



Distortionary tax and general equilibrium analysis of welfare maximized private and public goods consumption

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Abstract

There is a debate on economic efficiency and the improvement of economic welfare. In this research, I have fixed the value of rich and poor with equal weight with an assumption that the consumption tax is the source of government expenditure for public goods. This paper optimized improvement of equality and private consumption share of public goods with the prime revenue of consumption tax. This optimization process has also been analyzed in accordance with the theoretical assumption of Roy's identity and Marshallian ordinary market demand function for justification of equity and welfare. Finally, this process compared implicitly to the process of Kuznets pattern economic development with taking assumption of distortionary consumption tax to penetrate the relationship between long term economic growth and economic welfare. The empirical evidence found in my earlier publication that consumption tax is welfare augmented in long run if the source of government expenditure is consumption tax to produce public goods. This welfare maximized general equilibrium evidences have potential opportunity of welfare augmented resource mobilization in a developing country where consumption tax is the source of prime revenue.

Key words: Welfare maximization, consumption tax, private and public goods, Roy's identity

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Introduction

We know from Kuznets hypothesis is that as economic growth increase income inequality will be worsen in the initial stage and then increase and after a certain turning point it will decrease. There are theoretical presumptions of this hypothesis, that why economic growth increase in initial stage and then declined after a certain turning point. A number of different economic explanations for this relationship have been clarified involving variety of factors such as productivity changes, differential savings behavior, exploitations of workers, and so forth. The inverted U hypothesis can be derived with an assumption that the economy can be divided into two different sectors with different

pectoral income distributions and that there is a monotonic increase in the relative population of one of the sectors over time.

So far these assumptions are empirically justified in a country undergoing economic development and are consistent with many models of economic development theory such as Lewis-Fei-Rains surplus labour model, Dual economic models and Harrier-Todaro migration model. It is logical to consider that the economic development process of a country has been continuing without any intervention of the government policies if we consider these economic theories as an assumption of inverted U hypothesis.

But the real situation is different. Because we have a procedure of value judgment of the people as well as the income of the people has been taxed, and accordingly government has a role for infrastructure development, keeping the equal functional distribution of income to sustain the economic welfare. One country may keep the economic welfare with equal distribution in long time with U turning shape if the government considers re-distributive policy to the stage of economic development.

Now a day, the vital issue of public policy is to determine the size of the government, the activity to sustain the stability of the society, and improvement of economic welfare. This research optimized the theoretical model stating the arguments of economic efficiency under the assumption of consumption tax.

Finally, this process integrated implicitly to the process of Kuznets pattern long term economic development with taking assumption of distortionary consumption tax as an inverted U hypothesis. To the literacy of inverted U hypothesis relating inequality and economic development, Sherrman (1976), produced a theoretical model in relationship between inequality and income with the consistent of Lewis-Fei-Rains labour surplus models, Daul economy models and also satisfied the assumption of Kuznets hypothesis as a means of labour movement to the stage of economic development.

In Kuznet's literature, Saith (1983), Ahluwallia (1976), Rum (1988), Kanbar and Saidur (1993) have produced papers on inequality and economic growth to justify Kuznets hypothesis. Mushinski (2001), Ram (1989), Jha (1996), Hyunsub *et al.*, (2001) and Minami (1998) have also been published remarkable papers in relation between inequality and economic growth with the data of USA, UK and Germany. Wodon (1999) produced regarding survey of growth, inequality and poverty but he did not analyzed the long term relationship between inequality as Kuznets hypothesis suggested. Islam (2002) examined Kuznet's hypothesis to satisfy the pattern in developing country. He found evidence that by capturing structural change in a regression model

may satisfy the Kuznets hypothesis in developing country. But these literatures did not emphasis integrated theoretical aspects of welfare maximization. But I have analyzed here the equity and welfare maximized value judgment of private and public goods consumption with the implicit assumption of inverted U hypothesis of welfare maximization.

Since we know that Kuznets hypothesis is a stylized fact and this hypothesis presumed GDP as a proxy of income. This income is also presumed as identical demand for the society. In view of this, this paper has examined the welfare maximized consumption of private and public goods with the fixed assumption of distortionary consumption tax.

Materials and Methods

Let we consider that $W = E(U(y_1), \dots, U(y_n))$, where W represents utilitarian economic welfare. E is the function of U , that the positive change of y improves economic welfare. $E(U)$ represents the improvement function of equality. Here, y repents individual income or consumption. In this section, I have postulated four hypothesis of theoretical assumptions for welfare maximized consumption of private and public goods as following:

1. Taxing on consumption is distortionary only for bundles of private goods consumption

Definition and hypothesis: It is very difficult for the policy maker to provide emphasis on consumption tax for revenue maximization with a view to economic welfare. As we know that consumption tax is distortionary for welfare maximization. Production of public goods required public finance. Public finance organized by collecting revenue through consumption tax or tax on sales is not welfare maximized only for production bundles of private goods. We have theoretical evidence that consumer can achieve highest level of utility from income tax instead of consumption tax.

2. Consumption tax is welfare maximized for consumption of private and public goods

Definition and hypothesis: In this stage, I have maximized the consumption of private and public goods by adding consumption tax into the optimization process. Since the assumption of the public goods consumption is depending on the behavior of private goods consumption, therefore, we may presume that the national production of public goods depends on the individual behavior of private goods consumption. In a nutshell, we may say that as the private goods consumption increase, the consumption of public goods will also increase. The market price of private good is competitive and accordingly fixed by market. Same as the price for the public goods is fixed as the rate of consumption tax is fixed.

The preference of public goods consumption is non quasi-linear. As I discussed earlier, the individual budget constraint is the source of individual distribution of income. $y = c + \rho g + i$. Where: ρ is consumption tax rate, c is private consumption of an individual, g is the amount of public good consumption of an individual, i is the private savings of an individual. Each individual maximize their consumption of private and public goods in accordance with the aggregate constraint of identical income. Anyway, this assumption is hypothetical to reach the optimal level of equity through income distribution in a society where an individual may own all of national resource or less. But the people attain the distribution of national income with equal weight as wages and salaries.

Hence, the individual resource constraint will be in accordance with the distribution of national income. But in this situation, inequality of income may prevail in a society.

3. Consumption of public goods can't improve income equality without direct transfer

Definition and hypothesis: Now if we consider that the government expenditure has fixed only the production

of public goods. And accordingly, the rich and the poor attain the same utility from the production of public goods. In this case, we do not have the scope to think about the improvement of equality of income of a society under this assumption. The reason behind is that from this quasi linear preference, the rich and poor attain the same utility from the consumption of public goods but pay different prices. However, they attain the same price on private consumption and substitute difference price for consumption of public goods. But by this process, marginal direct income of the people does not change equally. In view of this, we may assume that the government is rational for the value of the production process of public goods. The price is also fixed by market mechanism. The peoples are also agreed on this price adjustment of public goods production.

It is very difficult to explain market price of private and public consumption in together as quasi linear choice of public goods consumption, but we may restrict this assumption as non quasi linear by considering the assumption of fixed market price for both private and public goods. This non quasi linear preference is applicable in the developing countries where government has been depending on the poor or average income peoples for their contribution of various consumption taxes. This non quasi linear assumption is also satisfies the Pareto optimization and the condition of economic efficiency. Finally, the above evidence proved that identical demand and consumption of private and public good improves equality. But the indirect utility from public good has no relation for improvement of equality except the assurance of fixed welfare of the people.

There is debate on evaluation of private and public consumption. In practice, public choice has been reflecting on the process of government taxation and production of public goods is a part of that social choice. But in theory, it has different assumption, in case of private goods we all consume different amounts of goods but pay the same price, but in public goods

case, we all consume same amount of good but have different prices in case of quasi linear preference. To determine the optimal condition of public good, we need to know each agent's willingness to pay for the public goods.

4. Welfare analysis of Roy's identity for private and public goods consumption

Definition and hypothesis: In this section, a theoretical proposition has been taken under the assumption of private and public goods consumption through taking assumptions of Roy's identity and Marshallain ordinary market demand function. These theories are considered to justify the relationship between equality, indirect utility and expenditure function. Here, I have fixed the assumption that consumption tax is the source of producing public goods. Since Hicksian demand is not observable, therefore, the equations integrated on the presumption of indirect utility and expenditure function.

Before going to analysis, the process of maximizing direct and indirect utility, we need to fix the argument for consistence of the above presumption. Let we

presume that $\sum_{i=1}^n \left(\frac{c}{g}\right)_i = \sum_{i=1}^n x_i$ is a cardinal process that

people are willing to substitute against private consumption to public goods consumption. Every consumption bundle of x consists of private and public goods. It is presumed that every marginal increase of private goods consumption will increase public goods consumption. The government will maximize the revenue through spending consumption tax. To go more details, let us prove hypothesis with equity assumption.

Results

In this section the postulated theoretical model of definition and hypothesis has been proved by the optimization techniques as following:

1. Taxing on consumption is distortionary only for bundles of private goods consumption

Proposition and Proof: Suppose we wish to tax a utility maximizing consumer to obtain a certain amount of revenue. Initially, the consumer's budget constraint is $p_1x_1 + p_2x_2 = y$. but after we impose a tax on sales of good 1, the consumer's budget constraint becomes $(p_1 + t)x_1 + p_2x_2 = y$. If we denote assumption after-tax, level of consumption by (x_1^*, x_2^*) , then the revenue collected by the tax is tx_1^* . Now if we decide to collect this same amount of revenue by a tax amount on income. The budget constraint of consumer would then be $p_1x_1 + p_2x_2 = m - tx_1^*$. This is a line with slope $-p_1 / p_2$ that passes through (x_1^*, x_2^*) . Notice that since the budget line cuts the indifference curve through (x_1^*, x_2^*) , the consumer can achieve a higher level of utility from an income tax, even though they both generate the same revenue (Varian 1992). Hence, theoretical assumption proved that consumer can get highest level of satisfaction from tax on income rather than tax on consumption if the bundle of consumption is private goods. We find that tax on consumption of private goods is distortionary and accordingly not welfare augmented.

2. Consumption tax is welfare maximized for consumption of private and public goods

Proposition and Proof: In view of this, we may maximize the optimal consumption of private and public goods with the constraint resource of consumption tax. The assumption is that all private and public goods are produced by the price of consumption tax.

Here, $\max U(c, g, \lambda)$

$$\text{st } \bar{p}_1 c + \bar{p}_2 g \leq [Y] - - 2 . 1$$

Where, c is private goods consumption, g is the amount of public goods consumption, p_1c is price of private goods consumption, p_2g is the price of public good consumption. Then, the individual maximize utility through the constraint of consumption tax:

$$U = f(c, g) + \lambda[Y] - \bar{p}_1 c - \bar{p}_2 g = 0 \quad \text{--- 2.2}$$

$$\frac{\partial U}{\partial c} = f'(c) - \lambda \bar{p}_1 = 0 \quad \text{--- 2.3}$$

$$\frac{\partial U}{\partial g} = f'(g) - \lambda \bar{p}_2 = 0 \quad \text{--- 2.4}$$

$$\frac{\partial U}{\partial \lambda} = [Y] - \bar{p}_1 c - \bar{p}_2 g = 0 \quad \text{--- 2.5}$$

By dividing the both equation of 2.3 and 2.4,

$$U^* = \frac{f'(c)}{f'(g)} = \frac{\lambda \bar{p}_1}{\lambda \bar{p}_2} \quad \text{--- 2.6}$$

Or, we may say, $U^* = \frac{f'(c)}{f'(g)} = \frac{\bar{p}_1}{\bar{p}_2} \quad \text{--- 2.7}$

As otherwise, this is an identity of income distribution:

$E(U) = (U(Y) = E(U^*(\frac{c_n}{g_n}) \geq 0)$, in terms of equity of private and public goods consumption.

3. Consumption of public goods can't improve income equality without direct transfer

Proposition and Proof: Here, v represents the indirect utility of public goods consumption and accordingly U represents direct utility maximized by income y .

$$\bigcap_{i=1}^n V_i = u \in U \mid u \in V_i, \forall i \in \{1, \dots, n\}$$

Same as,

$$\bigcap_{i=1}^n U(y)_i = u \in U \mid u(y) \in U(Y)_i, \forall i \in \{1, \dots, n\}$$

But, $\bigcap_{i=1}^n V_i \notin \bigcap_{i=1}^n U(y)_i$

From the above analysis, we found that people cannot attain the direct utility from public goods, but poor people can attain the same indirect utility as the rich people consume from public goods. Therefore, producing more and more public goods without attaining poor's direct utility by government transfer will increase income inequality of marginal poor people.

4. Welfare analysis of Roy's identity for private and public goods consumption

Proposition and Proof: The following simple observation leads to four important identities:

$e(p, v(p, m)) \equiv m$. The maximum expenditure necessary to reach utility $v(p, m)$ is m .

$v(p, e(p, u)) \equiv u$. The maximum utility from income $e(p, u)$ is u .

$$x_i(p, m) \equiv h_i(p, v(p, m)).$$

The Marshallian demand at income m is the same as the Hicksian demand at utility $u(p, m)$

$h_i(p, u) \equiv x_i(p, e(p, u))$. The Hicksian demand at utility u is the same as the Marshallian demand at income $e(p, u)$.

Let we presume that, if $x(p, m)$, and then we may write:

$$E(U) = x_i(p, y) = \frac{-\partial v(p, y)}{\frac{\partial p_i}{\partial v(p, y)} \dots} \text{for } \dots i = 1 \dots n. \quad \text{--- 4.1}$$

i.e. provided that right hand side is well defined and that $p \gg 0, y \gg 0$.

Let we prove the equation 4.1 with the assumption of the relationship between indirect utility and expenditure against equality. Suppose that x^* maximizes utility at (p^*, y^*) ; that is,

$$x^* = x(p^*, y^*);$$

$$\text{Let } u^* = u(x^*) = u(x(p^*, y^*)) = v(p^*, y^*),$$

$$u^* \equiv v(p, e(p, u^*)) \quad \text{--- 4.2}$$

that is no matter, what prices are, if consumer have minimum income to get utility u^* at those prices, then the maximum utility he can get is u^* . Since we know that this is an identity, we can differentiate it to get:

$$0 = \frac{\partial v(p^*, y^*)}{\partial p_i} + \frac{\partial v(p^*, y^*)}{\partial y} \times \frac{\partial e(p, u^*)}{\partial p_i} \quad \text{--- 4.3}$$

$$\text{or, } x_i^* = \frac{\partial e(p, u^*)}{\partial p_i} = \frac{\frac{-\partial v(p^*, y^*)}{\partial p_i}}{\frac{\partial v(p^*, y^*)}{\partial y}} \quad \text{--- 4.4}$$

In this case, as this is true for all p^*, y^* and since $x^* = x(p^*, y^*)$, the theorem is proved. In addition, we add more an equality assumption $E(U)$ to the right hand side to check the relationship between equality, indirect utility and expenditure function.

$$E(U) = x_i^* = \frac{\partial e(p, u^*)}{\partial p_i} = \frac{\frac{-\partial v(p^*, y^*)}{\partial p_i}}{\frac{\partial v(p^*, y^*)}{\partial y}} \quad \text{--- 4.5}$$

The equations 4.1 and 4.4 both have the same argument with different assumption to prove the relationship between indirect utility against income and expenditure theoretically.

Now we may conclude a solution for the assumption of equality between indirect utility and expenditure function, as change of price and income by dividing both the equations.

$$E(U) = \left(\frac{x_i}{x_i^*} \right) = \frac{\frac{\frac{-\partial v(p, y)}{\partial p_i}}{\frac{\partial v(p, y)}{\partial y}}}{\frac{\frac{-\partial v(p^*, y^*)}{\partial p_i}}{\frac{\partial v(p^*, y^*)}{\partial y}}} = \frac{\partial e(p, u^*)}{\partial p_i} \quad \text{--- 4.6}$$

$$\left(E(U) + \frac{y}{p_i \left(-\frac{1}{p_i} \frac{y^*}{\partial y^*} \right)} = \frac{x_i}{x_i^*} + \frac{y}{p_i \left(-\frac{1}{p_i} \frac{y^*}{\partial y^*} = 0 \right)} \right) \quad \text{--- 4.7}$$

$$E(U) = \left(\frac{x_i}{x_i^*} \right) = \frac{\partial e(p, u)}{\partial p_i} \geq 0 \quad \text{--- 4.8}$$

So we can write:

$$E(U) = \left(\frac{c_i}{g_i} \right) = \frac{e(p, u)}{\partial p_i} \geq 0 \quad \text{--- Where } i = 1, 2, \dots, n \quad \text{--- 4.9}$$

Since increase of private good consumption is due to increase of consumption tax, then the theorem is proved. We may say now that increase of consumption tax will increase government expenditure and accordingly improve equality (Varian 1992).

Discussion

The relationship between equality and consumption share of private and public goods has been tested by the consumption pattern of inverted U hypothesis. The Kuznets pattern derived the relationship between inequality and economic growth with an assumption of inverted U hypothesis. This pattern has a lot of limitation specially this pattern can not explain the welfare aspect of private and public goods consumption. In this research, the inverted U hypothesis has been tested by consumption bundle of private and public goods as a proxy of economic growth. We may presume that there are two goods produced in the economy. One is private goods and another is public goods. The empirical study found evidence that the consumption of private and public goods has a positive relation for improvement of equality. By these theoretical and empirical evidence, we may conclude that in case of direct and indirect utility, both have the positive relation between improvement of equality and consumption of private and public goods. All of this theoretical assumption satisfies the initial condition of general consumption pattern of inverted U hypothesis.

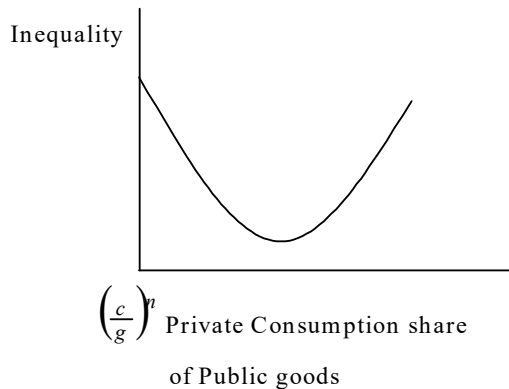
Since the Kuznets hypothesis has a vivid explanation to the relationship between long term inequality (equality) and growth, but this hypothesis has a limitation to explain the welfare situation of the developing country. Even, this hypothesis cannot explain the role of government, and the impact of the government policy for the production of public goods in a consistence

model. Therefore, a different pattern is produced to overcome the theoretical limitation of Kuznets pattern. Moreover, inverted U pattern of consumption of private and public goods is theoretically consistent with individual consumption behavior and aggregate production process of economic growth.

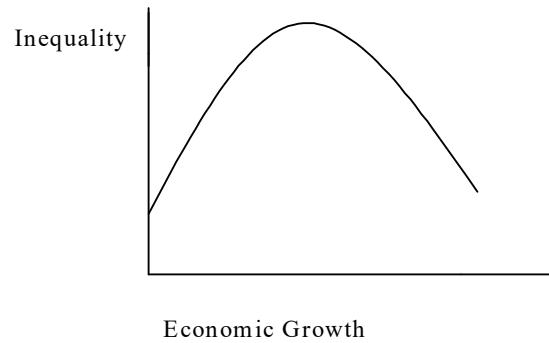
The patterns of government expenditure of industrialized and developing countries are not same. The industrialized countries normally emphasized increasing of income tax for production of public

goods. But in developing countries, government has no ample scope to increase income tax due to effect in slow economic growth. Even, the process of structural adjustment is more efficient in industrialized country rather than poor and developing country. If the structural adjustment is more efficient, and can capture the inefficiency of the market and government resource distribution, then there is a possibility to satisfy the Kuznets hypothesis in developing country.

Model:-A : Inequality (Equality) and Private Consumption Share of Public Goods



Model: B: Kuznets Pattern



But we may trust on welfare maximized private consumption share of public goods which is also a new pattern of theoretical presumption that we may increase economic welfare through the stage of economic development.

Empirical Evidence:

$$Model : g_{i,t} = \alpha + \beta_1 Z_{i,t} + \beta_2 \sum_{k=1}^n V_{k,i,t} + \gamma_i + \delta_{i,t}$$

Where g represents the Gini Coefficient as dependent variable, Z represents the independent variable of identical consumption between private and public goods substitution, V_k represents various independent

environmental Variables of GDP share and γ represent the group dummy variables of specific effects. The various environmental variables have been considered here to capture the quadratic equation since all independent variables are the share of national income. The empirical evidence found a consistence result in panel estimation. Both in the pooled, fixed and random effect. All the result are significant, the reason behind that the private consumption and the open trade captured the inefficiency of public production or consumption. This result is consistent with the assumption of long term increase of private and public consumption as well as improvement of economic welfare. See details for the proof of empirical evidence

of the author's published article for aforesaid theoretical assumptions. (Islam, 2007).

Conclusion

This research concludes the evidence of the long term relationship between improvement of economic welfare and consumption in both private and public goods with the consistence theoretical model and empirical evidence published earlier by the author. In this research, the long term relationship between improvement of equality and consumption to the stage of economic development has been satisfied in context of developing countries. But due to structural adjustment, the Kuznet's hypothesis in relation to improvement of inequality and growth has not been satisfied. Kuznet's hypothesis captures all the inefficiency of government and market mechanism in long run and improves economic welfare. But this hypothesis cannot explain clearly the relationship between theoretical process of economic welfare and consumption, even in the role of government in improving public consumption and economic welfare to the stage of economic development. Satisfying long run inverted U relationship between inequality (equality) and identical consumption share of private share of public goods will improve the welfare of developing states in both private and public sector in long run.

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