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QUESTION-BEGGING ARGUMENTS AS ONES THAT DO NOT EXTEND KNOWLEDGE

Rainer Ebert*

Abstract

In this article, I propose a formal criterion that distinguishes between deductively valid arguments that do and do not beg the question. I define the concept of a Never-failing Minimally Competent Knower (NMCK) and suggest that an argument begs the question just in case it cannot possibly assist an NMCK in extending his or her knowledge.

An argument is a set of claims, {P_1, P_2, …, P_n, C}, all but one of which, P_1, P_2, …, and P_n, are intended to provide rational support for the remaining one, C. P_1, P_2, …, and P_n are the *premises* of the argument, C is the *conclusion*.[1] The premises and the conclusion may be the meanings of declarative sentences in English, Bangla, or any other natural language,[2] mathematical statements, or any other propositions that are asserted to be true. For the purpose of this article, we will restrict ourselves to deductive arguments whose elements, P_1, P_2, …, P_n, and C, can be successfully translated into sentences

* Visiting Research Fellow, Centre de Recherche en Éthique, Montréal, Canada
Email: rainerebert@gmail.com, www.rainerebert.com

of Propositional Logic (PL) – also known as *Sentential Logic*.[3] A generic such argument can be represented as follows:

(I)
(P$_1$) φ_1
(P$_2$) φ_2
(P$_n$) φ_n
(C) γ ($\varphi_1, \varphi_2, \ldots, \varphi_n$, and γ are sentences of PL.)

A *good* argument need not be rhetorically effective, and bad arguments at times are, which is unfortunate. For an argument to be a good argument *qua* argument, however, it is necessary that it is valid, and has premises that are all true. An argument is valid, if, and only if, its conclusion is entailed by its premises, $P_1, P_2, \ldots, P_n \models C$. In other words, (I) is valid, if, and only if, every interpretation that makes φ_1, φ_2, …, and φ_n all true also makes γ true.[4] If an argument is valid, and if furthermore its premises are all true, it is sound.

Soundness is necessary for an argument to be good, but is it sufficient? Though it is sometimes claimed that all sound arguments are good argument, that does not seem to be true. Here are two arguments that are certainly sound, but almost as certainly bad, each with its PL translation:

(II)	(P)	Plato is dead.	(P)	p (p: Plato is dead.)
	(C)	Plato is dead.	(C)	p
(III)	(P$_1$)	$3 > 1$	(P$_1$)	q (q: $3 > 1$)
	(P$_2$)	$\sqrt{2}$ is irrational.	(P$_2$)	r (r: $\sqrt{2}$ is irrational.)
	(C)	$3 > 1$	(C)	q

The premises of these arguments are true and entail their respective conclusions, which is why (II) and (III) are indeed sound. Arguments of this kind, however, are often criticized as *begging the question*. Loosely speaking, an argument begs the question if the "conclusion is taken for granted in the premises."[5] A more precise definition of the fallacy of begging the question has been offered by Robert Hoffman, who distinguishes between three kinds of arguments:[6]

> "(A) Here, begging the question consists in inferring the truth of the putative conclusion not from that of some other proposition, but from its own truth, posited as a premise. Accordingly, the same proposition is asserted twice, so the condition of there being at least two propositions [in an argument] is unsatisfied and there is no argument at all. When the putative argument does comprise two or more propositions, then either (B) the conclusion follows from some proposition other than itself, in which case the argument is valid and does not beg the question (though it may contain unnecessary premises), or (C) the conclusion follows only from itself posited as a premise, in which case the other premises are unnecessary and the putative argument, as in (A), is not an argument at all."[7]

(II) and (III) are of the kinds described in (A) and (C), respectively, and not arguments at all, according to Hoffman – because they beg the question, which is "the error of taking oneself to be presenting an argument when one is merely asserting the truth of some proposition."[8] Only arguments of the kind described in (B) are not question-begging, says Hoffman, and hence "real" arguments. An example for such an argument can easily be constructed by rewriting (III) as follows:

(IV)	(P)	$3 > 1$ and $\sqrt{2}$ is irrational.	(P)	q & r
	(C)	$3 > 1$	(C)	q

Here, the conclusion follows from a proposition other than itself, and hence the argument is of the B-kind and supposedly

does not beg the question. This is implausible because (III) and (IV) are obviously equivalent. Asserting the truth of "$3 > 1$" and asserting the truth of "$\sqrt{2}$ is irrational" is asserting the truth of "$3 > 1$ and $\sqrt{2}$ is irrational." Similarly, in PL, claiming that q and claiming that r just *is* claiming that q & r.[9]

The Sense in which Every Valid Argument is Circular

In every deductively valid argument, of course, the conclusion is implicit in the premises, just not always as obviously as in (IV). How could the conclusion be entailed by the premises otherwise? Maybe the lesson from Hoffman's proposal is that hiding the conclusion in a premise-conjunct[10] is not hiding it enough, which would suggest this alternative proposal:

(#) An argument begs the question, if, and only if, its conclusion is equivalent to a premise-conjunct.[11]

While (#) is attractive at first glance, it loses much of its initial appeal once we understand that (#) implies that *every* valid argument is logically equivalent to a question-begging argument. To see this implication, we will use the Disjunctive Normal Form Theorem of PL, which states that every sentence of PL is equivalent to a PL sentence which is in disjunctive normal form (DNF).[12] A PL sentence is in DNF, if, and only if, it "is either a simple conjunction or a disjunction […] of simple conjunctions"[13] – where a simple conjunction is a sentence letter by itself, a negated sentence letter by itself, or a conjunction of sentence letters and negated sentence letters.[14] Applying the DNF Theorem to (I), and with $\{q_1, \ldots, q_k\}$ being the set of all sentence letters contained in $\varphi_1, \varphi_2, \ldots, \varphi_n$, and γ, we find that

$\varphi_1 \& \varphi_2 \& \ldots \& \varphi_n | == | \&_1 \vee \&_2 \vee \ldots \vee \&_m$ and $\gamma | == | \&_1^* \vee \&_2^* \vee \ldots \vee \&_l^*$,

where $\&_i$, $1 \le i \le m$, and $\&_j^*$, $1 \le j \le l$, are conjuncts of subsets of {q1, …, qk, ~q1, …, ~qk}. If γ is not logically true, $\varphi_1, \varphi_2, \ldots, \varphi_n| = \gamma$, if, and only if, $(\forall i)(\exists j)(\&_j^*$ is a conjunctive component of $\&_i)$. Otherwise, $\varphi_1, \varphi_2, \ldots, \varphi_n| = \gamma$ holds trivially.

Next, let us define DNF+ as the form a premise-conjunct or the conclusion of a PL argument is in, if, and only if,

- it is in DNF,
- each of its disjuncts contains each sentence letter or its negation but not both (which means that each disjunct contains as many conjuncts as there are sentence letters, k),
- and none of its disjuncts is equivalent to another.

If a premise-conjunct or a conclusion of a PL argument is in DNF and not logically false, then it is logically equivalent to a PL sentence which is in DNF+. In particular, $\&_1 \vee \&_2 \vee \ldots \vee \&_m$, if not logically false, can be brought into DNF+ by going through the following steps for all $1 \le i \le m$, starting with $i = 1$, and removing duplicate disjuncts in the end.

1. If $\&_i$ contains each sentence letter or its negation but not both, continue with $i + 1$.
2. If $\&_i$ contains a sentence letter and its negation, remove $\&_i$ and continue with $i + 1$.
3. For all $1 \le a \le k$, if $\&_i$ contains neither q_a nor $\sim q_a$, add $(q_a \vee \sim q_a)$ to $\&_i$ using a conjunction.
4. Repeatedly apply the distribution rule to bring the sentence resulting from the previous step into DNF and then continue with $i + 1$.

Analogously, we can bring $\&_1^* \vee \&_2^* \vee \ldots \vee \&_l^*$ into DNF+, if the conclusion of the argument is not logically false. The disjuncts in the DNF+ equivalents of $\&_1 \vee \&_2 \vee \ldots \vee \&_m$ and $\&_1^* \vee \&_2^* \vee \ldots \vee \&_l^*$ correspond to interpretations that make these equivalents true. Therefore, $\varphi_1, \varphi_2, \ldots, \varphi_n| = \gamma$, if, and only if,

$$\varphi_1 \& \varphi_2 \& \ldots \& \varphi_n| == |\alpha_1 \vee \alpha_2 \vee \ldots \vee \alpha_d \quad \text{and}$$
$$\gamma| == |\alpha_1 \vee \alpha_2 \vee \ldots \vee \alpha_e,$$

where α_c, $1 \le c \le e$, are the DNF+ disjuncts and $d \le e$. Note that, for every interpretation, one and only one α_c is true. Trivially,

$$\alpha_1 \vee \alpha_2 \vee \ldots \vee \alpha_d| == |(\alpha_1 \vee \alpha_2 \vee \ldots \vee \alpha_d \vee \alpha_{d+1} \vee \ldots \vee \alpha_e) \& (\alpha_1 \vee \alpha_2 \vee \ldots \vee \alpha_d)$$

Therefore, $\varphi_1 \& \varphi_2 \& \ldots \& \varphi_n| == |\gamma \& (\alpha_1 \vee \alpha_2 \vee \ldots \vee \alpha_d)$. The original argument is logically equivalent to the following argument:[15]

(P) $\gamma \& (\alpha_1 \vee \alpha_2 \vee \ldots \vee \alpha_d)$
(C) γ

For every valid argument, the conclusion is equivalent to a premise-conjunct of the argument in this form. Therefore, assuming (#), every valid argument is logically equivalent to a question-begging argument.

This or similar lines of reasoning have led some authors to conclude that begging the question cannot be understood in terms of the logical structure of arguments, and accordingly is a rhetorical, pragmatic, or informal fallacy, whereas others, including Sextus Empiricus and John Stuart Mill,[16] concluded that *all* deductively valid arguments beg the question. I will propose a formal criterion that charts a middle way between these two positions and allow us to make a meaningful distinction between good and bad deductively valid arguments. More specifically, it will separate arguments of the kind exemplified by (II) to (IV) from arguments that are more useful (for the lack of a better word). My criterion is objective or "formal" insofar it does not refer to specific individuals; it does, however, presuppose a certain *kind* of person, the Never-failing Minimally Competent Knower. I think this small concession in terms of formal purity is tolerable, especially given the fact that the vast majority of people are reasonably close to being of this kind.

Before going into detail, here is the general idea: By constructing an argument, "[w]e are trying to get at the truth, to know something."[17] We are trying to learn something new, achieve knowledge that we did not have prior to engaging with the argument. In other words, the purpose of an argument is to provide us with a justification for believing its conclusion that we possibly have not had yet. Hence, if knowledge of the premises presupposes the conclusion in the sense that one is not justified in believing the premises unless one is justified in believing the conclusion, the argument fails to fulfill its purpose. Accordingly, an argument could be said to "beg the question" if knowledge of the premises entails being justified in believing the conclusion. Before I can spell out this account of question-begging in more detail, I first need to introduce some basic elements of epistemic logic as well as the concept of a Never-failing Minimally Competent Knower.

Basic Elements of Epistemic Logic

The traditional analysis of knowledge as justified true belief has prevailed in Western philosophy for most of its history,[18] until the publication in 1963 of Edmund L. Gettier's seminal

three-page article, "Is Justified True Belief Knowledge?"[19] In it, Gettier offers two counterexamples to the traditional analysis that are generally regarded as compelling. For the purpose of this article, however, I will pretend that it is still possible to be a traditionalist with regard to knowledge and hope that what I say will motivate a similar analysis of what a good argument is on the basis of more sophisticated accounts of knowledge, such as the many forms of reliabilism that have been developed since the publication of Gettier's article.

If p is a proposition and S an individual capable of knowledge, like you and I, let "$K_S(p)$" stand for "S knows that p," "$B_S(p)$" for "S believes that p," and "$J_S(p)$" for "S has a good reason to believe that p."[20] Using this notation, the traditional analysis of knowledge can be represented as $K_S(p) \equiv p \& B_S(p) \& J_S(p)$. Therefore,

(1) $K_S(p) \supset p$, (*factuality of knowledge*)

(2) $K_S(p) \supset B_S(p)$, and

(3) $K_S(p) \supset J_S(p)$.

Note that, in general, equivalence transformations under an epistemic operator change the truth value of the overall statement. For example, $B_S(p\&(p \supset q))| \neq B_S(p\&q)$. If S believes that p entails q, initially does not believe that q, and comes to believe that p at time t, it will take S some time to form a belief that q, if S infers q from p at all. Accordingly, at time t, $B_S(p\&(p \supset q))\&{\sim}B_S(q)$.

Most people spent their whole life without ever hearing about the Swabian village of Adelmannsfelden, and never come to believe that Franziska von Hohenheim was born there (p). For most S, neither $B_S(p)$, nor $B_S({\sim}p)$. Hence,

$B_S(p) \vee B_S(\sim p)$ is not a logical truth.[21] Given that there are infinitely many propositions and we have limited capacities and lifetimes, we in fact never form beliefs about most propositions. There is also an upper limit on the number of beliefs we can hold about a given proposition. We seem to be constituted such that we cannot simultaneously believe that p, and that non-p. That is, for all S and all p, $\sim(B_S(p) \& B_S(\sim p))$ at any given point in time, which is a consequence of the following two widely accepted principles of epistemic logic:[22]

(4) $B_S(\varphi) \supset \sim B_S(\sim\varphi)$
(5) $B_S(\sim\varphi) \supset \sim B_S(\varphi)$

It is further uncontroversial that knowledge distributes over conjunctions, and that a belief is always known by the one who holds it:

(6) $B_S(\varphi \& \vartheta) \supset B_S(\varphi) \& B_S(\vartheta)$
(7) $B_S(\varphi) \supset K_S(B_S(\varphi))$

Having introduced some basic elements of epistemic logic, we can now move on to define a Never-failing Minimally Competent Knower.

The Never-failing Minimally Competent Knower

If it is possible that a proposition is true, and if that proposition entails another proposition, then it must also be possible that that other proposition is true. "It is possible that" is what Fred I. Dretske calls a *fully penetrating operator*. In general, an operator of PL, O, is fully penetrating, if, and only if, it has the following property: if p entails q, then O(p) entails O(q).[23] Not all operators are fully penetrating. Take the operator "It is strange that," for example. "Peter is married to his dog" entails

that Peter is married, and yet, while it is strange that Peter is married to his dog, it is not strange that he is married. The set of strange propositions is not closed under entailment. The same is true for knowledge. If S knows that p, and p entails q, it is not necessarily true that S knows that q. S may simply not know that p entails q and hence fail to form a belief that q. Since belief is a necessary condition for knowledge, we see how one may fail to know something that is entailed by something else one does know. One may think this changes if it is further assumed that S knows that p entails q and believes that q. Even then, however, S may fail to know that q. S may not bother to deduce q from p, despite the fact that S knows that p, and that p entails q. S may believe that q on some other ground instead, and that ground may not justify S's belief that q, disqualifying it from being knowledge. For example, suppose Sadia struggled in class for most of the semester, but then turned things around and excelled in the final exam. She got a 90 on the final exam, and she knows that, as the results have just been released. She also knows that, if she got a 90 on the final exam, she will pass the class with a B. But she does not bother to infer that she will pass the class with a B from the fact that she got a 90 on the final exam. Instead, she believes that she will pass the class with a B on the basis of a coin toss. The reason why Sadia does not know that she will pass the class with a B is that she is not justified in her belief that she will pass the class with a B, as tossing a coin as a belief-forming process is epistemically defective. Accordingly, J_S too is not a fully penetrating operator, and analogous counterexamples show that no epistemic operator is.[24]

Even though the actual reason on the basis of which Sadia believes that she will pass the class with a B is a bad reason, a good reason is readily available to her, assuming she has a minimal understanding of logic – and the same is true more generally. If S believes and is justified in believing that p, and that p entails q, the premises of a valid argument, and if S further competently infers q from his or her justified beliefs that p, and that p entails q, then S has a good reason to believe that q.[25] Why would we be interested in valid arguments otherwise?

In general, I suggest that, for any set of propositions, $\{\varphi_1, \varphi_2, \ldots, \varphi_n\}$, if its elements jointly entail γ, S believes and has a good reason to believe that φ_1, φ_2, …, and φ_n, and S competently infers γ from φ_1, φ_2, …, and φ_n, then S has a good reason to believe that γ:

(8) $(\varphi_1, \varphi_2, \ldots, \varphi_n | = \gamma), B_S(\varphi_1), \ldots, B_S(\varphi_n), J_S(\varphi_1), \ldots, J_S(\varphi_n)$,
(S competently infers γ from $\varphi_1, \varphi_2, \ldots, \varphi_n$) $| = J_S(\gamma)$

Since $K_S(p) \equiv p \& B_S(p) \& J_S(p)$, it follows that

(9) $(\varphi_1, \varphi_2, \ldots, \varphi_n | = \gamma), K_S(\varphi_1), \ldots, K_S(\varphi_n)$,
(S competently infers γ from $\varphi_1, \varphi_2, \ldots, \varphi_n$),
$B_S(\gamma) | = K_S(\gamma)$.

This accords with the common idea that we can increase our knowledge through inference from propositions we already know – *deduction transmits knowledge*.

Sadia's failure to know that she will pass the class with a B is the result of her failure to perform an inference in her mind of which she knows that it holds. Let us define a Never-failing Minimally Competent Knower (NMCK) as a person who never fails in this way. If a NMCK knows that $p \supset q$ and believes with good reason that the antecedent is true, he or she – merely *in virtue of* understanding what it means for one

statement to entails another – has a good reason to believe the consequent to be true.[26] NMCKs are masters of the most fundamental principles of reasoning and understand the most basic logical structure of what they know. Whenever they know a sentence to be true, they realize the most obvious consequences of its PL main connective for what they could possibly be justified in believing in. More specifically, S is a NMCK, if, and only if, the following conditions hold:

(10) $K_S(\varphi \supset \vartheta) \supset [J_S(\varphi) \supset J_S(\vartheta)]$
(11) $K_S(\sim\varphi) \supset \sim J_S(\varphi)$
(12) $K_S(\varphi) \supset \sim J_S(\sim\varphi)$
(13) $K_S(\varphi \& \vartheta) \supset J_S(\varphi) \& J_S(\vartheta)$
(14) $K_S(\varphi \vee \vartheta) \supset \sim[J_S(\sim\varphi) \& J_S(\sim\vartheta)] \& [J_S(\sim\varphi) \supset J_S(\vartheta)] \& [J_S(\sim\vartheta) \supset J_S(\varphi)]$
(15) $K_S(\varphi \equiv \vartheta) \supset [(J_S(\varphi) \supset J_S(\vartheta)) \& (J_S(\vartheta) \supset J_S(\varphi))]$
(16) $K_S(\varphi) \supset J_S(\varphi \vee \vartheta) \& J_S(\vartheta \vee \varphi)$

(10) states that, for all S that are NMCKs, the set of justified propositions is closed under known entailment.[27] (6) and (13) jointly imply $K_S(\varphi \& \vartheta) \supset [K_S(\varphi) \& K_S(\vartheta)]$. It is plausible to assume that the reverse, $[K_S(\varphi) \& K_S(\vartheta)] \supset K_S(\varphi \& \vartheta)$, holds as well (NMCKs always "put one and one together"). Since bivalence seems to be hardwired into our brains, one may want to include $J_S(\varphi \vee \sim\varphi)$ in the list as well.

The Epistemic Value of Arguments

Earlier, I argued that an argument fails to fulfill its epistemological purpose if knowledge of the premises presupposes the conclusion in the sense that one is not justified in believing the premises unless one is justified in believing the conclusion. For an NMCK, this suggests the following definitions of epistemically bad and good arguments:

> A deductively valid argument is an epistemically bad argument ("begs the question") if it is sufficient for an NMCK to know the premises and believe that the conclusion is true in order to know the conclusion. In such a case, $K_S(\varphi_1), \ldots, K_S(\varphi_n)| = J_S(\gamma)$ and nothing is gained with regard to extending one's knowledge from going through and understanding the argument.
>
> In contrast, a deductively valid argument is epistemically good if $K_S(\varphi_1), \ldots, K_S(\varphi_n), B_S(\gamma)| \neq K_S(\gamma)$. An epistemically good argument is a *guide* for NMCKs, which may be more or less difficult to follow, on how to utilize their existing knowledge to justify beliefs they are not already justified in holding.

Given these definitions, (II) and (III) trivially are epistemically bad arguments, and so is (IV), due to (13). Furthermore, every instance of the modus ponens begs the question. In contrast, the following are examples of epistemically good arguments, each of them with the reason why they fail to be question-begging:

(V)	(P)	~~p
	(C)	p
	→	$K_S(\sim\sim p)\,| = \sim J_S(\sim p)\,| \neq J_S(p)$
(VI)	(P$_1$)	$\sim p$
	(P$_2$)	$p \supset q$
	(C)	$\sim p$
	→	$K_S(\sim q), K_S(p \supset q)| = \sim J_S(q), J_S(p) \supset J_S(q)$ $\,| = \sim J_S(p)\,| \neq J_S(\sim p)$
(VII)	(P$_1$)	q
	(P$_2$)	$p \supset (q \supset r)$
	(P$_3$)	p
	(C)	r
	→	$K_S(q), K_S(p \supset (q \supset r)), K_S(p)$ $\,| = J_S(q), J_S(p) \supset J_S(q \supset r), J_S(p)$ $\,| = J_S(q), J_S(q \supset r)\,| \neq J_S(r)$

The difference between (V), (VI), and (VII) on the one hand and (II), (III), and (IV) on the other hand is that the former guide us through a valid inference that is not hardwired into our brains, while the latter merely state the cognitively nearly inescapable.

I believe this account has a fair level of intuitive plausibility when applied to (II) to (VII), but it is not free of problems. For example, consider the following argument, which consists of a number of modi ponentes strung together:

(VIII)	(P$_1$)	p_1
	(P$_2$)	$p_1 \supset p_2$
	(P$_3$)	$p_2 \supset p_3$
	…	
	(P$_n$)	$p_{n-1} \supset p_n$
	(C)	p_n

Using (3) and (10), we find that knowledge of the premises of this argument entails $J_S(p_1)$, $J_S(p_1) \supset J_S(p_2)$, …, and $J_S(p_{n-1}) \supset J_S(p_n)$ and hence $J_S(p_n)$. Therefore, it is sufficient for an NMCK to know the premises of (VIII) and believe that p_n in order to know the conclusion. I have argued that in such a case nothing is gained with regard to extending one's knowledge from going through and understanding the argument. This seems reasonable for $n = 1$ insofar, in this case, (VIII) merely states the meaning of the entailment symbol and, accordingly, is of little use to an NMCK who, by definition, unfailingly performs simple inferences. For $n > 1$, however, (VIII) seemingly can well be a useful guide through a valid inference. Image you have known P$_2$, P$_3$, …, and P$_n$ for a long time and then come to know P$_1$. If you have a minimal understanding of P_2, this new piece of knowledge will provide you with a good reason to believe that p_2. A belief that p_3, however, does not seem to be justified unless and until you actually form a belief that p_2. In order to accommodate this thought, we could deny that the set of justified propositions is closed under known entailment and replace (10) by

$$(17) \quad K_S(\varphi \supset \vartheta) \supset [K_S(\varphi) \supset J_S(\vartheta)].$$

The modified account resulting from this move renders (VIII) fallacious for $n = 1$, while arguments of this form with $n > 1$ are recognized as potentially helpful guides for NMCKs. This shows that an epistemological approach to the fallacy of begging the question can handle a string of modi ponentes. Before it can emerge as a serious theory, however, my proposal has to be subjected to further scrutiny, as there may well be other argument forms that reveal additional problems.

Conclusion

The conclusion of every deductively valid argument is somehow contained in its premises. I showed exactly *how*. That has led some philosophers to believe that begging the question is not a formal fallacy, as otherwise every valid argument would be fallacious. I argued that this conclusion has been drawn too quickly and proposed a formal criterion that distinguishes between good and bad valid arguments. The criterion is motivated by the fundamental function of arguments – *the extension of knowledge*. I introduced some basic elements of epistemic logic and sketched an epistemological account of begging the question. For simplicity's sake, I built that account on the traditional analysis of knowledge as justified true belief. I showed that it handles

exemplary cases of question-begging arguments well and is flexible when confronted with objections. Further work, however, is needed, and I hope that I said enough to make my approach seem worthy of the effort of doing that work.[28]

References

1. Sometimes the notion of an argument is defined such that an argument can have more than one conclusion. It is always possible to divide up an argument with more than one conclusion into several arguments that each have just one conclusion, by separately combining each conclusion with all premises.
2. A declarative sentence is a complete and grammatically well-formed sentence that makes a claim.
3. Such a translation requires that the premises and the conclusion can, in a sufficiently detailed manner, be divided up into their respective sub-statements, which are replaced by sentence letters, and the logical connectives between them – while preserving the argument's deductive potential. For a discussion of why that requirement sometimes cannot be met, see Grandy & Osherson 2010, Chapter 8.
4. An interpretation assigns a truth-value to each sentence letter.
5. Stevenson & Waite 2011, p. 1073.
6. This is in response to Richard Robinson, who argues that begging the question is not a fallacy at all: "Don't tell me that my premise is true and my conclusion follows, but all the same my argument is bad because it 'begs the question'. That's absurd" (Robinson 1971, p. 116).
7. Hoffman 1971, p. 51.
8. Hoffman 1971, p. 51.

9. Another problem with Hoffman's proposal is that he does not offer an account of propositional identity. As David H. Sanford notes, "neither orthographic identity nor mutual strict implication does the trick" (Sanford 1972, p. 197). While the former is obviously not necessary, the latter does not seem to be sufficient insofar it condemns too many single-premise arguments.
10. A premise-conjunct is a conjunctive component of the conjunct of all premises. For example, consider an argument with two premises, one of which, p & q, has the conjunction as its main connective. The other premise is r. The premise-conjuncts of this argument then are p, q, r, p & q, p & r, q & r, and p & q & r.
11. John Woods and Douglas Walton consider this proposal in Woods & Walton 1975.
12. Cf. Grandy & Osherson 2010, p. 103, Corollary (61).
13. Grandy & Osherson 2010, p. 102, Definition (58).
14. Cf. Grandy & Osherson 2010, p. 101, Definition (57).
15. This result can also be understood set-theoretically. Consider the interpretation set I, whose 2^k elements are the k-tupels corresponding to the rows on the left-hand side of a truth table. In this picture, a sentence of PL corresponds to one and only one subset of I. Let $p \subseteq I$ and $q \subseteq I$. $p \models q$, if, and only if, $p \subseteq q$. Further, $p \subseteq q$, if, and only if, $p = q \cap p$, and $q \cap p$, if, and only if, q & p. Therefore, $p \models q$, if, and only if, $(q \;\&\; p) \models q$.
16. Cf. Sextus Empiricus 1933, pp. 195-197, and Mill 1843, p. 120.
17. Robinson 1971, p. 115.
18. The claim that the prevailing conception of knowledge before Gettier was that of knowledge as justified true belief is disputed, cf. Dutant 2015, and Le Morvan 2017.
19. Gettier 1963.

20. That is, S would be justified in believing that p if S believed that p.
21. The same is true for J_S and K_S, for analogous reasons.
22. (4) and (5) do not entail that one cannot hold contradicting beliefs, $\not\models [B_S(\varphi) \supset \sim B_S(\vartheta)$, if $\varphi \equiv \sim\vartheta]$. For example, a person may come to believe both that $p \& q$, and that $\sim p \vee \sim q$, without realizing the contradiction. That may be because the person is unfamiliar with formal logic, or because the two beliefs were formed in entirely different and unrelated contexts and the person has never considered both beliefs side by side.
23. Cf. Dretske 1970, p. 1007.
24. Cf. Dretske 1970, p. 1009.
25. This claim has been disputed. For an introduction to the relevant literature, see Collins 2020, and Luper 2020.
26. NMCKs always ("automatically") perform obvious inferences. If the antecedent of a known entailment is believed for a good reason, the "automatic" act of inference will provide a NMCK with a good reason to believe that the consequent is true. Note that this is not to say that he or she will necessarily believe that the consequent is true.
27. NMCKs have mastered one of the most basic lessons of logic, the modus ponens.
28. To be clear, I did *not* show that there is no rhetorical, pragmatic, or informal fallacy that can reasonably be identified with what is called the fallacy of begging the question. It might very well be true that what is wrong with arguments that are commonly said to beg the question has components in two dimensions really, one rhetorical, pragmatic, or informal, the other epistemological.

Bibliography

Collins, John M., “Epistemic Closure Principles,” *The Internet Encyclopedia of Philosophy*, URL = <https://iep.utm.edu/epis-clo/> [accessed on December 24, 2020].

Dretske, Fred I., “Epistemic Operators,” *The Journal of Philosophy* 67 (1970), pp. 1007-1023.

Dutant, Julien, “The Legend of the Justified True Belief Analysis,” *Philosophical Perspectives* 29 (2015), pp. 95-145.

Gettier, Edmund L., “Is Justified True Belief Knowledge?”, *Analysis* 23 (1963), pp. 121-123.

Grandy, Richard, and Daniel Osherson, *Sentential Logic for Psychologists* (September 28, 2010), URL = <https://www.princeton.edu/~osherson/primer.pdf> [accessed on December 21, 2020].

Hoffman, Robert, “On Begging the Question at Any Time,” *Analysis* 32 (1971), p. 51.

Le Morvan, Pierre, “Knowledge before Gettier,” *British Journal for the History of Philosophy* 25 (2017), pp. 1216-1238.

Luper, Steven, “Epistemic Closure,” *The Stanford Encyclopedia of Philosophy* (Summer 2020), Edward N. Zalta (ed.), URL = <https://plato.stanford.edu/archives/sum2020/entries/closure-epistemic/> [accessed on December 24, 2020].

Mill, John Stuart, *A System of Logic* (London: Longmans Green, 1843).

Robinson, Richard, “Begging the Question, 1971,” *Analysis* 31 (1971), pp. 113-117.

Sanford, David H., “Begging the Question,” *Analysis* 32 (1972), pp. 197-199.

Sextus Empiricus, *Outlines of Pyrrhonism*, vol. II., trans. by R. G. Bury (London: Loeb Classical Library, 1933).

Stevenson, Angus, and Maurice Waite (eds.), *Concise Oxford English Dictionary* (Oxford: Oxford University Press, 2011).

Woods, John, and Douglas Walton, “Petitio Principii,” *Synthese* 31 (1975), pp. 107-127.

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