier

Table 1: Showing DFBETA and DFFITS values when different firms are omitted

$FIRM$	1	2	3	4	5	6	7	8	9	10
Coeff	DFBETA									
β_0	-35.3571	9.7088	-5.8883	3.7104	-4.0557	5.3188	-0.0839	-0.5232	-3.3611	9.0865
β_1	-0.0025	0.0113	-0.0096	-0.0004	0.0065	-0.0009	0.0007	-0.0002	0.0018	-0.0019
β_2	0.1575	-0.0114	-0.0148	-0.0019	-0.0332	-0.0035	-0.0042	-4.7E-05	-0.0057	-0.0071
$TIME$	DFFITS									
1	-1.7973	6.7415	-6.4156	1.0246	-2.2672	1.3608	-0.0949	-0.1925	-1.1354	1.9349
2	-1.0507	6.7489	-6.4181	0.9940	-2.3329	1.3601	-0.1116	-0.2157	-1.1451	1.9349
3	-0.5739	6.9006	-6.4267	1.0004	-2.4197	1.3593	-0.1276	-0.2327	-1.1490	1.9349
4	-0.4877	7.3218	-6.8727	1.0395	-2.5547	1.3585	-0.1548	-0.2223	-1.1666	1.9349
5	-0.4379	7.4114	-6.9491	1.0279	-2.6096	1.3575	-0.1808	-0.2203	-1.1670	1.9349
6	-0.4104	7.2479	-7.1004	1.0255	-2.6448	1.3572	-0.2051	-0.2291	-1.1819	1.9349
7	-0.2020	7.2386	-7.4940	1.0349	-2.6798	1.3566	-0.2296	-0.2241	-1.1906	1.9349
8	0.1674	7.3484	-8.0712	1.0490	-2.7323	1.3552	-0.2474	-0.2311	-1.2024	1.9349
9	-0.1431	7.3846	-8.1786	1.0354	-2.7555	1.3540	-0.2629	-0.2409	-1.2133	1.9349
10	-0.4437	7.3635	-8.2053	1.0250	-2.7998	1.3538	-0.2740	-0.2432	-1.2174	1.9349
11	-0.1682	7.1996	-8.0771	1.0115	-2.8264	1.3539	-0.2848	-0.2526	-1.2376	1.9349
12	0.3921	6.9674	-8.1243	1.0205	-2.8555	1.3532	-0.2929	-0.2530	-1.2306	1.9349
13	2.6791	7.3334	-8.3746	1.0481	-2.8791	1.3516	-0.3069	-0.2442	-1.2494	1.9349
14	3.6227	7.4525	-8.0620	1.0457	-2.8973	1.3501	-0.3154	-0.2560	-1.2628	1.9349
15	3.5670	7.5241	-7.4318	1.0678	-2.9191	1.3473	-0.3245	-0.2520	-1.2685	1.9349
16	3.7734	7.5326	-7.5559	1.0675	-2.9357	1.3434	-0.3362	-0.2552	-1.2714	1.9349
17	3.3140	7.4123	-7.7245	1.0763	-2.9499	1.3418	-0.3462	-0.2611	-1.2778	1.9349
18	3.8035	7.5771	-7.7845	1.1133	-2.9721	1.3385	-0.3541	-0.2780	-1.2875	1.9349
19	3.7775	7.4953	-7.7843	1.1164	-2.9920	1.3354	-0.3630	-0.3182	-1.3006	1.9349
20	4.6994	7.4333	-7.8145	1.1400	-3.0017	1.3286	-0.3741	-0.3324	-1.3112	1.9349

Table 2: Showing DFBETA values when time points are omitted

TIME	DFBETA		
	β_0	β_1	β_2
1	6.72E-07	3.73E-10	-2.37E-09
2	1.24E-06	1.88E-09	-7.47E-09
3	8.64E-07	3.50E-09	-1.04E-08
4	6.51E-07	1.06E-10	-7.69E-10
5	4.28E-07	7.48E-10	-2.18E-09
6	7.43E-07	2.39E-09	-6.53E-09
7	1.12E-06	3.05E-09	-7.86E-09
8	1.32E-06	1.25E-09	-3.23E-09
9	9.57E-07	1.78E-09	-4.45E-09
10	1.07E-06	2.24E-09	-6.38E-09
11	6.98E-07	2.86E-09	-7.12E-09
12	6.15E-07	4.43E-09	-9.28E-09
13	8.38E-07	6.52E-10	3.30E-09
14	9.61E-07	-1.73E-10	6.76E-09
15	-7.39E-08	8.43E-12	8.34E-09
16	-6.47E-07	8.42E-11	1.13E-08
17	-2.41E-06	2.38E-09	1.35E-08
18	-4.80E-06	1.88E-09	2.65E-08
19	-1.43E-05	4.94E-09	5.41E-08
20	-1.94E-05	2.61E-10	9.05E-08

Table 3: Showing DFFITS values when time points are omitted

FIRM	1	2	3	4	5	6	7	8	9	10
TIME					DFFITs					
1	0.0009	0.0008	0.0007	0.0007	0.0003	0.0007	0.0004	0.0007	0.0004	0.0007
2	0.0023	0.0024	0.0022	0.0022	0.0000	0.0016	0.0007	0.0021	0.0006	0.0015
3	0.0032	0.0032	0.0030	0.0030	-0.0012	0.0016	0.0002	0.0030	0.0002	0.0014
4	0.0007	0.0010	0.0007	0.0008	0.0006	0.0007	0.0007	0.0008	0.0007	0.0007
5	0.0012	0.0016	0.0012	0.0010	-0.0002	0.0007	0.0002	0.0009	0.0004	0.0006
6	0.0033	0.0037	0.0032	0.0025	-0.0015	0.0014	-0.0005	0.0025	0.0000	0.0012
7	0.0047	0.0051	0.0049	0.0033	-0.0016	0.0020	-0.0007	0.0032	0.0001	0.0017
8	0.0050	0.0047	0.0039	0.0021	0.0003	0.0019	0.0008	0.0025	0.0011	0.0018
9	0.0039	0.0044	0.0041	0.0024	-0.0009	0.0015	-0.0002	0.0025	0.0005	0.0014
10	0.0038	0.0048	0.0043	0.0031	-0.0017	0.0018	-0.0009	0.0029	0.0002	0.0016
11	0.0039	0.0047	0.0045	0.0034	-0.0023	0.0015	-0.0016	0.0032	-0.0006	0.0012
12	0.0064	0.0066	0.0068	0.0051	-0.0034	0.0024	-0.0024	0.0048	-0.0010	0.0013
13	0.0060	0.0056	0.0051	0.0025	0.0029	0.0023	0.0025	0.0026	0.0027	0.0013
14	0.0042	0.0050	0.0044	0.0028	0.0038	0.0028	0.0036	0.0030	0.0037	0.0015
15	0.0041	0.0044	0.0037	0.0019	0.0034	0.0019	0.0030	0.0018	0.0030	0.0000
16	0.0048	0.0051	0.0044	0.0022	0.0040	0.0023	0.0035	0.0017	0.0035	-0.0008
17	0.0072	0.0091	0.0066	0.0039	0.0046	0.0029	0.0036	0.0020	0.0038	-0.0029
18	0.0082	0.0092	0.0081	0.0049	0.0070	0.0032	0.0058	0.0015	0.0063	-0.0057
19	0.0121	0.0119	0.0122	0.0077	0.0108	0.0018	0.0083	0.0021	0.0090	-0.0135
20	0.0125	0.0122	0.0129	0.0089	0.0132	0.0034	0.0117	0.0005	0.0118	-0.0168

6 Concluding Remarks

In this paper we have studied the deletion diagnostics for panel data. Methods to deal with an outlier in a unit, or at a particular time-point or for a particular unit at a particular time-point are derived. It is obvious that an outlier in one of the three cases may not be an outlier in another case and hence the identification needs to be done in the proper perspective. This has been illustrated through our example.

The error structure assumed in both the derivations and the illustrations is as in (2.2). This structure is usually preferred when there is reason to believe that there is a unit effect as reflected by the changing intercepts of the different units. However, if the panel data is thought to have additionally a time effect, then a two-way error model of the form, $u_{it} = \mu_i + \lambda_t + v_{it}$, needs to be applied.

Extension to the AR(1) error structure (4.1) has been discussed in Section 4. Such an error structure is preferred when the response exhibits first order autocorrelation. These results can be extended to a general

Table 4: Showing DFFITS values when a single time point corresponding to a particular firm is omitted

FIRM	1	2	3	4	5	6	7	8	9	10
$TIME$					DFFITs					
1	1.2315	2.2176	-0.7154	2.5626	3.1465	3.0014	2.7192	3.7005	0.8720	2.4903
2	-7.8844	10.0847	-7.2090	1.1645	3.6734	3.1895	1.5031	1.6398	0.2736	2.2395
3	-14.8923	9.4871	-11.1829	-0.4414	5.0540	2.7953	1.9118	0.0787	0.2756	2.3267
4	-5.8559	-2.4377	-8.8338	2.5084	2.4724	2.7564	1.9369	0.4800	-0.0677	2.5687
5	-13.6336	-8.8507	-10.6929	-0.3071	0.1926	1.5215	0.6676	0.4007	-0.7839	2.3338
6	-5.7553	2.9702	-8.0535	0.7397	0.1054	1.6858	0.8602	0.0819	-0.6035	2.2491
7	-1.7672	10.8122	-3.2049	1.3742	0.8806	3.1467	1.4994	2.7786	-0.1234	2.3834
8	5.2392	10.4471	-4.7018	1.8855	-2.3754	2.6870	0.0902	1.4330	0.1530	2.5352
9	1.8598	4.5048	-10.6247	0.1385	-0.0518	0.7088	0.3394	-0.4288	-1.7371	2.4462
10	4.0061	-0.3988	-10.5100	0.2725	-2.6147	0.7505	2.4359	-0.6367	0.5032	2.4804
11	-0.5776	-2.0349	-9.6442	1.5721	-2.5331	1.4643	-0.7964	-1.7988	-1.5721	2.4562
12	6.8041	11.8158	-5.8792	-1.0533	-3.9221	1.6723	-1.0310	-0.4493	-0.2772	2.5063
13	2.3456	13.9535	-4.1049	1.0083	-4.9020	1.3461	-1.1925	1.1716	-0.6873	2.6999
14	-2.3226	22.8354	-5.0459	2.0509	-4.0501	2.1550	-1.5254	-0.8905	-2.6905	2.8178
15	-6.4542	11.3410	-8.0524	1.2340	-5.1648	1.1758	-1.9222	-2.0387	-3.4512	2.5851
16	-1.2671	12.3591	-10.6249	1.8667	-6.7983	-0.5223	-3.2194	-2.5090	-2.5222	2.4958
17	-4.2802	21.3092	-9.8570	5.5988	-5.9516	0.9747	-2.0422	-1.0951	-2.3530	2.4586
18	1.1537	25.0638	-11.3688	2.3148	-6.6486	0.5007	-1.6677	-1.3593	-3.4419	2.6691
19	13.8365	17.5512	-13.3026	0.8957	-7.2683	1.9932	-2.0548	-3.7934	-4.2126	2.8011
20	22.3682	2.3989	-17.0622	1.7803	-8.3269	0.5505	-1.2773	-6.7147	-5.9036	2.6516

autoregressive model of order p (AR(p)), or a moving average model of order q (MA(q)) or even an autoregressive moving average model of orders p and q (ARMA(p, q)), depending on the nature and lag of the autocorrelations. A simulation study to validate the results can also be carried out in subsequent studies.

References

Baltagi, B.H. (2008), *Econometric Analysis of Panel Data*, 4^{th} ed., John Wiley, New York.

Banerjee, M. and Frees, E.W. (1997), "Influence diagnostics for linear longitudinal models," *Journal of the American Statistical Association*, 92, 999-1005.

Banerjee, M. (1998), "Cook's Distance in linear longitudinal models," *Communications in Statistics, Theory and Methods*, 27, 2973-2983.

Beach, C.M. and Mackinnon, J.G. (1978), "A maximum likelihood procedure for regression with autocorrelated errors," *Econometrica*, 46, 51-58.

Belsley, D.A., Kuh, E. and Welsch, R.E. (1980), *Regression Diagnostics*. John Wiley, New York.

Chatterjee, S. and Hadi, A.S. (1988), *Sensitivity Analysis in Linear Regression*. John Wiley, New York.

Cook, R.D. (1986), "Assessment of local Analysis," *Journal of the Royal Statistical Society, Series B*, 48, 133-169.

Diggle, R., Liang, P. and Zeger, S. (1994), *Analysis of Longitudinal data*. Oxford University Press.

Greene, W.H. (2005), *Econometric Analysis,* 5^{th} *ed.*. Pearson Education, Singapore.

Grunfeld, Y. (1958), *The Determinants of Corporate Investment*, Ph.D. thesis, Department of Economics, University of Chicago.

Haslett, J. (1999), "A Simple derivation of deletion diagnostic results for general linear model with correlated errors," *Journal of the Royal Statistical Society, Series B*, 61, 603-609.

Haslett, J. and Hayes, K. (1998), "Residuals for linear model with general covariance structure," *Journal of the Royal Statistical Society, Series B*, 60, 201-215.

Kim, S.W. and Huggins, R. (1998), "Diagnostics for autocorrelated regression models," *Australian and New Zealand Journal of Statistics*, 40, 65-71.

Ouwens, J.N.M., Tan, F.E.S. and Berger, M.P.F. (1999), "Local influence for repeated measures generalized linear mixed models," *Proceedings of the 14th International Workshop on Statistical Modelling, Graz (eds H. Friendl, A. Berghold and G. Kauermann)*, 308-316.

Schall, R., and Dunne, T.T. (1991), "Diagnostics for Regression-ARMA Time Series," *Directions in Robust Statistics & Diagnostics, (ed. W. Stahel and S. Weisberg)*, Part 2, 205-221.

Sen Roy, S and Guria, S.N. (2004), "Regression diagnostics in an autocorrelated model," *Brazilian Journal of Probability and Statistics*, 18, 103-112.

Tan, F.E.S., Ouwens, J.N.M. and Berger, M.P.F. (2001), "Detection of Influential Observations in Longitudinal Mixed Effects Regression Models," *Journal of the Royal Statistical Society, Series D*, 50, 271-284.

Tsai, C.L. and Wu, X. (1992), "Assessing local influence in linear regression models with first-order autoregressive or heteroscedastic error structure," *Statistics and Probability Letters*. 14, 247-252.

Yang, K. and Chang, Y.C. (2006), "A note on model diagnostics in longitudinal data analysis," *Computational Statistics*, 21, 571-587.

Zhu, H., Ibrahim, J.G. and Cho H. (2012), "Perturbation and scaled Cook's Distance," *Annals of Statistics*, 40, 785-811.

Received: January 19, 2024

Accepted: May 29, 2024