Jagannath University Journal of Science
Volume 10, Number I, Jun. 2023, pp. 27−42
https://jnu.ac.bd/journal/portal/archives/science.jsp
ISSN 3005-4486 (Online), ISSN 2224-1698 (Print)

The Characteristic Behavior and Bifurcation of the Cubic Map

Research Article

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DOI: https://doi.org/10.3329/jnujsci.v10i1.71163

Received: 21 June 2023, Accepted: 30 November 2023

ABSTRACT

Important characteristics preserved from the standard 1-dimensional cubic map are studied here. Many important features of the original 1-dimensional cubic map have survived, and their behavior is being studied here. Attracting, repelling, and neutral fixed points are analyzed. The use of the map as an aid in the study of period doubling bifurcation has been depicted. On the other hand, map can display an exorbitance of additional behaviors. It can be seen that nearby spots on trajectories move closer together and further apart as time progresses. These are the paths that never seem to settle into regular orbits or stop moving altogether. Modifying the starting conditions even slightly can shift the course of evolution. In reality, patterns drive chaotic systems despite their seemingly nonlinear and unpredictable behavior. Exploring the chaotic behavior of the cubic equation by varying the governing parameters, finding Bifurcation diagrams, etc., are all subtopics of this work, but finding the cubic map is the main focus.

Keywords: *Fixed Points, Stability, Bifurcation, Chaos, Period doubling, Cubic map*

1. Introduction

Studies of dynamical systems often focus on periodic change. Problems with the Solar System's enduringness and continuing development inspired the late 19th-century development of the theory of dynamical systems by Prajapati*et al*. (2019). Finding solutions to these problems has spawned a robust academic discipline with widespread implications in fields as diverse as physics, biology, meteorology, astronomy, economics (Sarmah*et al.*,2014,Das*et al.*, 2010, May, 1974), and many others (Hamacher, 2012, McCartney, 2012, Philominathan*et al*., 2011). Scientists in the mathematical field often work with maps that are not linear to investigate the myriad identifying traits and properties linked to the disordered state of the systemby Kuznetsov*et al.* (1996). Despite the quadratic map's apparent simplicity, the dynamics it exhibits are surprisingly nuancedby May (1976). May (1979) analyzed the generalized cubic map is

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much more challenging. Oscillatory processes, such as those with a high number of fixed points or chaotic behaviorby Mahecha *et al.* (2006), have been the topic of a considerable lot of research in recent years, particularly in light of the most recent downturnby Sridhar (2011) and Zhang *et al*. (2016). In addition to analyzing the system in its natural state, the bifurcation analysis seeks to quantify the complexity of the system's dynamic behavior after periodic forcing is applied. Abashar*et al.* (2011) examined the Stroboscopy Poincare maps and one-dimensional limit cycle bifurcations are common techniques. Comprehensive map explanations are available in (Gallas, 1983 and Collet *et al.* 1980). The nonlinearity of logical maps is represented by a quadratic function, whereas the nonlinearity of cubic maps is represented by a cubic function. Both the standard Feingenbaum scaling and the period-doubling bifurcation eventually result in total anarchy for the two models (Feigenbaum 1978 and 1979*).* For many problems, the distribution of solutions and the topology of the manifolds on which they lie undergo drastic changes as the parameter approaches its critical value. Changes of this magnitude could have far-reaching consequences. In this context, bifurcation refers to the phenomenon itself, whereas bifurcation values and "bifurcation points" refer to specific values for the relevant parameters. Like other dynamical systems, it has been shown that the properties of fixed points on iterated maps can vary as a function of the system's control parameters. One method for explaining period-doubling sequences is the Feingenbaum scaling by Grebogi*et al.*(1982). Grebogi*et al.* (1983) depicted the previously unrelated chaotic bands were only pulled together by a crisis involving a number of chaotic attractors. There was no alternative explanation that made sense.Murray (2001) and Agarwal*et al.* (1997) created the richer dynamics and more impressive computing outputs, it should be emphasized that the discrete dynamical system is superior to the continuous one. Remember this, because it's important. This method also works well and is valid when used to models of chemical oscillatory reactions (Kapral, 1991, Pearson 1991, Floudas*et al.,* 2004). This is why, in addition to the bifurcation analysis, It is also considered the stability analysis of the discrete-time model of the system (Zafar *et al.*, 2017,Xu*et al.*, 2013, Yu*et al.* 2001). Golovin*et al. (*2008),Din (2018) and Leach*et al*.(1992) explain their work on a chemical oscillatory model in discrete time. Higher-order Hopf normal form expansions are needed to improve cycle approximations or study more complex dynamical systems. The harmonic balance approach generates higher-order approximations in the frequency domain. Because nonlinear maps often have weak and strong resonances, extrapolating this conclusion to discrete time is risky by Robinson (1990). The higher-order harmonic balance enables for sufficient accuracy to be attained while estimating the invariant cycle. Mees (1981) and Moiola*et al.* (1996), who also studied continuous-time systems, used approaches quite similar to this one. Farmers can choose from a wide variety of pest control methods (Freedman, 1976, Van*et al.*, 1988), each of which may reduce the financial losses incurred as a result of insect pests. Recent mathematical models have considered resource exhaustion. Recent academic research has generated such models. Most of this research (Qin*et al.*, 2014, Yang*et al.*, 2016, Tian*et al.,* 2019) examined how control measures that are non-deterministic and impulsive affect pest management. These articles study nonlinear impulsive functions independent of all parameters except natural enemy population density. Operator theoretic methods (Zhao*et al.*, 2009, Zhang*et al.*, 2008, Zhao *et al.*, 2011, Wang*et al.*, 2014) simplify nontrivial periodic solutions to problems involving fixed points and bifurcations.The main objective is to have an understanding of how the dynamics of the cubic map shift depending on the value of the parameter.

2. Governing form of Cubic Map

Assume the 1-dimensional representation of the cubic form.

$$f_{\zeta}(x) = f(x,\zeta) = -x^3 + \zeta x\ , \quad -2 \le x \le 2 \tag{1}$$

Where ζ is a parameter value and $0 \le \zeta \le 3$.

Bimodal map describes a recursive dynamical system that operates over quantifiable time steps.

This is what is meant when talking about a bimodal map. This type of system can be identified by its indefinitely many unstable periodic hotspots and a chaotic pattern of behavior. This kind of mechanism is also completely uncontrollable. Furthermore, a system of this kind has the possibility of containing an endless number of cyclic nodes. This type of map is also called a cubic map in numerous fields all over the world. The term bimodal map is commonly used when discussing cubic map within the framework of cartography. This term refers to the same category of map as the one it has been discussing.

An approach that predicts up to n alternative steady orbits for a particular set of parameters in maps with n critical points reveals that the cubic's behaviors are far more probable to be chaotic than the quadratic's. This arises from the fact that the idea can be used to analyze maps with a limitless amount of critical nodes. From the hypothesis that maps with n critical points can sustain up to n steady distinct positions, this finding can be deduced as an obvious consequence. This theory predicts that, for maps with n critical points, the number of unique stable orbits may range from zero to n, based on the quantities that are entered for the parameters. The results of this investigation have given evidence that the dynamics of the cubic are more complex than those of the quadratic. This warning is the outcome of my investigation. When looking at maps, if there is just one hump, this suggests that there is only one stable state, which is also known as an attractor, but there is only one of them. On the other hand, it's not impossible for maps that are generally cubic to have two distinct kinds of attractor patterns. This is something that can happen. A number of authors have provided an in-depth explanation of the procedure that can lead to the development of a pair of alternative stable orbits by making use of the cubic map.

3. Solution of the Cubic Map

$$f_{\zeta}(x) = f(x,\zeta) = -x^3 + \zeta x$$

The first statement, which is one with which we are already familiar, is the statement that $y = x$.

Now,

$$f(x,\zeta) = -x^3 + \zeta x$$

$$\Rightarrow -x^3 + \zeta x = x$$

$$\Rightarrow -x^3 + \zeta x - x = 0$$

$$\Rightarrow x(-x^2 + \zeta - 1) = 0$$

Either $x = 0$ or, $\Rightarrow -x^2 + \zeta - 1 = 0$

$$\Rightarrow -x^2 = 1 - \zeta$$

$$\Rightarrow x^2 = \zeta - 1$$

$$\Rightarrow x = \pm\sqrt{\zeta - 1} \quad (2)$$

4. Analysis of Fixed Points

In mathematical concepts, a value can be considered fixed if it does not change due to the procedure that is carried out on it. A fixed point is characterized as this specific value.

The values that ζ can take on are what decide where each of the there are three stable spots located within the space..

$$x = 0, +\sqrt{\zeta - 1}, \text{ and } -\sqrt{\zeta - 1}. \quad (3)$$

Case 1: When $\zeta = 1$, then $x = 0$ is the only fixed point for $f(x,\zeta)$.

Case 2: When $\zeta = 1.5$, then $x = 0, \sqrt{0.5}$, and $-\sqrt{0.5}$ is the fixed point for $f(x,\zeta)$.

Case 3: When $\zeta = 2$, then $x = 0, +1,$ and -1 is the fixed point for $f(x,\zeta)$.

Case 4: When $\zeta = 2.5$, then $x = 0, \sqrt{1.5}$ and $-\sqrt{1.5}$ is the fixed point for $f(x,\zeta)$.

Case 5: When $\zeta = 3$, then $x = 0, \sqrt{1.5}$ and $-\sqrt{1.5}$ is the fixed point for $f(x,\zeta)$.

5. Enunciation in Case the Fixed Points $x = 0, +\sqrt{\zeta - 1},$ and $-\sqrt{\zeta - 1}.$ are Attracting, Repelling, or Neutral

It has been demonstrated that the traits of anchor points on repeated maps can shift in appearance based on the settings of the technique's control factors. This is similar to what has been discovered in the past with regard to other dynamical systems.

Differentiating (1) we have

$$f'_{\zeta}(x) = -3x^2 + \zeta$$

$$\left|f'_{\zeta}(x)\right| = \left|-3x^2 + \zeta\right|$$

$$\left|f'_{\zeta}\left(\pm\sqrt{\zeta - 1}\right)\right| = \left|-3\left(\pm\sqrt{\zeta - 1}\right)^2 + \zeta\right|$$

$$\left|\Rightarrow f'_{\zeta}\left(\pm\sqrt{\zeta - 1}\right)\right| = \left|-3(\zeta - 1) + \zeta\right|$$

$$\Rightarrow \left|f'_{\zeta}\left(\pm\sqrt{\zeta - 1}\right)\right| = \left|-3\zeta + 3 + \zeta\right|$$

$$\Rightarrow \left|f'_{\zeta}\left(\pm\sqrt{\zeta - 1}\right)\right| = \left|-2\zeta + 3\right|$$

(i) When $\zeta = 0$, then

$\left|f'_{\zeta}(x)\right| = \left|-3.0^2 + 0\right| = 0 < 1$ so by definition, 0 is an attracting fixed point.

$$\Rightarrow \left|f'_{\zeta}\left(\pm\sqrt{\zeta - 1}\right)\right| = \left|-2\zeta + 3\right| = \left|-2.0 + 3\right| = 3 > 1$$

So by definition, $x = \pm\sqrt{\zeta - 1}$ is repelling fixed point.

(ii) When $\zeta = 1$, then

$\left|f'_{\zeta}(x)\right| = \left|-3.0^2 + 1\right| = 1$ so by definition, 0 is a neutral fixed point.

When $\zeta = 1$, then

$$\left|f'_{\zeta}\left(\pm\sqrt{r - 1}\right)\right| = \left|-2\zeta + 3\right| = \left|-2.1 + 3\right| = 1$$

So by definition, $x = \pm\sqrt{\zeta - 1}$ is neutral fixed point.

(iii) When $0 < \zeta < 1$, then $\left|f'_{\zeta}\right| = \left|\zeta\right| < 1$ so by definition, 0 is an attracting fixed point.

When $0 < \zeta < 1$, take $\zeta = 0.5$ then

$$\left|f'_{\zeta}\left(\pm\sqrt{r - 1}\right)\right| = \left|-2 \times 0.5 + 3\right| = \left|-1 + 3\right| = 2 > 1$$

So by definition, $x = \pm\sqrt{\zeta - 1}$ is repelling fixed point.

(iv) When $1 < \zeta < 2$, then $\left|f'_{\zeta}\right| = \left|\zeta\right| > 1$ so by definition, 0 is a repelling fixed point.

When $1 < \zeta < 2$, take $\zeta = 1.5$ then

$$\left|f'_{\zeta}\left(\pm\sqrt{r - 1}\right)\right| = \left|-2 \times 1.5 + 3\right| = \left|-3 + 3\right| = 0 < 1$$

So by definition, $x = \pm\sqrt{\zeta - 1}$ is attracting fixed point.

(v) When $\zeta = 2$, then $\left|f'_{\zeta}\right| = \left|\zeta\right| > 1$ so by definition, 0 is a repelling fixed point.

When $\zeta = 2$, then

$$\left|f'_{\zeta}\left(\pm\sqrt{r - 1}\right)\right| = \left|-2 \times 2 + 3\right| = \left|-1\right| = 1$$

So by definition, $x = \pm\sqrt{\zeta - 1}$ is neutral fixed point.

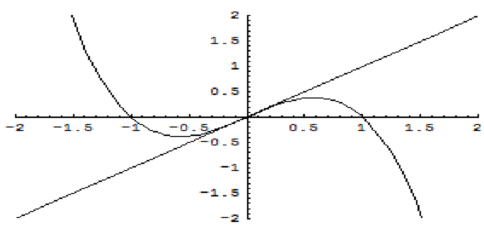
(vi) When $2 < \zeta < 3$, then $\left|f'_{\zeta}\right| = \left|\zeta\right| > 1$ so by definition, 0 is a repelling fixed point.

When $2 < \zeta < 3$, take 2.5, then

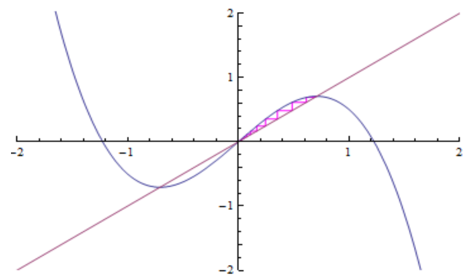
$$\left|f'_{\zeta}\left(\pm\sqrt{r - 1}\right)\right| = \left|-2 \times 2.5 + 3\right| = \left|-2\right| = 2 > 1$$

So by definition, $x = \pm\sqrt{\zeta - 1}$ is a repelling fixed point.

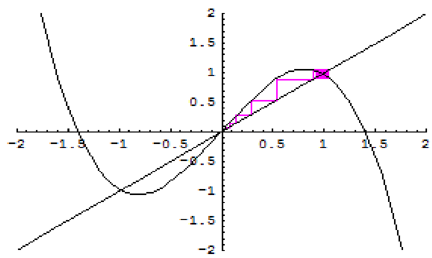
To visualize these scenarios across a broad range of ζ values, we employ the "cobweb" graphical analysis here.



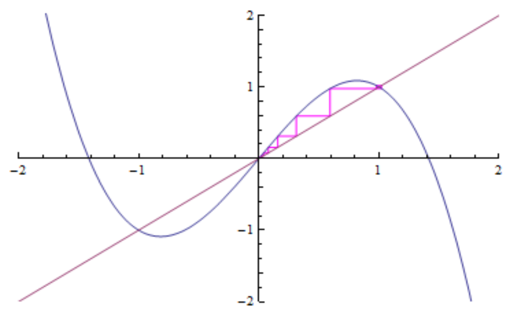
(a) When $\zeta = 1.$



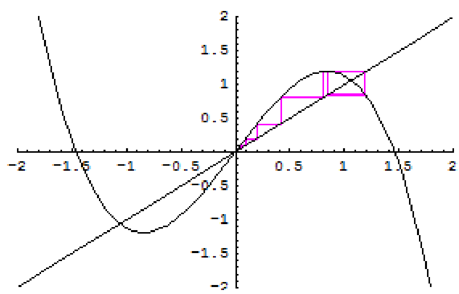
(b) When $\zeta = 1.5.$



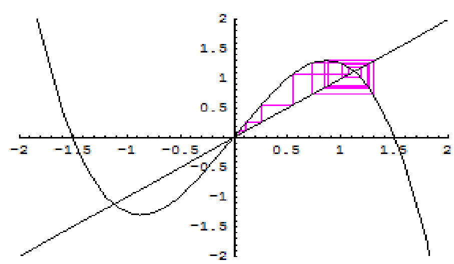
(c) When $\zeta = 1.96.$



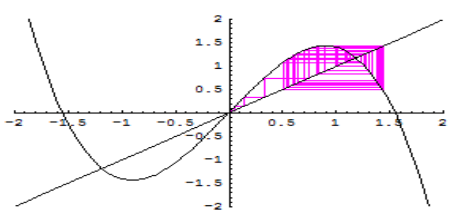
(d) When $\zeta = 2.$



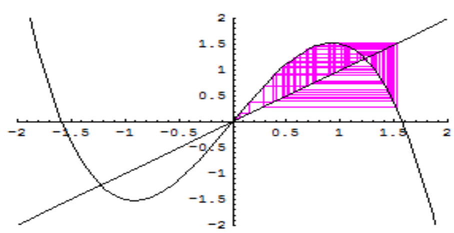
(e) When $\zeta = 2.12.$



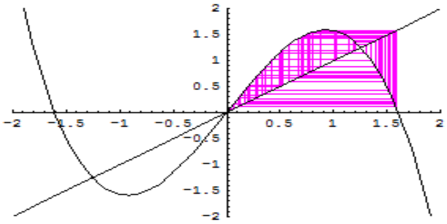
(f) When $\zeta = 2.25.$



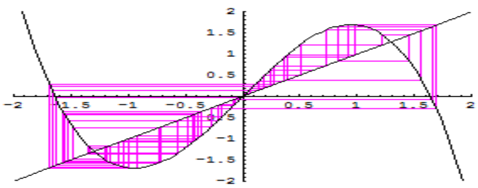
(g) When $\zeta = 2.40.$



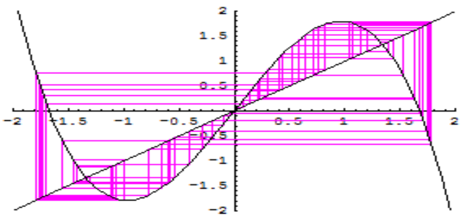
(h) When $\zeta = 2.50.$



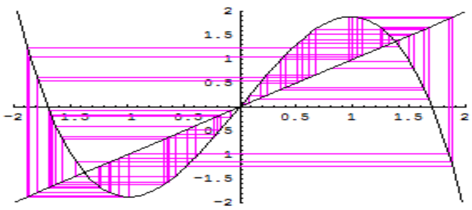
(i) When $\zeta = 2.57$.



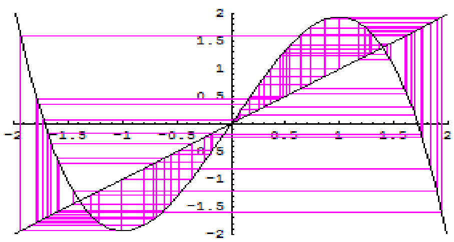
(j) When $\zeta = 2.68$.



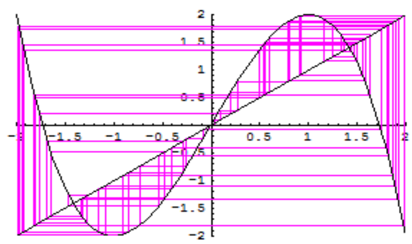
(k) When $\zeta = 2.79$.



(l) When $\zeta = 2.88$.



(m) When $\zeta = 2.94$.



(n) When $\zeta = 3$.

Figure 1. Cobweb graph of the equation (1) different values of ζ.

(a) When $\zeta = 1$, the cobweb graph demonstrates that the fixed points at 0 and $\pm\sqrt{\zeta - 1}$ are neutral fixed point.

(b) When $\zeta = 1.5$, the cobweb graph demonstrates that the fixed points at 0 is a repelling fixed point and $\pm\sqrt{\zeta - 1}$ is an attracting fixed point respectively..

(c) When $\zeta = 1.96$, the cobweb graph demonstrates that the fixed points at 0 is a repelling fixed point and $\pm\sqrt{\zeta - 1}$ is an attracting fixed point.

(d) When $\zeta = 2$, the cobweb graph demonstrates that the fixed points at 0 is a repelling fixed point and $\pm\sqrt{\zeta - 1}$ is a neutral fixed point.

(e) When $\zeta = 2.12$, the cobweb graph demonstrates that the fixed points at 0 and $\pm\sqrt{\zeta - 1}$ are repelling fixed point.

(f) When $\zeta = 2.25$, the cobweb graph demonstrates that the fixed points at 0 and $\pm\sqrt{\zeta-1}$ are repelling fixed point.

(g) When $\zeta = 2.40$, the cobweb graph demonstrates that the fixed points at 0 and $\pm\sqrt{\zeta-1}$ are repelling fixed point.

(h) When $\zeta = 2.50$, the cobweb graph demonstrates that the fixed points at 0 and $\pm\sqrt{\zeta-1}$ are repelling fixed point.

(i) When $\zeta = 2.57$, the cobweb graph demonstrates that the fixed points at 0 and $\pm\sqrt{\zeta-1}$ are repelling fixed point.

(j) When $\zeta = 2.68$, the cobweb graph demonstrates that the fixed points at 0 and $\pm\sqrt{\zeta-1}$ are repelling fixed point.

(k) When $\zeta = 2.79$, the cobweb graph demonstrates that the fixed points at 0 and $\pm\sqrt{\zeta-1}$ are repelling fixed point.

(l) When $\zeta = 2.88$, the cobweb graph demonstrates that the fixed points at 0 and $\pm\sqrt{\zeta-1}$ are repelling fixed point.

(m) When $\zeta = 2.94$, the cobweb graph demonstrates that the fixed points at 0 and $\pm\sqrt{\zeta-1}$ are repelling fixed point.

(n) When $\zeta = 3$, the cobweb graph demonstrates that the fixed points at 0 and $\pm\sqrt{\zeta-1}$ are repelling fixed point.

6. Dynamics of the Cubic Map

Due to their sensitivity to the initial conditions, the orbits of neighboring seeds end up behaving very differently from one another after a few repetitions. These deviations are not always negligible. In many situations, the solution to a mathematical equation presents a formidable challenge. The task may appear to be impossible. Computing approximations to mathematical problem solutions is the most common use of computers in the scientific community. Scientists have often been unable to make predictions based on the results produced by computers, despite major improvements in the speed and precision of calculation. Despite the fact that both of these facets of calculation have witnessed tremendous progress, this remains the case. Due to the nature of the situation, they are forced to go through a significant amount of mental suffering. Because of its inherent simplicity, the cubic map serves as an excellent starting point for conversations about chaos. The anarchic state is depicted in a straightforward manner by the cubic map for ζ -values. One of the defining characteristics of chaotic systems is their sensitivity to the parameters with which they are first seeded.

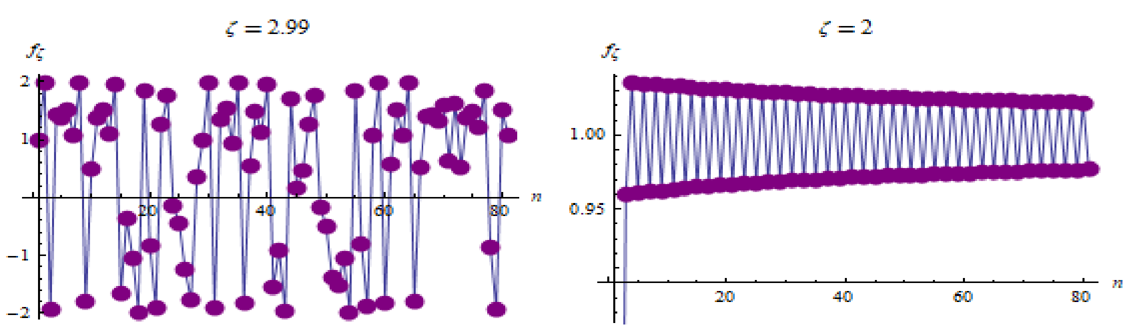


Figure 2. Dynamics of the cubic map (1) for iteration number, $n = 80$ and $\zeta = 2.99, 2$.

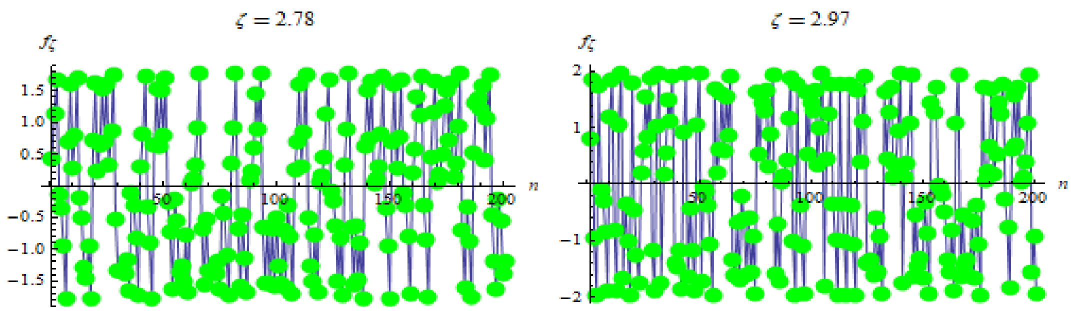


Figure 3. Dynamics of the cubic map (1) for iteration number, $n = 200$ and $\zeta = 2.78, 2.97.$

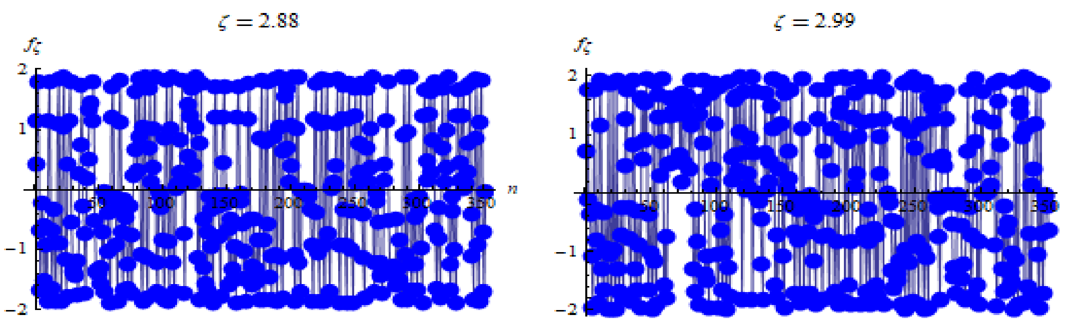


Figure 4. Dynamics of the cubic map (1) for iteration number, $n = 350$ and $\zeta = 2.88, 2.99.$

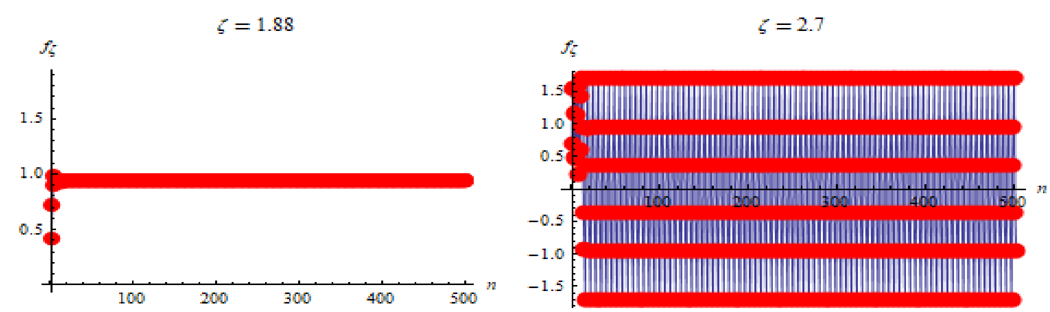


Figure 5. Dynamics of the cubic map (1) for iteration number, $n = 500$ and $\zeta = 1.88, 2.7.$

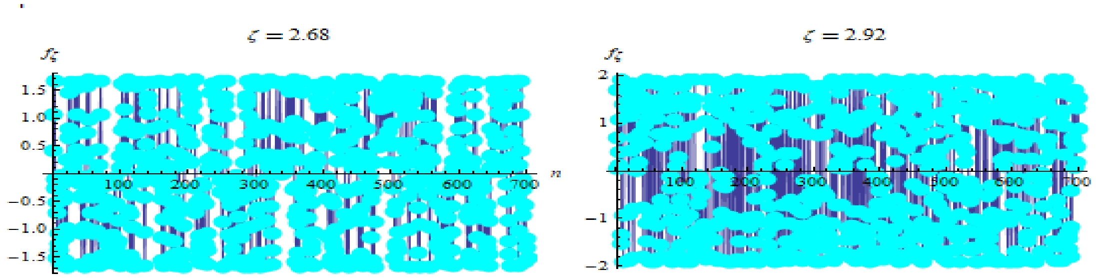


Figure 6. Dynamics of the cubic map (1) for iteration number, $n = 700$ and $\zeta = 2.68, 2.92.$

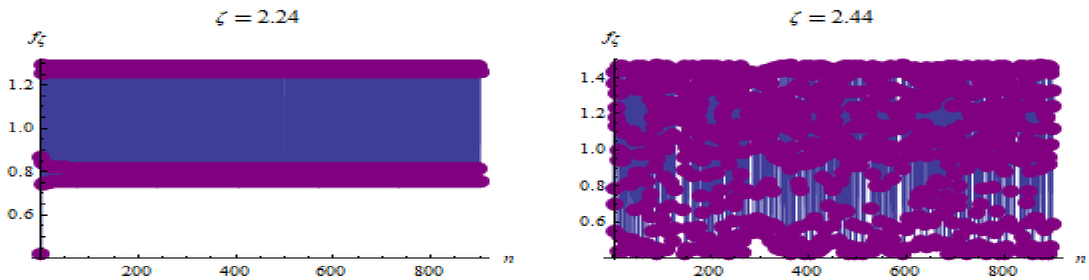


Figure 7. Dynamics of the cubic map (1) for iteration number, $n = 900$ and $\zeta = 2.24, 2.44.$

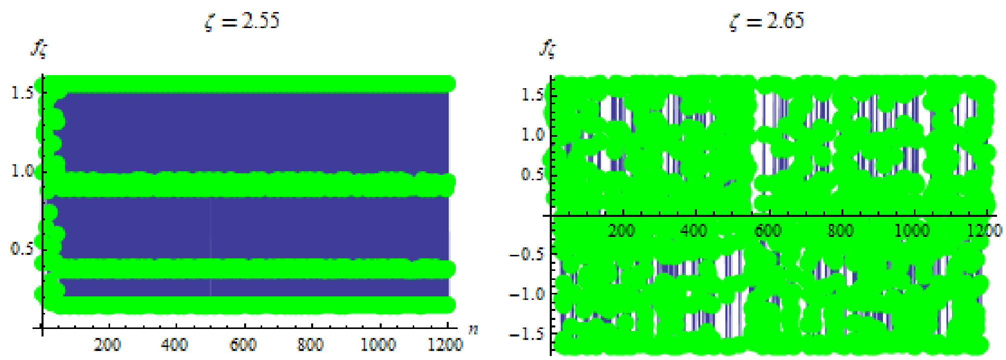


Figure 8. Dynamics of the cubic map (1) for iteration number, $n = 1200$ and $\zeta = 2.55,\ 2.65.$

Many various parameter values were employed, and the oscillations, both periodic and non-periodic, that occurred across the many repetitions are shown in fig. 2-8. Figure 2-8 depicts an abnormal circumstance that leads to anarchy, and it's worth mentioning in passing. Depending on what is used as a starting point, the previously stated statistics can be recalculated in either an upward or downward direction. Short bursts of periodicity might emerge when the level of chaos rises to a particular threshold. Long-term periodic behavior in a deterministic system is said to be chaotic if its occurrence is highly dependent on the initial conditions. It seems unlikely that a long-term trajectory will ever fail to reach a stable site or a periodic orbit. Certain intervals of increasing numbers may exhibit chaotic behavior, punctuated by brief periods of periodic behavior. A image representing a plot of a time series displays the iterated values. The values have become increasingly erratic over time, as depicted by this graph.

7. Bifurcation Analysis of the Cubic Map (1)

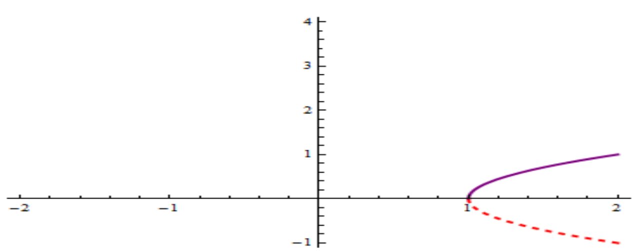


Figure 8. Bifurcation diagram of the equation (2).

When $\zeta = 3$, consequently, the response to issue (1) is going to be

$$x = 0 \text{ and } x = \pm\sqrt{3} \tag{4}$$

$x = \pm\sqrt{3}$ is stable (sink) and $x = 0$ is unstable(a source).

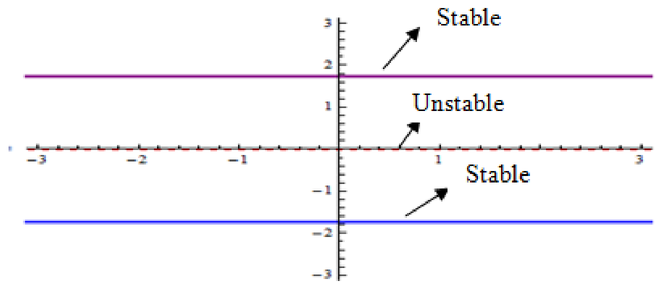


Figure 9. Bifurcation diagram of the equation (4).

When $\zeta = 1$ then $x = 1, -1$ stable (sink) and $x = 0$ (a source) are the three possible solutions to the equation (1).

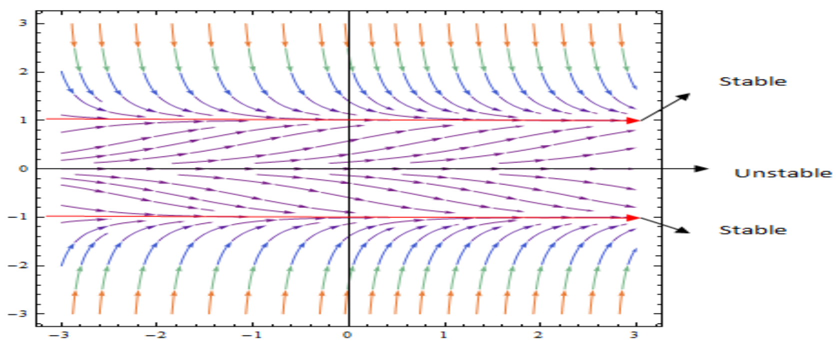


Figure 10. Bifurcation diagram of the equation (1) as $\zeta = 1, x = -1, 1.$

When $\zeta = 1.5$ then $x = 1.22474, -1.22474$ stable (sink) and $x = 0$ (a source) are the three possible solutions to the equation (1).

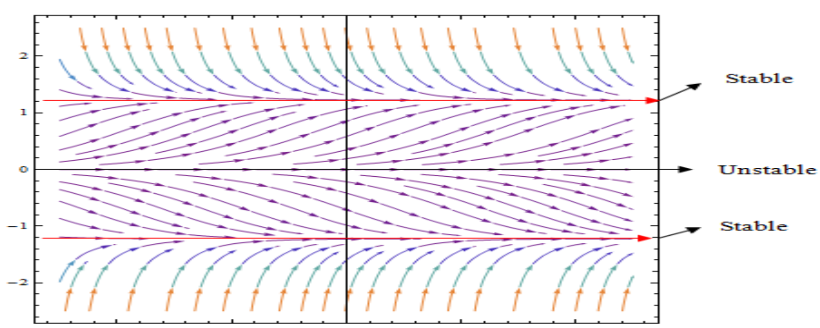


Figure 11. Bifurcation diagram of the equation (1) as $\zeta = 1.5, x = -1.22474, 1.22474.$

When $\zeta = 2$ then $x = 1.414, -1.414$ stable (sink) and $x = 0$ (a source) are the three possible solutions to the equation (1).

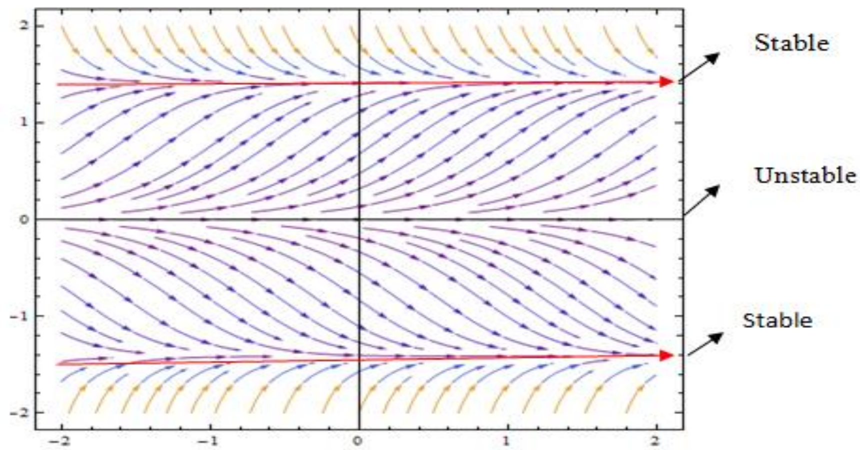


Figure 12. Bifurcation diagram of the equation (1) as $\zeta = 2, x = -1.414, 1.414.$

When $\zeta = 2.5$ then $x = 1.58114, -1.58114$ stable (sink) and $x = 0$ (a source) are the three possible solutions to the equation (1).

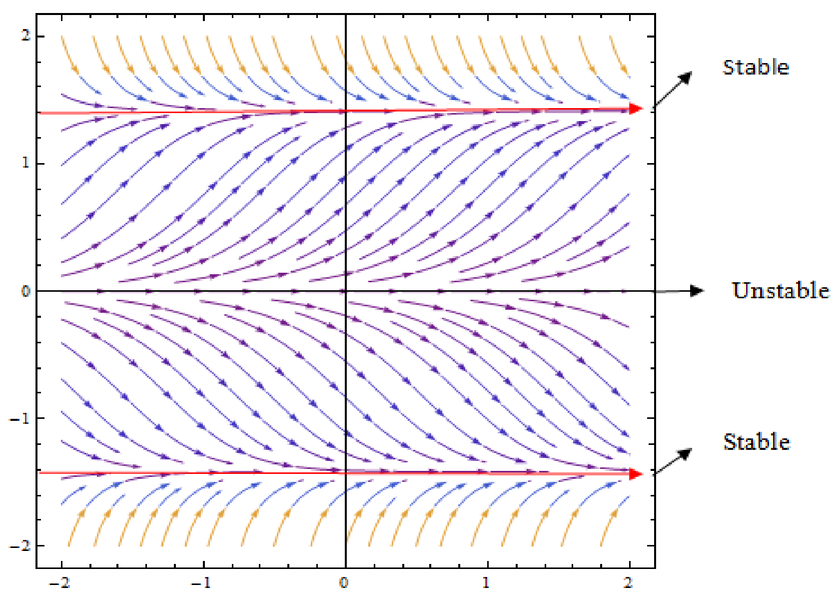


Figure 13. Bifurcation diagram of the equation (1) as $\zeta = 2.5, x = -1.58114, 1.58114.$

When $\zeta = 2.79$ then $x = 1.67033, -1.67003$ stable (sink) and $x = 0$ (a source) are the three possible solutions to the equation (1).

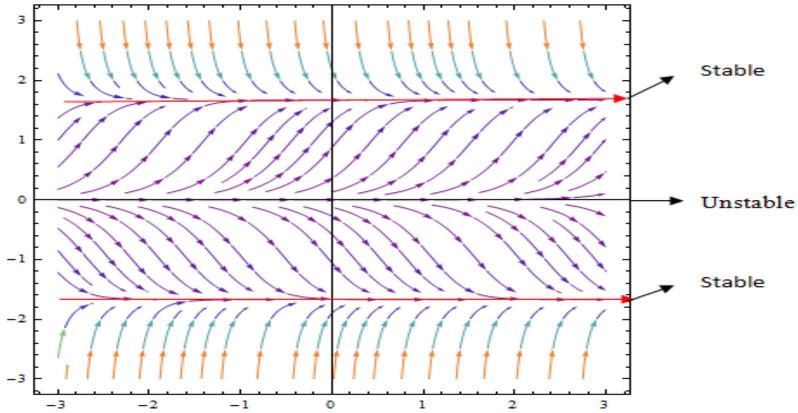


Figure 14. Bifurcation diagram of the equation (1) as $\zeta = 2.79, x = -1.67033, 1.67033.$

When $\zeta = 3$ then $x = 1.732, -1.732$ stable (sink) and $x = 0$ (a source) are the three possible solutions to the equation (1).

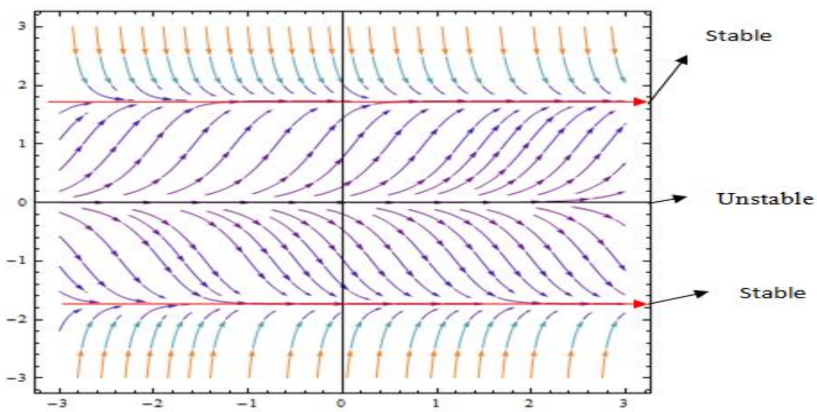
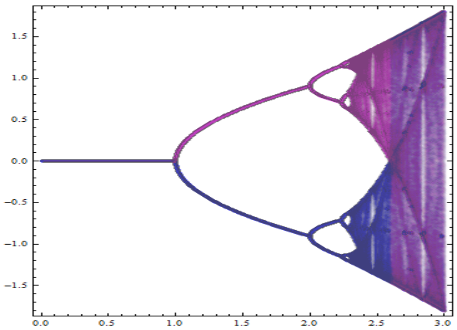


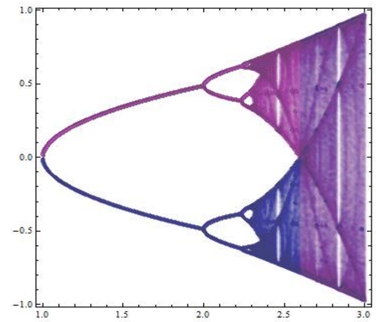
Figure 15. Bifurcation diagram of the equation (1) as $\zeta = 3, x = -1.732, 1.732.$

8. Period Doubling Bifurcation of Cubic Map

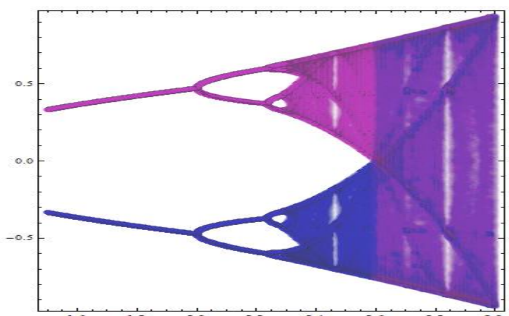
It suggests that there was a division in a dynamic system, when the variation in the control parameter causes a significant change in the qualitative behavior of the system. Stated simply, the system will exhibit distinct qualitative behaviors at the onset of a bifurcation. This happens every time there is a change in the control parameter's value. The process of splitting a system's behavior into two distinct zones, each of which has a definite parameter value at which the transition occurs, is known as bifurcation. The process of bifurcation, which splits a system's behavior into two separate zones, is also referred to as bifurcation. Almost invariably, a shift in one of the system's parameters will be reflected in a similarly gradual shift in the available options to address the issue. This can be attributed to the closed-loop nature of the system. In the vast majority of situations, this is the situation. Nonetheless, a large number of problems exhibit a dramatic change in the number of solutions and a large modification in the structure of the manifolds that include those solutions as a parameter approaches certain critical levels. This happens in a variety of circumstances. This is something that could occur in a number of contexts. This occurs in a sizable portion of situations with a diverse range of issues.



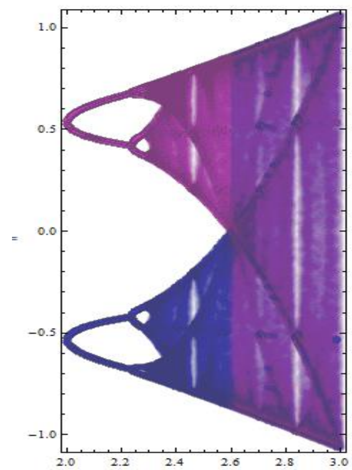
(a) $0 \leq \zeta \leq 3$



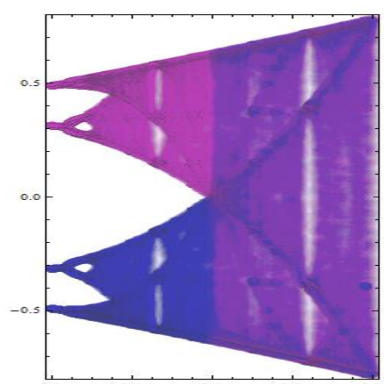
(b) $1 \leq \zeta \leq 3$



(c) $1.5 \leq \zeta \leq 3$



(d) $2 \leq \zeta \leq 3$



(e) $2.2 \leq \zeta \leq 3$.

Figure 16. Bifurcation orbit diagram of the cubic map.

From the interval, $0 \le \zeta \le 3$, the trajectory's behavior changes for the better, and it stays that way all the way to the finish. The trajectories begin to stabilize into a pattern of change that occurs between two locations when $\zeta = 1$ from Fig.16 (a). Between those two locations, this pattern of change manifests itself. As a result of their mutual attraction, these two points stay put to form what is known as a two cycle. This allows us to confidently assert that the trajectories of the cubic map endure a period doubling splitting at the value of $\zeta = 1$. This occurs because the parameter $\zeta = 1$ at this time. From the interval, $1 \le \zeta \le 3$, the Fig.16 (b) contains two cycle when $\zeta = 1$. From the interval, $1.5 \le \zeta \le 3$, the Fig. 16 (c) contains four-cycle when $\zeta = 2$. From the interval, $2 \le \zeta \le 3$, the figure Fig. 16 (d) contains "four cycle" when $\zeta = 2$ and the interval, $\zeta = 2.2$, the figure (d) contains "eight cycle". For the interval, $2.2 \le \zeta \le 3$, the figure Fig. 16 (e) contains "sixteen cycle" as $\zeta = 2.4$.

When periods are doubled, there is a path leading to chaos.

Table 1. Bifurcation points and periods of the cubic map.

Bifurcation points	**Periods of the cubic map**
$\zeta_1 = 1$	2
$\zeta_2 = 2$	4
$\zeta_3 = 2.2$	8
$\zeta_4 = 2.4$	16
$\zeta_5 = 2.6$	32
$\zeta_6 = 2.8$	64

The system is now organized in a 4-cycle pattern, which is an improvement over its previous 2-cycle structure. As a result, the four cycle is generated once the derivative that is put in place of f^4 is At the culmination of the second cycle, when the derivation of was obtained at the fixed value, f^2. Period duplication is a method by which a is carried out several times, which leads to cycles of periodicity 2 when the derivative of $2^3, 2^4$ …...etc. till the extent is reached at which the derivative ceases to exist of $\zeta = \zeta\infty$, where period doublings start piling up to use another name for it. This point ($\zeta = \zeta\infty$) is where it reaches the limit of the number of period doublings that may be performed and chaotic situation occur. That is, when period is infinitive (∞) then chaotic situation occur. The procedure comes to an end once it reaches this point. Because the periodicity is 2^∞, it may be deduced that the iterate of the map have transitioned into a periodic state.

9. Conclusion

In a cubic map, some equilibrium points are stable while others are unstable. With this map, we can pinpoint several distinct locations. It has become apparent that some fixed points have an alluring characteristic, whilst others are repellent, and a third group has an essentially neutral personality. The bifurcations that the cubic map's control parameter can generate set it apart from other maps. The period is said to have doubled when a bifurcation occurs and the cubic map converges to a fixed point at the bifurcation. Once the bifurcation graph reaches the maximum number of period doublings that can be performed at the point $\zeta = \zeta_\infty$ the system enters a chaotic state. It is inevitable that an unstable circumstance will arise once the period is infinitive. When we reach this point in the process, we will have successfully finished everything that needed to be done. A time series graphic depicting the iterated values shows how the values gradually get more chaotic over time.

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