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Modeling and Stability Analysis of the Dynamics of Prey-Predator Population

Research Article

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ABSTRACT

The aim of this work is to investigate the numerical study of prey-predator model and give a thorough overview of the existing research. Here we see instances that involve stability analysis, phase plane, and phase portrait, among others. Furthermore, a numerical simulation is shown to illustrate the results of the prey-predator model. This is to provide light on the procedures followed in order to arrive at the conclusions. The one-prey, one-predator principle is discussed in this article from multiple theoretical perspectives. Finally, for various parameter values and time intervals, we have demonstrated the phase portraits and the dynamic behavior of the pre-predator model.

Keywords: *Prey-predator systems, Population, Species, Stability, Equilibrium Points*

1. Introduction

The fast evolution of many systems, brought about by physical, chemical, and biological phenomena, results in their abrupt dynamical properties. These events can be represented by impulsive differential equations (*Lsksmikantham et al. 1980; Bainov et al. 1989*). The splitting theory of uninterrupted systems of dynamics has made a well-known analytical advance (*Leine et al. 2000; Guckenheimer et al. 1983*). Significant progress has been achieved in this area of research. Studying on impulsive differential systems mostly focuses on stability properties rather than bifurcation patterns (Liu et al. 2003; Lu et al. 2002). Many writers have studied predator-prey dynamics with a fixed harvesting rate, quota for either species, or both. Brauer et al. 1979; Braue F et al. 1979; Brauer et al. 1981; Brauer et al. 1981 explored predator-prey models with constant harvesting and quotas of both species. The subject of the long-term viability of particular species ranks high among the most basic and consequential issues in ecology. In order to keep

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ecological models stable, it is crucial to create appropriate controls. This is because there are often major shifts occurring in real-world ecosystems. Researchers have come a long way in the past few years in their comprehension of the control issue linked to biodynamic systems Lansun (1988). The connection between prey and food sources in these communities can be better understood through the application of mathematical models. Lotka and Volterra's pioneering work in the field has led to a surge in interest in predator-prey research in mathematical ecology, population dynamics, and other applied fields Volterra (1926). When it came to epidemiology, Kermack et al. (1927) were trailblazers who created ground-breaking techniques. Although they complement one another, epidemiological and ecological research is distinct disciplines. However, a great deal of commonality exists across the two fields as well. As a result, eco-epidemiology, which merges the two fields, has gained traction in recent decades by Raid et al. (2012).In ecology, the interaction between predators and prey is among the most crucial types of interdependent partnerships. Biological networks and the ecological system as a whole rest on this link, as is universally acknowledged. Despite its high regard as the most trustworthy model of predator-prey dynamics, the Lotka-Voltera model has an inherent instability by Liu et al. 2009.

Learning about the dynamics between predators and prey is a major focus for bio mathematicians (Kent et al. 2003; Wang et al. 2011; Gao et al. 2013). The concept has been supported and strengthened by a large body of research up to this time (Celik et al. 2009; Hsu et al. 2001). According to some experts, a new prey-predator paradigm could be formed by zooplankton, phytoplankton, and fish (Chakraborty et al. 2012). The model's parameters dictate the predators' dynamical behaviours in prey-predator systems. Oscillations, stable states, and bifurcations are examples of events that fit this description (Faria et al. 1995). Several ecologists and mathematicians have investigated the population dynamics of this system (Ma et al. 2012; Pei et al. 2008). Research into the dynamics of predator-prey systems is made possible using mathematical population models. They released the Lotka-Volterra model, a simple predator-prey model. Predation links are not as common as partnerships between other kinds of animals. Since the relationship was unaffected by any significant outside forces, their population changes could be predicted (Bentout et al. 2021; Bentout et al. 2021; Mezouaghi et al. 2022). The notion that all organisms in an ecosystem must fight for a finite resource is the primordial premise of all forms of competition. When different species compete for the same resource, this is called "intraspecific competition" or "interspecific competition (Djilali et al. 2021; Agiza et al. 2009). A dynamic and intricate web of relationships between predators and their prey is exposed by this dynamic ecological web. These systems are crucial to the ecosystem's well-being. These systems are affected by factors including as population size, resource availability, and predator-prey interactions. The differential and difference equations are used in many models that account for the dynamics of each population. When there is no overlap between generations in a population, discrete-time models outperform continuous-time ones (Din et al. 2017; Zhao et al. 2018; Fang et al. 2018; Khan et al. 2019). Studies in ecology have looked at predator-prey relationships as well as herbivore-plant interactions (Liu et al. 2007; Freedman et al. 1984). The distinctive dynamical dynamics of three-species models first baffled theoretical ecologists. Consequently, that happened. As the number of dimensions and differential equations keeps growing, theorists and experimenters will encounter many new challenges. Considering the prevalence of three-species groups, this must be carefully considered. More and more natural domains are exhibiting three-species systems (Erbe et al.1986; Kumar et al. 1989; Kuznetsov et al. 1996). There are a number of examples of this, such as a plant, an omnivore, and a parasite (Maiti et al. 2005), or a plant, a pest, and a predator (Srinivasu et al. 2007). Whenever a vertebrate senses the potential danger posed by a predator, it triggers one of its several antipredator reactions. Examples of these reactions include changes in the body's function, foraging behaviour, environmental use, and alertness level (Sarkar et al. 2020; Peacor et al. 2013; Khajanchi et al. 2017; Pettorelli et al. 2011). For example, birds may not try to return to their nests if they feel threatened, which means their young are defenceless. This might increase their chances of survival in the near term, but it could hurt the population in the long run (Tiwari et al. 2020).It is possible to gain a more thorough understanding of the complex ecological dynamics at play when predators' scavenging behaviours are considered alongside prey's adaptation behaviour (Bhattacharyya et al. 2022; Zhou et al. 2019; Hamdallah et al. 2021). An improved ecological representation of the species might be achieved by incorporating prey-refuging strategy switching into the already-existing Filippov model (Li et al. 2021). Certainly, this is doable. The density of predators and prey in the ecosystem is compared to identify this flipping. These results demonstrate the importance of threshold-based refuge seeking judgements (Liu et al. 2009), which are vital for the ongoing maintenance of ecological stability and the safeguarding of species. Interdependent ecological systems, such as those involving predators and prey, are dynamic and intricate. Maintaining ecological balance is next to impossible in the absence of these systems. Population size, resource availability, predator-prey ratio, and other variables all play a role in how these systems function. The article examines the dynamics of system with one prey and one predator. Stability study is essential to understanding predator and prey population dynamics because it illuminates their long-term behaviour. Stability studies can determine if predator and prey populations would converge or fluctuate. How stable these systems are may reveal predator-prey dynamics and how they affect ecosystem stability. Overall, this articleis introduced predator-prey systems, their behaviour, and how stability analysis can illuminate and predict their future.

2. Formulation of One Prey-One Predator Model

Assume two species (tiger and deer) occupy the same ecosystem as prey (deer) and predator (tiger). Let N_1 be the tiger population and N_2 be the deer population. In the absence of the deer, the tiger population is decreased by the rate $-\rho N_1$.But when deer are present in the environment then,it seems reasonable that the number of encounters or interactions between these two species is jointly proportional to their population N_1 and N_2, that is proportionally to the product $N_1 N_2$. Thus the when deer are present then tigers are added to the system at a rate $\alpha N_1 N_2$. With the tiger population decimated, a logistic model may now be used to describe the behaviour of the deer. This means that during this time period, the deer population's birth and mortality rates will change by an amount equal to $\mu(\delta - N_2)$. So, with no tigers the differential equation model for the deer's is $N_2' = \mu(\delta - N_2)N_2$. If there is a nonzero tiger population, then there will be an additional death rate for the deer population so that the difference between the birth rate and the death rate is equal to $\mu(\delta - N_2) - \beta N_1$. The constants μ, δ and β must be determined from careful observations of the populations. The new mathematical model is given by,

$$\left.\begin{aligned} N_1' &= (-\rho + \alpha N_2)N_1; \qquad & N_1(0) = N_{10} \\ N_2' &= (\mu(\delta - N_2) - \beta N_1)N_2; & N_2(0) = N_{20} \end{aligned}\right\} \tag{1}$$

Here, μ is the growth rate constant for deer, δ stands for the maximum number of deer that a certain ecosystem can support, β is the death rate constant due to tiger predation, ρ is the death rate constant for tigers, and α is the efficiency of tiger predation (i.e., the proportion of encounters between a tiger and a deer that result in a successful kill).

3. Stability of One Prey-One Predator Model

To begin, we think about (1), a system of nonlinear mathematical equations. The main goal of this effort is to find the stable (equilibrium) positions. In order to make this work,

$$P = (-\rho + \alpha N_2)N_1 = 0$$

$$Q = (\mu(\delta - N_2) - \beta N_1)N_2 = 0$$

The points of equilibrium are so found

$$(N_1, N_2) = \left\{(0,0),(0,\delta),\left(-\frac{\mu(\rho-\delta\alpha)}{\beta\alpha}, \frac{\rho}{\alpha}\right)\right\}$$

(N_1, N_2) means Predator-prey population.

Table 1: Behaviour of species along equilibrium points.

Equilibrium Points	**Predator (N_1)**	**Prey (N_2)**	**Behaviour of Species**
$E_1(0,0)$	0	0	Both species extinct
$E_2(0,\delta)$	0	δ	Predator goes extinct Prey prevails
$E_3(\frac{-(\mu(\rho-\delta\alpha)}{\beta\alpha}, \frac{\rho}{\alpha})$	$\frac{-(\mu(\rho-\delta\alpha)}{\beta\alpha}$	$\frac{\rho}{\alpha}$	Both species co-exist

It is possible to define the matrix of Jacobian form as:

$$J_{(u,v)} = \begin{pmatrix} P_{N_1} & P_{N_2} \\ Q_{N_1} & Q_{N_2} \end{pmatrix}$$

Where, $P = (-\rho + \alpha N_2)N_1$ and $Q = (\mu(\delta - N_2) - \beta N_1)N_2$.

Now differentiation of both P and Q with respect to N_1 and N_2 respectively and as a result, and the matrix that is known as the Jacobian is

$$J_{(u,v)} = \begin{pmatrix} \alpha N_2 - \rho & \alpha N_1 \\ \beta N_2 & \mu(\delta - N_2) - \mu N_2 - \beta N_1 \end{pmatrix}$$

Case 1: For the fixed point $(0,0)$

$$J_{(0,0)} = \begin{pmatrix} -\rho & 0 \\ 0 & \mu \end{pmatrix}$$

The eigenvalues for this particular instance are $-\rho$ and $\delta\mu$, which are real and one of them is negative and the other is positive since ρ, δ and μ is all positive constants. Then $(0,0)$ are saddle point and therefore, the trivial equilibriums always unstable.

Case 2: For the fixed point $(0,\delta)$

$$J_{(0,M)} = \begin{pmatrix} \delta\alpha - \rho & 0 \\ -\delta\beta & -\delta\mu \end{pmatrix}$$

The eigenvalues for this particular instance are $\delta\alpha - \rho(\geq 0, \because \delta > \rho)$ and $-\delta\mu$, which are real and one of them is negative and the other is positive since ρ, δ and μ are all positive constants. Then δ is saddle point and therefore, the equilibriums always unstable.

Case 3: For the fixed point $(\frac{-(\mu(\rho-\delta\alpha)}{\beta\alpha}, \frac{\rho}{\alpha})$

$$J_{(0,M)} = \begin{pmatrix} 0 & \frac{-(\mu(\rho-\delta\alpha)}{\beta} \\ \frac{-\beta\rho}{\alpha} & \frac{-\mu\rho}{\alpha} \end{pmatrix}$$

In this case the eigenvalues are $\frac{-\rho\mu + \sqrt{\rho\mu(-4\delta\alpha^2 + 4\rho\alpha + \rho\mu)}}{2\alpha}$ and $\frac{-\rho\mu - \sqrt{\rho\mu(-4\delta\alpha^2 + 4\rho\alpha + \rho\mu)}}{2\alpha}$, if the real component of the two eigenvalues is negative, we can find out if the state of equilibrium points is stable.

Proposition 1: *The equilibrium states* $E_1(0,0)$ *;* ρ *,* α *,* N_1, N_2 μ *,* δ *>0 of prey-predator model (1) is unstable as* $\lambda_1 = -\rho$ *and* $\lambda_2 = \delta\mu$ *.*

<u>Proof</u>: From model (1) we get, $E_1(0,0)$ to recapitulate the stability setting

$$\left.\begin{aligned} N_1 &= 0 + \omega_1(t) \\ N_2 &= 0 + \omega_2(t) \end{aligned}\right\} \quad (2)$$

From the equation (1) and (2) we get,

$$\Rightarrow \begin{aligned} N_1' &= (-\rho + \alpha\omega_2(t))\omega_1(t) \\ N_2' &= (\mu(\delta - \omega_2(t)) - \beta\omega_1(t))\omega_2(t) \end{aligned}$$

$$\Rightarrow \begin{aligned} N_1' &= -\rho\omega_1 + \alpha\omega_1\omega_2 \\ N_2' &= \mu\delta\omega_2 - \mu\omega_2{}^2 - \beta\omega_1\omega_2 \end{aligned}$$

$$\Rightarrow \begin{aligned} N_1' &\approx -\rho\omega_1 \\ N_2' &\approx \mu\delta\omega_2 \end{aligned}$$

In matrix form this can be written as,

$$\begin{pmatrix} N_1' \\ N_2' \end{pmatrix} \approx \begin{pmatrix} -\rho & 0 \\ 0 & \delta\mu \end{pmatrix} \begin{pmatrix} \omega_1 \\ \omega_2 \end{pmatrix}$$

The characteristic matrix is

$$\begin{pmatrix} -\rho & 0 \\ 0 & \delta\mu \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

The characteristic equation is

$$\begin{vmatrix} -\rho - \lambda & 0 \\ 0 & \delta\mu - \lambda \end{vmatrix} = 0$$

$$\Rightarrow (-\rho - \lambda)(\delta\mu - \lambda) = 0$$

Either $-\rho - \lambda = 0 \Rightarrow \lambda = \rho$ or $\delta\mu - \lambda = 0 \Rightarrow \lambda = \delta\mu$.

Proposition 2: *The equilibrium states* $E_2(0,\delta)$ *;* ρ *,* α *,* N_1, N_2 μ *,* δ *>0 of prey-predator model (1) is unstable as* $\lambda_1 = -d + \alpha\delta$ *and* $\lambda_2 = -\delta\mu$ *.*

<u>Proof</u>: From model (1) we get, $E_2(0,M)$ to recapitulate the stability setting

$$\left.\begin{aligned} N_1 &= 0 + \omega_1(t) \\ N_2 &= \delta + \omega_2(t) \end{aligned}\right\} \quad (3)$$

From the equation (1) and (3) we get,

$$\Rightarrow \begin{aligned} N_1' &= (-\rho + \alpha(\delta + \omega_2)\omega_1 \\ N_2' &= (\mu(\delta - \delta - \omega_2) - \beta\omega_1)(\delta + \omega_2) \end{aligned}$$

$$\Rightarrow \begin{aligned} N_1' &= -\rho\omega_1 + \alpha\delta\omega_1 + \alpha\omega_1\omega_2 \\ N_2' &= (-\mu\omega_2 - \beta\omega_1)(\delta + \omega_2) \end{aligned}$$

$$\Rightarrow \begin{aligned} N_1' &= -\rho\omega_1 + \alpha\delta\omega_1 + \alpha\omega_1\omega_2 \\ N_2' &= (-\mu\delta\omega_2 - \mu\omega_2{}^2 - \delta\beta\omega_1 - \beta\omega_1\omega_2) \end{aligned}$$

$$\Rightarrow \begin{aligned} N_1' &= (-\rho + \alpha\delta)\omega_1 \\ N_2' &= -\delta\beta\omega_1 - \mu\delta\omega_2 \end{aligned}$$

In matrix form this can be written as,

$$\begin{pmatrix} N_1' \\ N_2' \end{pmatrix} \approx \begin{pmatrix} -\rho+\alpha\delta & 0 \\ -\beta\delta & -\mu\delta \end{pmatrix} \begin{pmatrix} \omega_1 \\ \omega_2 \end{pmatrix}$$

The characteristic matrix is

$$\begin{pmatrix} -\rho+\alpha\delta & 0 \\ -\beta\delta & -\mu\delta \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

The characteristic equation is

$$\begin{vmatrix} -\rho+\alpha\delta-\lambda & 0 \\ -\beta\delta & -\mu\delta-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (-\rho+\alpha\delta-\lambda)(-\mu\delta-\lambda) = 0$$

Either $(-\rho+\alpha\delta-\lambda) = 0 \Rightarrow \lambda = -\rho+\alpha\delta$ or $-\mu\delta-\lambda = 0 \Rightarrow \lambda = -\mu\delta.$

Proposition 3: *The equilibrium states* $E_3(\frac{-(\mu(\rho-\delta\alpha)}{\beta\alpha}, \frac{\rho}{\alpha})$ *;* $\rho, \alpha, N_1, N_2\ \mu, \delta$ *>0 of prey-predator model(1) is unstable as* $\lambda_1 = \frac{-\rho\mu + \sqrt{\rho\mu(-4\delta\alpha^2 + 4\rho\alpha + \rho\mu\ )}}{2\alpha}$ *and* $\lambda_2 = \frac{-\rho\mu - \sqrt{\rho\mu(-4\delta\alpha^2 + 4\rho\alpha + \rho\mu\ )}}{2\alpha}$.

Proof: From model (1) we get, $E_3(\frac{-(\mu(\rho-\delta\alpha)}{\beta\alpha}, \frac{\rho}{\alpha})$ to recapitulate the stability setting

$$\left. \begin{aligned} N_1 &= \frac{-(\mu(\rho-\delta\alpha)}{\beta\alpha} + \omega_1(t) \\ N_2 &= \frac{\rho}{\alpha} + \omega_2(t) \end{aligned} \right\} \quad (4)$$

From the equation (1) and (4) we get,

$\Rightarrow$

$$N_1' = (-\rho+\alpha(\frac{\rho}{\alpha}+\omega_2))(\frac{-(\mu(\rho-\delta\alpha)}{\beta\alpha}+\omega_1)$$

$$N_2' = (\mu(\delta-(\frac{\rho}{\alpha}+\omega_2)) - \beta(\frac{-(\mu(\rho-\delta\alpha)}{\beta\alpha}+\omega_1))(\frac{\rho}{\alpha}+\omega_2)$$

$\Rightarrow$

$$N_1' = \alpha\omega_2(\frac{-(\mu(\rho-\delta\alpha)}{\beta\alpha}+\omega_1)$$

$$N_2' = (\mu\delta - \frac{\mu\rho}{\alpha} + \mu\omega_2 + \frac{\mu(\rho-\delta\alpha)}{\alpha} - \beta\omega_1)(\frac{\rho}{\alpha}+\omega_2)$$

$\Rightarrow$

$$N_1' = \frac{-\mu(\rho-\delta\alpha)}{\beta}\omega_2 + \alpha\omega_1\omega_2)$$

$$N_2' = (\frac{\mu\delta\rho}{\alpha} - \frac{\mu\rho^2}{\alpha^2} + \frac{\mu\rho(\rho-\delta\alpha)}{\alpha^2} - \frac{\beta\rho}{\alpha}\omega_1 + \mu\delta\omega_2 + \mu\omega_2^{\ 2} + \frac{\mu(\rho-\delta\alpha)}{\alpha}\omega_2 - \beta\omega_1\omega_2)$$

$$\Rightarrow \begin{aligned} N_1' &\approx \frac{-\mu(\rho-\delta\alpha)}{\beta}\omega_2 \\ N_2' &\approx -\frac{\beta\rho}{\alpha}\omega_1 + \frac{\mu\rho}{\alpha}\omega_2 \end{aligned}$$

One way to express this in matrix form is as follows:

$$\begin{pmatrix} N_1' \\ N_2' \end{pmatrix} \approx \begin{pmatrix} 0 & \frac{-\mu(\rho-\delta\alpha)}{\beta} \\ -\frac{\beta\rho}{\alpha} & \frac{\mu\rho}{\alpha} \end{pmatrix} \begin{pmatrix} \omega_1 \\ \omega_2 \end{pmatrix}$$

The characteristic equation is

$$\left| \begin{pmatrix} 0 & \frac{-\mu(\rho-\delta\alpha)}{\beta} \\ -\frac{\beta\rho}{\alpha} & \frac{\mu\rho}{\alpha} \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right| = 0$$

$$\Rightarrow \begin{vmatrix} -\lambda & \frac{-\mu(\rho-\delta\alpha)}{\beta} \\ -\frac{\beta\rho}{\alpha} & \frac{\mu\rho}{\alpha} - \lambda \end{vmatrix} = 0$$

$$\Rightarrow \lambda^2 - \frac{\mu\rho}{\alpha}\lambda - \frac{\rho\mu(\rho - \delta\alpha)}{\alpha} = 0$$

$$\therefore \lambda = \frac{\frac{\mu\rho}{\alpha} \pm \sqrt{\left(-\frac{\mu\rho}{\alpha}\right)^2 + 4\frac{\rho\mu(\rho - \delta\alpha)}{\alpha}}}{2}$$

$$\Rightarrow \lambda = \frac{-\rho\mu \pm \sqrt{\rho\mu(-4\delta\alpha^2 + 4\rho\alpha + \rho\mu)}}{2\alpha}$$

Either

$$\lambda_1 = \frac{-\rho\mu + \sqrt{\rho\mu(-4\delta\alpha^2 + 4\rho\alpha + \rho\mu)}}{2\alpha} \quad \text{or}$$

$$\lambda_2 = \frac{-\rho\mu - \sqrt{\rho\mu(-4\delta\alpha^2 + 4\rho\alpha + \rho\mu)}}{2\alpha}$$

4. Numerical Simulation

Analytical investigations are always being considered incomplete unless numerical verification of the results is included. An example of the findings presented in this part of the article is the use of numerical simulation, which is detailed further on. The numerical simulation was implemented using MATLAB R2020a as the work environment. Based on the model that has been given, the phrase "predator" is linked to the symbol y (1) and the term "prey" is related to the symbol y (2) in the yprf2.m file. As an example, by plotting a solution, the rationale behind the two species' cyclical population patterns. Doing this is within reach. Discovering a state of dynamic equilibrium is possible at the site of this specific solution. At each particular time, there are systems on this phase plane that are located within these elliptical solutions and are in a limit cycle. On this phase plane, it can find these systems. This phase plane clearly displays the existence of these systems. To start in a limit cycle and, consequently, a stable solution, the system does not have to meet any specific requirements. The reason behind this is that the system has already achieved a stable solution. Regardless, it is reached one in the end. This is the polar opposite of that.

Now that we've considered that; this time, it has employed numerous parameters, we may move on to discussing the model graphs.

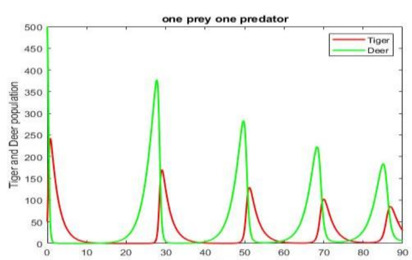

Fig.1: Dynamical Behaviour of prey-predator model 1); Assume $\rho = 0.5, \alpha = 0.01, \mu = 0.0005, \delta = 1000, \beta = 0.02.$

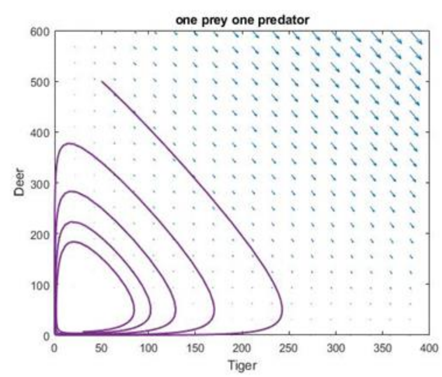

Fig. 1(a): Phase portrait of Fig. 1.

The phase plane plot does not depend on the timing of the comparison; it is used to compare the populations of predators and prey. Take the hypothetical situation where prey and predator animals as an example. If there are 500 preys and 50 predators at the outset, we can determine how the two species have changed through time. The chosen time interval is entirely up to the discretion of the user. Here are the settings: What follows is a graph showing the relationship between the predator and prey populations over time, using to = 0, tf = 90, and y0 = [50 500]. A dramatic increase in predators is accompanied by a decrease in prey, as is shown clearly in this graph. The observation of this is not difficult. This makes complete biological sense, since an increase in the predator-prey interaction leads to a rise in the total number of prey deaths when the predator population grows.

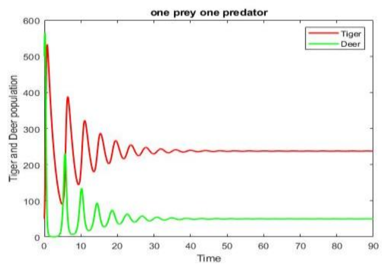

Fig. 2: Dynamical Behaviour of prey-predator model (1); Assume $\rho = 0.5, \alpha = 0.01, \mu = 0.005, \delta = 1000, \beta = 0.02.$

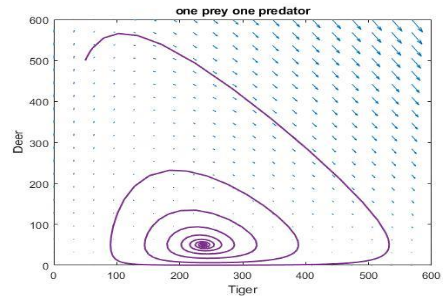

Fig. 2(a): Phase portrait of Fig. 2.

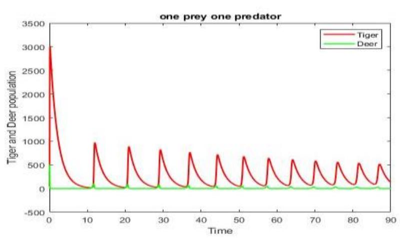

Fig. 3: Dynamical Behaviour of prey-predator model (1); Assume $\rho = 0.5, \alpha = 0.1, \mu = 0.005, \delta = 1000, \beta = 0.02.$

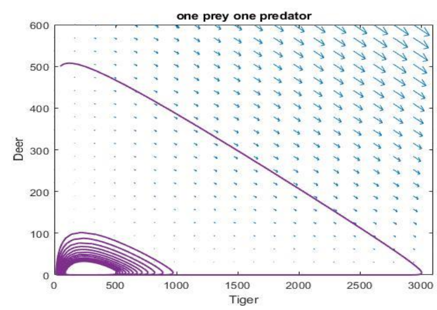

Fig. 3.(a): Phase portrait of Fig.3.

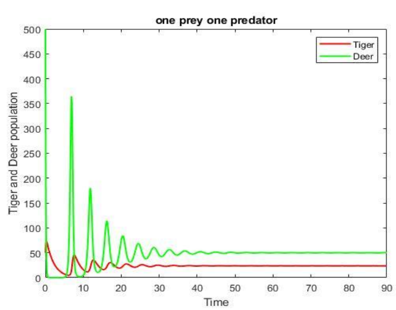

Fig. 4: Dynamical Behaviour of prey-predator model (1); Assume $\rho = 0.5, \alpha = 0.01, \mu = 0.005, \delta = 1000, \beta = 0.2.$

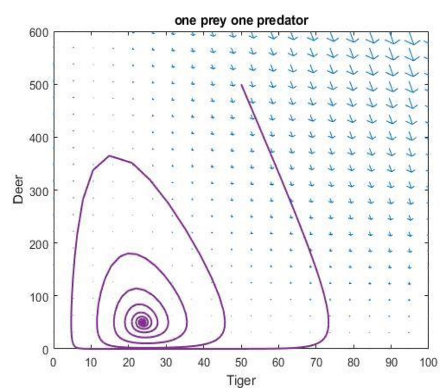

Fig. 4(a): Phase portrait of Fig. 4.

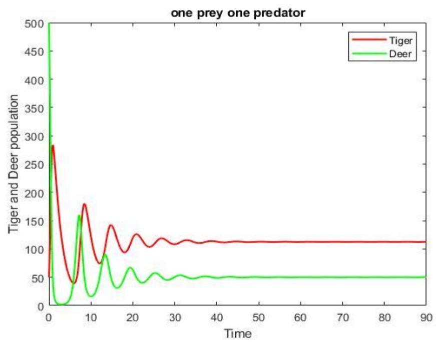

Fig. 5: Dynamical Behaviour of prey-predator model (1); Assume $\rho = 0.5, \alpha = 0.01, \ \mu = 0.005,$ $\delta = 500, \beta = 0.2.$

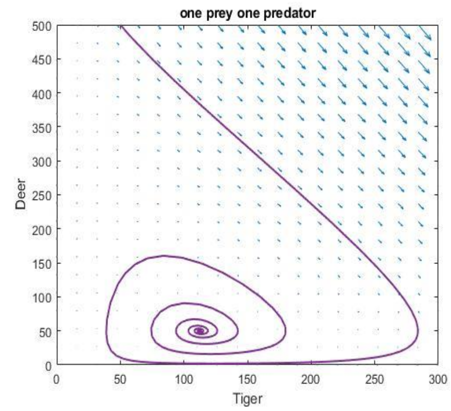

Fig. 5(a): Phase portrait of Fig. 5.

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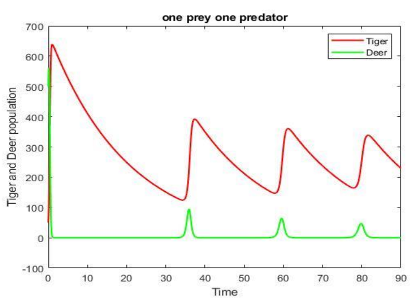

Fig. 6: Dynamical Behaviour **o**f prey-predator model (1); Assume $\rho = 0.05, \alpha = 0.01, \ \mu = 0.005,$ $\delta = 1000, \beta = 0.02.$

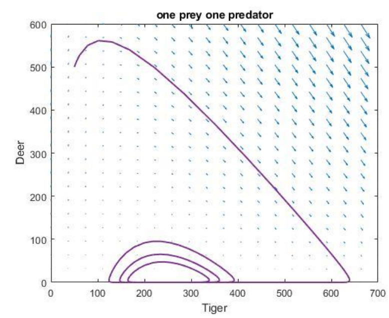

Fig. 6(a): Phase portrait of Fig. 6.

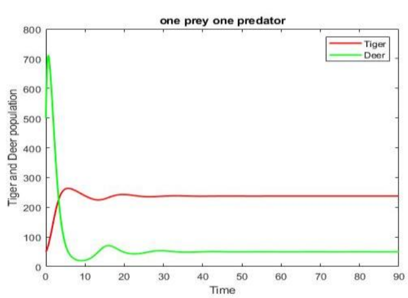

Fig. 7: Dynamical Behaviour of prey-predator model (1); Assume $\rho = 0.5, \alpha = 0.001, \ \mu = 0.0005,$ $\delta = 1000, \beta = 0.02.$

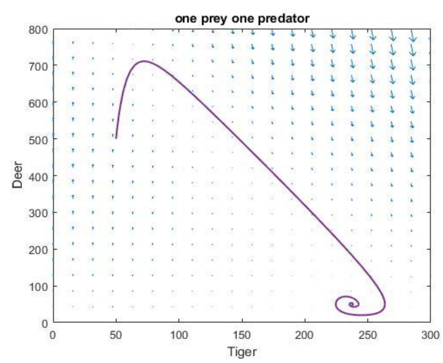

Fig. 7(a): Phase portrait of Fig.7.

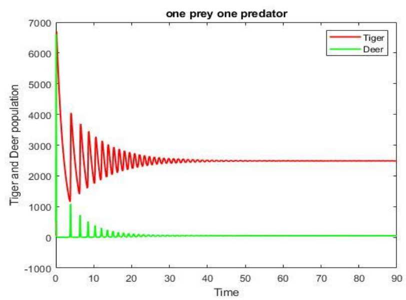

Fig. 8: Dynamical Behaviour of prey-predator model (1); Assume
$\rho = 0.5, \alpha = 0.01, \mu = 0.005,$
$\delta = 10000, \beta = 0.02.$

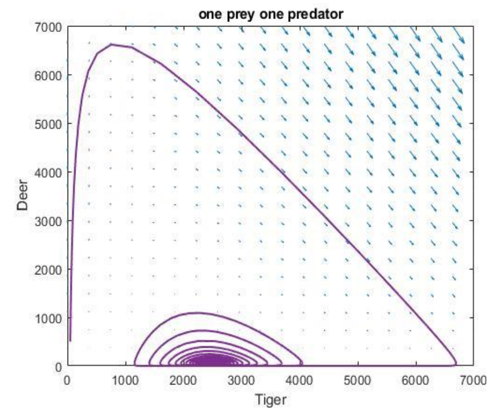

Fig. 8(a): Phase portrait of Fig. 8.

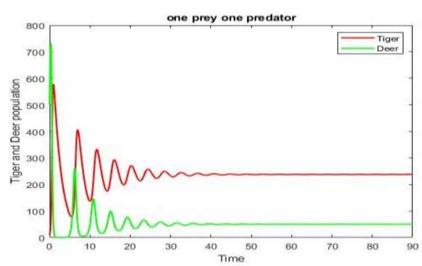

Fig. 9: Dynamical Behaviour of prey-predator model (1); Assume
$\rho = 0.5, \alpha = 0.01, \mu = 0.005,$
$\delta = 1000, \beta = 0.02, y_0 = [10, 500]$

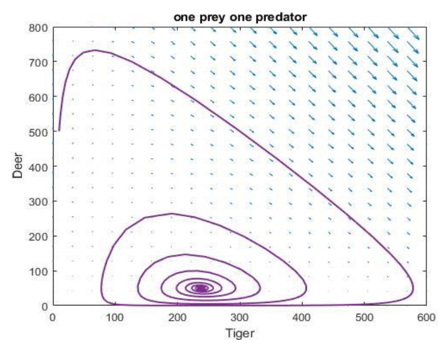

Fig. 9(a): Phase portrait of Fig. 9.

5. Results and Discussions

The line graphs provided depict the population of prey and predators over a 90-day period. The horizontal axis represents time in days, while the vertical axis represents the population of prey and predators. Both populations exhibit fluctuations throughout the 90-day timeframe. Upon examining the graphs, it becomes apparent that the populations of 'Tiger' and 'Deer' are highly influenced by various parameter values. Increasing the growth rate of deer leads to reduced oscillation in Fig.2. In Fig.3, enhancing the efficiency of tiger predation results in a significant increase in the tiger population, causing a noteworthy decrease in the deer population. Consequently, the tiger population declines, indicating an evident pattern. This process continues until the populations reach an equilibrium state. In Fig.4, raising the death rate of deer due to tiger predation leads to a substantial decrease in the deer population, as expected. Lowering the carrying capacity to 500 from 1000 in Fig. 5 causes the population to settle into a steady state with minimal oscillation. Decreasing the death rate constant for tigers in Fig.6 results in a lesser decrease in the tiger population throughout the graph. In Fig.7, reducing the efficiency of tiger predation causes the deer population to initially increase, followed by a quick transition to a steady state. Fig.8 demonstrates that increasing the carrying capacity tenfold leads to more pronounced oscillations before eventually reaching a steady state. Maintaining all parameters as in Fig.2, except for the initial condition, where the initial tiger population is set to 10 instead of 50, figure 9 produces nearly identical results to Fig. 2. Similarly, in Figure 10, preserving all parameters as in Fig. 2, except for the initial condition where the initial deer population is set to 1000 instead of 500, the outcome in figure 10 closely resembles figure 2. This implies that the initial conditions have minimal impact on the populations.

6. Conclusion

A model is nothing more than an abridged representation of a more complex system. Species interactions are one of biology's most fascinating driving forces. We need mathematical models that take interactions into consideration in order to simulate these processes. Every graph demonstrates how, when phase images are examined, our numerical solution is getting closer to the steady-state solution. Predator and prey ecological population cycles do not appear to be stabilizing very quickly or dramatically. The dynamic behavior is shown to be highly influenced by the real-world parameters as well as the parameter values. The species involved in this predator-prey relationship operate in relatively closed settings, which accounts for the paucity of available evidence.So, even though there weren't any major outside factors that could change the relationship, population variations still happened on a regular cycle. Techniques of linear ordinary differential equations allow us to characterise and interpret these population fluctuations quantitatively; these equations, however, do rely on certain simplifying assertions that exclude incalculable variables.While tiger predation improves, the tiger population grows, reducing the deer population. Tiger numbers are declining in a recognizable way as a result of this. This would continue until demographic equilibrium is reached. The stability of the system with regard to each equilibrium point is influenced by the values of the eigen parameters.

References

Agiza H, ELabbasy E, Metwally H, Elsadany A. 2009. *"Chaotic dynamics of a discrete prey–predator model with Holling type II,"Nonlinear Analysis*. *Real World Applications*, 10(1): 116–129.

Bainov DD, Simeonov PS. 1989. Systems with Impulse Effect: Stability Theory and Applications, Ellis Horwood Limited, Chichester, 1st edition.

Brauer F, Soudack AC. 1979. "Stability regions and transition phenomena for harvested predator–prey systems", *Journal of Mathematical Biology*, 7: 319–337.

Braue F, Soudack AC. 1979. "Stability regions in predator–prey systems with constant prey harvesting", *Journal of Mathematical Biology*, 8: 55–71.

Brauer F, Soudack AC. 1981. "Constant-rate stocking of predator–prey systems", *Journal of Mathematical Biology*, 11: 1–14.

Brauer F, Soudack AC. 1981. "Coexistence properties of some predator–prey systems under constant rate harvesting", *Journal of Mathematical Biology*, 12: 101–114.

Bentout S, Tridane A, Djilali S, Touaoula T. 2021. "Age-structured modeling of COVID-19 epidemic inthe USA, UAE and Algeria." *Alexandria Engineering Journal*, 60: 401–411.

Bentout S, Chen Y, Djilali S. 2021. "Global dynamics of an Seir model with two age structures and a nonlinear incidence". Acta Applicandae Mathematicae, 171(1): 1–27.

Bhattacharyya J, Piiroinen PT, Banerjee S. 2022. "Effects of predator-driven prey dispersal on sustainable harvesting yield", *International. Journal of Bifurcation and Chaos*, 32(11): 2250160.

Celik C, Duman O. 2009. "Allee effect in a discrete-time predator system", Chaos, Solitons and Fractals, 40(4): 1956-1962.

Chakraborty K., Jana S, Kar TK. 2012. "Global dynamics and bifurcation in a stage structured prey-predator fishery model with harvesting", *Applied Mathematics and Computation*. 218: 9271–9290.

Djilali S, Bentout S. 2021. "Pattern formations of a delayed diffusive predator–prey model with predator harvesting and prey social behaviour", *Mathematical. Methods in Applied Sciences*,44(11), 9128–9142.

Din Q. 2017. "Complexity and chaos control in a discrete-time prey–predator model", *Communications Nonlinear Science and Numerical Simulation*, 49: 113–134.

Erbe L, Freedman H, Rao VSH. 1986. "Three-species food-chain models with mutual interference and time delays", *Mathematical Biosciences* 80(1): 57–80.

Faria T, Magalhaes LT. 1995. Normal forms for retarded functional differential equations with parameters and Applications to Hopf bifurcations. Journal of Differential. Equation, 122: 181–200.

Fang Q, Li X. 2018. "Complex dynamics of a discrete predator–prey system with a strong Allee effect on the prey and a ratio-dependent functional response", *Advance Differential Equations*, 320(1): 1–16.

Freedman H, Waltman P. 1984. "Persistence in models of three interacting predator-prey populations", *Mathematical Biosciences* 68(2): 213–231.

Guckenheimer J, Holmes P. 1983. Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields, Springer-Verlag, New York, 1st edition.

Gao Y, Li B. 2013. "Dynamics of a ratio dependent predator-prey system with a strong Allee effect", *Discrete Continuous Dynamical Systems*, 18(9): 2283-2313.

Hsu SB, Hwang TW, Kuang Y. 2001. "Global analysis of the Mechaelis Menten type ratio-dependent predator-prey system", *Journal of Mathematical Biology*, 42(6): 489-506.

Hamdallah, SA, Arafa AA, Tang S, Xu Y. 2021. "Complex dynamics of a Filippov three-species food chain model', International *Journal of Bifurcation and Chaos***,** 31(05), 2150074.

Kermack WO, Mckendrick AG. 1927. "Contributions to the mathematical theory of epidemics", part 1, Proceedings of the Royal Society, 115 (5): 700–721.

Kent A, Doncaster, Sluckin CPT. 2003. "Conquences for predators of rescue and Allee effects on prey", *Ecological Modelling*, 162(3): 233-245.

Khan AQ. 2019. "Bifurcations of a two-dimensional discrete-time predator–prey model", *Advance Differential Equations*, 56(1): 1–23.

Kumar R, Freedman H. 1989. "A mathematical model of facultative mutualism with populations interacting in a food chain," *Mathematical Biosciences*, 97(2): 235–261.

Kuznetsov YA, Rinaldi S. 1996. Remarks on food chain dynamics. *Mathematical Biosciences*, 134(1): 1–33.

Khajanchi S. 2017. "Modeling the dynamics of stage-structure predator-prey system with Monod-Haldane type response function", *Applied Mathematical and Computation*, 302: 122–143.

Lsksmikantham V, Bainov DD, Simeonov PS. 1980. "Theory of Impulsive Differential Equations", *World Scientific*, Singapore, 3: 40-52.

Leine RI, Van CDH, Van De VBL. 2000. "Bifurcations in nonlinear discontinuous systems", *Nonlinear Dynamics*, 23: 105–164.

Liu XN, Chen LS. 2003. "Complex dynamics of Holling type II Lotka–Volterra predator-prey system with impulsive perturbations on the predator", *Chaos Solitons and Fractals*, 16: 311–320.

Lu ZH, Chi XB, Chen LS. 2002. "The effect of constant and pulse vaccination on SIR epidemic model with horizontal and vertical transmission", *Mathematical andComputer Modelling*, 36: 1039–1057.

Lansun C. 1988. *Models and Research Methods of Mathematical Ecology*, Science Press, Beijing. 1st edition.

Liu X, Xiao D. 2007. "Complex dynamic behaviours of a discrete-time predator–prey system", *Chaos Solitons and Fractals*, 32(1), 80–94.

Liu H, Li Z, Gao M, et al. 2009. "Dynamics of a host-parasitoid model with llee effect for the host and parasitoid aggregation", *Ecological Complexity*, 6(3): 337-345.

Li W, Huang L, Wang J. 2021. "Global asymptotical stability and sliding bifurcation analysis of a general Filippov-type predator-prey model with a refuge", *Applied Mathematical and Computation*. 405: 126263.

Liu Q, Liao X, Liu Y, Zhou S, Guo S. 2009. "Dynamics of an inertial two-neuron system with time delay", *Nonlinear Dynamics*, 58: 573–609.

Ma Z., Wang Y., Jiang G. 2012. "Bifurcation analysis of a linear Hamiltonian system with two kinds of impulsive control", *Nonlinear Dynamics*, 70: 2367–2374.

Mezouaghi A., Djilali S, Bentout S, Biroud K. 2022. "Bifurcation analysis of a diffusive predator–prey model with prey social behavior and predator harvesting", *Mathematical. Methods in Applied Sciences*, 45(2): 718–731.

Maiti A, Samanta G. 2005. "Deterministic and stochastic analysis of a prey-dependent predator prey system", *International Journal of Mathematical Education in Science*, 36(1): 65–83.

Pei Y, Zeng G, Chen L. 2008. "Species extinction and permanence in a prey–predator model with two-type functional responses and impulsive biological control", *Nonlinear Dynamics*. 52: 71–81.

Peacor SD, Peckarsky BL, Trussell GC, Vonesh JR. 2013. "Costs of predator-induced phenotypic plasticity: a graphical model for predicting the contribution of no consumptive and consumptive effects of predators on prey", *Oecologia*: 171, 1–10.

Pettorelli N, Coulson T, Durant SM, Gaillard JM. 2011. "Predation, individual variability and vertebrate population dynamics", *Oecologia*, 167: 305–314.

Raid K, NajiAN, Mustafa. 2012., "The Dynamics of an Eco-Epidemiological Model with Nonlinear Incidence Rate", *Journal of Applied Mathematics*, Article ID 852631: 24.

Srinivasu P, Prasad B, Venkatesulu M. 2007. "Biological control through provision of additional food to predators: a theoretical study", *Theoretical Population Biology*, 72(1): 111–120.

Sarkar K, Khajanchi, S. 2020. "Impact of fear effect on the growth of prey in a predator-prey interaction model", *Ecological Complexity*, 42: 100826.

Tiwari PK, Singh RK, Khajanchi S, Kang Y, Misra AK. 2020. "A mathematical model to restore water quality in urban lakes using phoslock", *Discrete Continuous Dynamical. Systems*, 26(6): 3143–3175.

Volterra V. 1926. "Variazioni e fluttauazionidelnumero d individui in species animals conviventii, Memoria. Reale, Accademia. *Nazionale deiLinciei* 2: 31–33.

Wang WX, Zhang YB, Liu CZ. 2011. "Analysis of a discrete time predator prey system with Allee effects", *Ecological Complexity*, 8(1): 81-85.

Zhou H, Wang X, Tang S. 2019. "Global dynamics of non-smooth Allee pest-natural enemy system with constant releasing rate", *Mathematical Biosciences and Engineering*,16(6): 7327–7361.