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Improvement on Calibration Weightings in Stratified Random Sampling

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Abstract

Calibration weightings is the process of formulating calibration constraints using a given distance measure to obtain expression of the calibration weights. One of the major limitations of the simple calibration technique by Deville and Sarndal (1992); is that the calibration weights obtained by this process may be negative and or extremely large. To overcome this challenge, this study develops a new framework for obtaining optimum calibration weightings using inverse exponentiation. A new calibration regression estimator of population mean is proposed in stratified random sampling. Properties of the new estimator are derived and its efficacy established through empirical comparisons with existing estimators. Results of analysis showed that the new estimator obtained by the new calibration weightings is more precise and highly efficient than calibration estimators obtained by the simple calibration technique by Deville and Sarndal (1992), under the same optimum conditions.

Keywords: Calibration weightings, Distance measure, Efficiency, Empirical comparisons, Inverse exponentiation, Variance deduction

AMS Classification: 62D05, 62G05, 62H12.

1. Introduction

In sample survey, one of the methods of sample selection is by stratifying the population. Stratification is one of the design instruments that gives increase precision. Calibration approach under stratified random sampling uses information from auxiliary variables to obtain optimum strata weights. The integration of auxiliary information has significant importance in formulating efficient estimators for population or subpopulation parameter estimation and it enhances efficiency in different sampling designs. Many authors have worked on the estimation of population parameters using the knowledge of auxiliary variables and have suggested different estimation methods for estimating population mean of the study variable. Work in this direction include; Bahl and Tuteja (1991), Singh and Kumar (2010), Diana *et al*. (2011), Malik and Singh (2012), Haq and Shabbir (2013), Clement *et al* (2014a), Lu *et al* (2014), Clement and Enang (2015), Lone and Tailor (2015), Clement (2016, 2017), Beevi *et al* (2017), Clement (2018a), Yadav *et al* (2019), Izunobi and Onyeka (2019), Clement and Inyang (2020), Zaman (2020), Clement (2020,2021), Clement *et al* (2021), Clement (2022), Inyang and Clement (2023) and Clement *et al* (2024a) among others.

The concept of calibration estimation in sample survey was introduced by Deville and Sarndal (1992). They used auxiliary information to obtain weighting system using a given distance measure and a set of calibration constraints. They observed that each distance measure has a corresponding set of calibration weights and an estimator. Hence, the strength of the formulated calibration constraints determines the efficiency of the resulting calibration estimator(s). The process of improving the efficiency of the study variable by deriving mathematical expression for the calibration weights through the formulated calibration constraints on a given distance measure is called calibration weightings. Survey estimation under calibration is discussed in Arnab and Singh (2005), Kott (2006), Kim (2010), Rao *et al.* (2012), Clement *et al*. (2014b), Koyuncu and Kadilar (2016), Clement and Enang (2017), Clement (2018b), Enang and Clement (2020), Clement and Inyang (2021), Clement and Etukudoh (2023), Clement and Enang (2024) and Clement *et al* (2024b) among others.

In calibration estimation theory, the calibration weights are formulated such that a given distance measure is minimized subject to some specified constraints related auxiliary variable information. However, the calibration weights defined by minimizing a distance measure under some given constraints may be negative and or extremely large. This is not acceptable if the calibration weights are used in large scale sample surveys. In addition, it can affect the precision (or accuracy) of parameter estimate(s) of interest. In the progression for improvement in calibration estimation, this study sought to address this limitation by suggesting a new approach to calibration estimation in stratified random sampling based on a new formulated calibration constraints using inverse exponentiation. The aim is to get reasonable calibration weights that will optimize the efficiency of calibration estimators.

2. Sample Design and Procedure

Consider a finite population of size (N) such that $U = (U_{1,}U_2, \ldots, U_N)$. Let (X) and (Υ) be the supplementary and study variables respectively taking values X_i and Y_i on the ith unit $U_i (i = 1, 2, \ldots, N)$ of the population. It is assumed that every information on the population mean $(\bar{X})$ of the supplementary variable (X) is known and $(x_i, y_i) \geq 0$, (since survey variables are generally non-negative). Let a sample of size (n) be selected by simple random sampling without replacement (SRSWOR) based on which the mean $(\bar{x})$ for the supplementary variable (X) and the mean $(\bar{y})$ for the study variable (Υ) are obtained.

Let the population $[U = (U_{1,}U_2, \ldots, U_N)$ of size $(N)]$ be partitioned into H strata with N_h units in the hth stratum where a simple random sample of size n_h is selected without replacement. Let the total population size be $N = \sum_{h=1}^{H} N_h$ and the sample size $n = \sum_{h=1}^{H} n_h$, respectively. Associated with the ith element of the hth stratum are y_{hi} and x_{hi} with $x_{hi} > 0$ being the covariate; where y_{hi} is the y value of the ith element in stratum h, and x_{hi} is the x value of the ith element in stratum h, $h = 1,2, \ldots, H$ and $i = 1,2, \ldots, N_h$.

Let the stratum weights be $W_h = N_h/N$ and the sample fraction be $f_h = n_h/N_h$. Let the hth stratum means of the supplementary variable X and the study variable Y $\left(\bar{x}_h = \sum_{i=1}^{n_h} x_{hi}/n_h \,; \bar{y}_h = \sum_{i=1}^{n_h} y_{hi}/n_h \right)$ be the unbiased estimator of the population mean $\left(\bar{X}_h = \sum_{i=1}^{N_h} x_{hi}/N_h \,; \bar{Y}_h = \sum_{i=1}^{N_h} y_{hi}/N_h \right)$ of X and Y respectively, based on n_h observations.

$S_{hx}^2 = \frac{1}{N_h-1}\sum_{i=1}^{N_h}(x_{hi} - \bar{X}_{hi})^2$; $S_{hy}^2 = \frac{1}{N_h-1}\sum_{i=1}^{N_h}(\bar{y}_h - \bar{Y}_h)^2$, $S_{hxy} = \frac{1}{N_h-1}\sum_{i=1}^{N_h}(x_{hi} - \bar{X}_{hi})\ (y_h - \bar{Y}_h)$, $S_{hx_ix_j} = \frac{1}{N_h-1}\sum_{i=1}^{N_h}(x_{hi} - \bar{X}_{hi})\ (x_{hj} - \bar{X}_{hj})$ $\bar{x}_{i,st} = \sum_{h=1}^{H} W_h\,\bar{x}_{hi}$ and $\bar{y}_{st} = \sum_{h=1}^{H} W_h\,\bar{y}_h$. Let $\bar{x}_h = \frac{1}{n_h}\sum_{i=1}^{n_h} x_{hi}$ and $s_{hx}^2 = \frac{1}{n_h-1}\sum_{i=1}^{n_h}(x_{hi} - \bar{x}_h)^2$ be the sample mean and variance for the supplementary variable. Let $\bar{y}_h = \frac{1}{n_h}\sum_{i=1}^{n_h} y_{hi}$ and $s_{hy}^2 = \frac{1}{n_h-1}\sum_{i=1}^{n_h}(y_{hi} - \bar{y}_h)^2$ be the sample mean and variance for the study variable.

Let define the relative errors terms as

$e_{hy} = \left(\frac{\bar{y}_h-\bar{Y}_h}{\bar{Y}_h}\right)$ so that $\bar{y}_h = \bar{Y}_h\left(1 + e_{hy}\right)$

$e_{hx} = \left(\frac{\bar{x}_h-\bar{X}_h}{\bar{X}_h}\right)$ so that $\bar{x}_h = \bar{X}_h(1 + e_{hx1})$

$e_{hs} = \left(\frac{s_{hx}^2-S_{hx}^2}{S_{hx}^2}\right)$, so that $s_{hx}^2 = S_{hx}^2(1 + e_{hs})$

Let define the expected values of the relative errors as

$\mathrm{E}\left(e_{hy}\right) = E(e_{hx}) = (e_{hs}) = 0$, $E\left(e_{hy}^2\right) = \gamma_h C_{hy}^2, E(e_{hx}^2) = \gamma_h C_{hx}^2, E(e_{hs}^2) = \gamma_h C_{hs}^2$

$E\left(e_{hy}e_{hx}\right) = \gamma_h\rho_{hyx}C_{hy}C_{hx}$, $E\left(e_{hy}e_{hs}\right) = \gamma_h\rho_{hys}C_{hy}C_{hs}$, $E(e_{hx}e_{hs}) = \gamma_h\rho_{hxs}C_{hx}C_{hs}$,

and $\gamma_h = \left(\frac{1}{n_h} - \frac{1}{N_h}\right)$ where the parameters wherever they appear are defined as follows

$\bar{y}_h$ -sample stratum mean of study variable

$\bar{Y}_h$ -population stratum mean of study variable

$\bar{x}_h$ - sample stratum mean of auxiliary variable

$\bar{X}_h$ - population stratum mean of auxiliary variable

s_{hx}^2 -sample stratum variance of auxiliary variable

S_{hx}^2 -population stratum variance of auxiliary variable

C_{hx}^2 -coefficient of variation of auxiliary variable

C_{hy}^2 -coefficient of variation of study variable

ρ_{hxy} -correlation coefficient between auxiliary variable and study variable

ρ_{hxs} -correlation coefficient between mean and variance of auxiliary variable

ρ_{hys} -correlation coefficient between mean of study variable and variance of auxiliary variable.

3. Calibration Estimation by Deville and Sarndal (1992) Technique

This section applies the simple calibration technique by Deville and Sarndal (1992) to estimation theory with respect to Koyuncu and Kadilar (2016) calibration regression estimator in stratified random sampling design.

3.1The Koyuncu and Kadilar (2016) Estimator

Koyuncu and Kadilar (2016), being motivated by Tracy et al. (2003), used the conventional calibration estimation technique by Deville and Sarndal (1992) to propose the following calibration regression estimator in stratified random sampling:

$$\bar{y}_{kk}^* = \sum_{h=1}^{H} \vartheta_h \bar{y}_h \tag{1}$$

using chi-square distance measure of the form

$$L(\vartheta_h, w_h) = \sum_{h=1}^{H} \frac{(\vartheta_h - w_h)^2}{w_h Q_h} \quad (2)$$

subject to the calibration constraints defined by

$$\sum_{h=1}^{H} \vartheta_h \bar{x}_h = \sum_{h=1}^{H} W_h \bar{X}_h \quad (3)$$

$$\sum_{h=1}^{H} \vartheta_h s_{hx}^2 = \sum_{h=1}^{H} W_h S_{hx}^{\prime 2} \quad (4)$$

$$\sum_{h=1}^{H} \vartheta_h = \sum_{h=1}^{H} W_h \quad (5)$$

obtained the calibration weights

$$\vartheta_h = W_h + W_h Q_h (\lambda_{10} \bar{x}_h + \lambda_{20} s_{hx}^2 + \lambda_{30}) \quad (6)$$

If (6) is substituted in [(3), (4), (5)] respectively and the resulting system of equations are solved; the values of the $\lambda_{i0} s$ are obtained.

If the $\lambda_{i0} s$ are substituted in (6) and the result is substituted in (1) respectively; the Koyuncu and Kadilar (2016) calibration regression estimator is obtained as

$$\bar{y}_{KK}^* = \sum_{h=1}^{H} W_h \bar{y}_h + \hat{\beta}_{h,10} \sum_{h=1}^{H} W_h (\bar{X}_h - \bar{x}_h) + \hat{\beta}_{h,20} \sum_{h=1}^{H} W_h (S_{hx}^2 - s_{hx}^2) \quad (7)$$

where $\hat{\beta}_{h,10}$ and $\hat{\beta}_{h,20}$ are the coefficients of regression defined by:

$$\hat{\beta}_{h,10} = \frac{T_{11}(b_{11}b_{12} - b_{13}^2) + T_{22}(b_{13}b_{14} - b_{11}b_{15}) + T_{33}(b_{13}b_{15} - b_{14}b_{12})}{b_{11}b_{12}b_{16} - b_{14}^2 b_{12} - b_{11}b_{15}^2 - b_{13}^2 b_{16} + 2b_{14}b_{13}b_{16}}$$

$$\hat{\beta}_{h,20} = \frac{T_{11}(b_{13}b_{14} - b_{11}b_{15}) + T_{22}(b_{11}b_{16} - b_{14}^2) + T_{33}(b_{14}b_{15} - b_{13}b_{16})}{b_{11}b_{12}b_{16} - b_{14}^2 b_{12} - b_{11}b_{15}^2 - b_{13}^2 b_{16} + 2b_{14}b_{13}b_{16}}$$

where $T_{11} = \sum_{h=1}^{H} w_h \bar{x}_h \bar{y}_h$, $T_{22} = \sum_{h=1}^{H} w_h s_{hx}^2 \bar{y}_h$, $T_{33} = \sum_{h=1}^{H} w_h \bar{y}_h$, $b_{11} = \sum_{h=1}^{H} W_h$, $b_{12} = \sum_{h=1}^{H} W_h s_{hx}^4$, $b_{13} = \sum_{h=1}^{H} W_h s_{hx}^2$, $b_{14} = \sum_{h=1}^{H} W_h \bar{x}_h$, $b_{15} = \sum_{h=1}^{H} w_h \bar{x}_h s_{hx}^2$, $b_{16} = \sum_{h=1}^{H} W_h \bar{x}_h^2$ **[**See Koyuncu and Kadilar (2016) for detail]

3.2 Estimation of Variance for Koyuncu and Kadilar (2016) Estimator

Koyuncu and Kadilar (2016) did not derive variance expression for their estimator **[**See Koyuncu and Kadilar (2016) for detail]. Here, the estimator of variance for the Koyuncu and Kadilar (2016) calibration regression estimator is derived using large sample approximation (LASAP) method. Expressing (7) in the relative error terms gives

$$\bar{y}_{KK}^* = \sum_{h=1}^{H} W_h \left[\bar{Y}_h (1 + e_{hy}) - \hat{\beta}_{h,10} \bar{X}_h e_{hx} - \hat{\beta}_{h,20} S_{hx}^2 e_{hs} \right]$$

So that

$$[\bar{y}_{KK}^* - \bar{Y}] = \sum_{h=1}^{H} W_h \left[\bar{Y}_h e_{hy} - \hat{\beta}_{h,10} \bar{X}_h e_{hx} - \hat{\beta}_{h,20} S_{hx}^2 e_{hs} \right] \quad (8)$$

Squaring both sides of (8) gives

$$[\bar{y}_{KK}^{*}-\bar{Y}]^{2}=\sum_{h=1}^{H}W_h^2\left[\bar{Y}_h^2e_{hy}^2+\hat{\beta}_{h,10}^2\bar{X}_h^2e_{hx}^2+\hat{\beta}_{h,20}^2S_{hx}^4e_{hs}^2-2\bar{Y}_h\hat{\beta}_{h,10}\bar{X}_he_{hy}e_{hx}\right.$$
$$\left.-2\bar{Y}_h\hat{\beta}_{h,20}S_{hx}^2e_{hy}e_{hs}+2\hat{\beta}_{h,10}\hat{\beta}_{h,20}\bar{X}_hS_{hx}^2e_{hx}e_{hs}\right] \quad (9)$$

Taking expectation of both sides of (9) gives

$$V[\bar{y}_{KK}^{*}]=\sum_{h=1}^{H}W_h^2\,\gamma_h[\bar{Y}_h^2C_{hy}^2+\bar{Y}_h^2\hat{\beta}_{h,10}^2\bar{X}_h^2C_{hx}^2+\hat{\beta}_{h,20}^2S_{hx}^4C_{hs}^2-2\hat{\beta}_{h,10}\bar{Y}_h\bar{X}_h\rho_{hxy}C_{hx}C_{hy}$$
$$-2\hat{\beta}_{h,20}\bar{Y}_hS_{hx}^2\rho_{hsy}C_{hy}C_{hs}+2\hat{\beta}_{h,10}\hat{\beta}_{h,10}S_{hx}^2\bar{X}_h\rho_{hxs}C_{hx}C_{hs}] \quad (10)$$

3.3 Optimality conditions for Koyuncu and Kadilar (2016) Estimator

In this section, optimality conditions for optimum performance of Koyuncu and Kadilar (2016) calibration regression estimator is deduced. Thus, setting $\frac{\partial\hat{V}[\bar{y}_{kk}^{*}]}{\partial\hat{\beta}_{h,10}}=0$ and $\frac{\partial\hat{V}[\bar{y}_{kk}^{*}]}{\partial\hat{\beta}_{h,20}}=0$ respectively gives

$$\hat{\beta}_{h,10}=\frac{\bar{Y}_h\rho_{hyx}C_{hy}C_{hx}-\hat{\beta}_{h,20}S_{hx}^2\rho_{hxs}C_{hx}C_{hs}}{\bar{X}_hC_{hx}^2} \quad (11)$$

$$\hat{\beta}_{h,20}=\frac{\bar{Y}_h\rho_{hys}C_{hy}C_{hs}-\hat{\beta}_{h,10}\bar{X}_h\rho_{hxs}C_{hx}C_{hs}}{S_{hx}^2C_{hs}^2} \quad (12)$$

Substituting (12) in (11) or (11) in (12), the optimum values of $\hat{\beta}_{h,10,opt}$ and $\hat{\beta}_{h,20,opt}$ are obtained respectively as

$$\hat{\beta}_{h,10,opt}=\frac{\bar{Y}_hC_{hy}\left(\rho_{hyx}-\rho_{hys}\rho_{hxs}\right)}{\bar{X}_hC_{hx}(1-\rho_{hxs}^2)} \quad (13)$$

$$\hat{\beta}_{h,20,opt}=\frac{\bar{Y}_hC_{hy}\left(\rho_{hys}-\rho_{hyx}\rho_{hxs}\right)}{S_{hx}^2C_{hs}(1-\rho_{hxs}^2)} \quad (14)$$

substituting the value of $\hat{\beta}_{h,10,opt}$ in (13) and $\hat{\beta}_{h,20,opt}$ in (14) for $\hat{\beta}_{h,10}$ and $\hat{\beta}_{h,20}$ in (7), an asymptotically optimum estimator (*AOE*) $\bar{y}_{KK,opt}^{*}$ of population mean is obtained for Koyuncu and Kadilar (2016) calibration regression estimator as

$$\bar{y}_{KK,opt}^{*}=\sum_{h=1}^{H}W_h\bar{y}_h+\frac{\bar{Y}_hC_{hy}\left(\rho_{hyx}-\rho_{hys}\rho_{hxs}\right)}{\bar{X}_hC_{hx}(1-\rho_{hxs}^2)}\sum_{h=1}^{H}W_h(\bar{X}_h-\bar{x}_h)$$
$$+\frac{\bar{Y}_hC_{hy}\left(\rho_{hys}-\rho_{hyx}\rho_{hxs}\right)}{S_{hx}^2C_{hs}(1-\rho_{hxs}^2)}\sum_{h=1}^{H}W_h(S_{hx}^2-s_{hx}^2) \quad (15)$$

Similarly, substituting the value of $\hat{\beta}_{h,10,opt}$ in (13) and $\hat{\beta}_{h,20,opt}$ in (14) for $\hat{\beta}_{h,10,}$ and $\hat{\beta}_{h,20}$ in (10), variance of the asymptotically optimum estimator (*AOE*) $\bar{y}_{KK,opt}^{*}$ [or minimum variance of $\bar{y}_{KK}^{*}$] for Koyuncu and Kadilar (2016) calibration regression estimator is obtained as

$$V_{opt}[\bar{y}_{KK}^{*}]=\sum_{h=1}^{H}W_h^2\gamma_h\bar{Y}_h^2C_{hy}^2(1-\rho_{xs}^2)^{-2}\{(1-\rho_{xs}^2)^2+\left(\rho_{hxy}-\rho_{hxs}\rho_{hsy}\right)^2$$
$$+\left(\rho_{hsy}-\rho_{hxs}\rho_{hxy}\right)^2-(1-\rho_{xs}^2)\left[2\rho_{hxy}\left(\rho_{hxy}-\rho_{hxs}\rho_{hsy}\right)+2\rho_{hsy}\left(\rho_{hsy}-\rho_{hxs}\rho_{hxy}\right)\right]$$
$$+2\rho_{hxs}\left(\rho_{hxy}-\rho_{hxs}\rho_{hsy}\right)\left(\rho_{hsy}-\rho_{hxs}\rho_{hxy}\right)\} \quad (16)$$

4. Calibration Estimation by Logarithmic Calibration Weightings

Deville and Sarndal (1992), suggested calibration estimation technique for minimising a distance measure between initial weights and final weights with respect to calibration constraints. However, the calibration weights defined by minimizing a distance measure under some given constraints may be negative and or extremely large. To overcome this limitation, this paper uses inverse exponentiation technique in formulating the calibration constraints to get reasonable calibration weights that will optimize the efficiency of calibration estimators.

4.1 Modified Koyuncu and Kadilar (2016) Estimator

Following Koyuncu and Kadilar (2016), a new calibration regression estimator of population mean is proposed using inverse exponentiation method as

$$\bar{y}^{*}_{New,KK} = \sum_{h=1}^{H} W_h^{*} log\bar{y}_h \tag{17}$$

where W_h^{*} are new calibration weights minimising the chi-square distance measure

$$L(W_h^{*}, W_h) = \sum_{h=1}^{H} \frac{(W_h^{*} - W_h)^2}{W_h Q_h} \tag{18}$$

subject to the logarithmic calibration constraints defined by

$$\sum_{h=1}^{H} W_h^{*} log\bar{x}_h = \sum_{h=1}^{H} W_h \, log\bar{X}_h \tag{19}$$

$$\sum_{h=1}^{H} W_h^{*} \, logs_{hx}^2 = \sum_{h=1}^{H} W_h \, logS_{hx}^{\prime 2} \tag{20}$$

$$\sum_{h=1}^{H} W_h^{*} = \sum_{h=1}^{H} W_h \tag{21}$$

The Lagrange function is given by

$$L(W_h^{*}, W_h)^{*} = \sum_{h=1}^{H} \frac{(W_h^{*} - w_h)^2}{W_h Q_h} - 2\lambda_{10}^{*} \left(\sum_{h=1}^{H} W_h^{*} log\bar{x}_h - \sum_{h=1}^{H} W_h log\, \bar{X}_h \right)$$
$$-2\lambda_{20}^{*} \left(\sum_{h=1}^{H} W_h^{*} log\, s_{hx}^2 - \sum_{h=1}^{H} W_h \, logS_{hx}^2 \right) - 2\lambda_{30}^{*} \left(\sum_{h=1}^{H} W_h^{*} - \sum_{h=1}^{H} W_h\right) \tag{22}$$

Minimizing the chi-square distance measure (18) subject to the calibration constraints in [(19), (20), (21)] respectively, calibration weights (W_h^{*}) is obtained as

$$W_h^{*} = W_h + W_h Q_h (\lambda_{10}^{*} log\bar{x}_h + \lambda_{20}^{*} \, logs_{hx}^2 + \lambda_{30}^{*}) \tag{23}$$

Substituting (23) into [(19), (20), (21)] respectively, a system of equations is obtained:

$$\begin{bmatrix} m_{16} & m_{15} & m_{14} \\ m_{15} & m_{12} & m_{13} \\ m_{14} & m_{13} & m_{11} \end{bmatrix} \begin{bmatrix} \lambda_{10}^{*} \\ \lambda_{20}^{*} \\ \lambda_{30}^{*} \end{bmatrix} = \begin{bmatrix} \mu_{10} \\ \mu_{20} \\ \mu_{30} \end{bmatrix} \tag{24}$$

Solving for the $\lambda_{i0}^{*}s$ in (24) gives

$$\lambda_{10}^{*} = \frac{\mu_{10}(m_{11}m_{12} - m_{13}^2) + \mu_{20}(m_{14}m_{15} - m_{11}m_{16})}{(m_{11}m_{12}m_{16} - m_{12}m_{14}^2 - m_{11}m_{15}^2 - m_{13}^2 m_{16} + 2m_{13}m_{14}m_{15})}$$

$$\lambda_{20}^{*} = \frac{\mu_{20}(m_{11}m_{16} - m_{14}^2) - \mu_{10}(m_{11}m_{15} - m_{13}m_{14})}{(m_{11}m_{12}m_{16} - m_{12}m_{14}^2 - m_{11}m_{15}^2 - m_{13}^2 m_{16} + 2m_{13}m_{14}m_{15})}$$

$$\lambda_{30}^{*} = \frac{\mu_{10}(m_{13}m_{15} - m_{12}m_{14}) + \mu_{20}(m_{14}m_{15} - m_{13}m_{16})}{(m_{11}m_{12}m_{16} - m_{12}m_{14}^2 - m_{11}m_{15}^2 - m_{13}^2 m_{16} + 2m_{13}m_{14}m_{15})}$$

where $m_{11} = \sum_{h=1}^{H} W_h Q_h$ $\quad m_{12} = \sum_{h=1}^{H} W_h Q_h \, (logs_{hx}^2)^2$ $\quad m_{13} = \sum_{h=1}^{H} W_h Q_h logs_{hx}^2$

$m_{14} = \sum_{h=1}^{H} W_h Q_h log\bar{x}_h$, $\quad m_{15} = \sum_{h=1}^{H} W_h Q_h (log\bar{x}_h)(logs_h^2)$, $\quad m_{16} = \sum_{h=1}^{H} W_h Q_h \, (log\bar{x}_h)^2$

$\mu_{10} = \sum_{h=1}^{H} W_h(log\bar{X}_h - log\bar{x}_h),\ \mu_{20} = \sum_{h=1}^{H} W_h(logS_{hx}^2 - logs_{hx}^2),\ \ \mu_{30} = 0$

If the $\lambda_{i0}^{*}s$ are substituted in (23) and subsequently in (17) while setting $Q_h = 1$, the proposed modification to Koyuncu and Kadilar (2016) calibration regression estimator of population mean in stratified random sampling is obtained as

$$\bar{y}_{New,KK}^{*} = \sum_{h=1}^{H} W_h\ log\bar{y}_h + \hat{\beta}_{h,10}^{*} \sum_{h=1}^{H} W_h(log\bar{X}_h - log\bar{x}_h) + \hat{\beta}_{h,20}^{*} \sum_{h=1}^{H} W_h(logS_{hx}^2 - logs_{hx}^2) \tag{25}$$

here $\hat{\beta}_{h,10}^{*}$ and $\hat{\beta}_{h,20}^{*}$ are the coefficients of regression and are given by

$$\hat{\beta}_{h,10}^{*} = \frac{\mu_{40}(\tau_{11}\tau_{12} - \tau_{13}^2) - \mu_{50}(\tau_{11}\tau_{15} - \tau_{13}\tau_{14}) + \mu_{60}(\tau_{13}\tau_{15} - \tau_{12}\tau_{14})}{(\tau_{11}\tau_{12}\tau_{16} - \tau_{12}\tau_{14}^2 - \tau_{11}\tau_{15}^2 - \tau_{13}^2\tau_{16} + 2\tau_{13}\tau_{14}\tau_{15})}$$

$$\hat{\beta}_{h,20}^{*} = \frac{\mu_{40}(\tau_{13}\tau_{14} - \tau_{11}\tau_{15}) - \mu_{50}(\tau_{11}\tau_{16} - \tau_{14}^2) + \mu_{60}(\tau_{14}\tau_{15} - \tau_{13}\tau_{16})}{(\tau_{11}\tau_{12}\tau_{16} - \tau_{12}\tau_{14}^2 - \tau_{11}\tau_{15}^2 - \tau_{13}^2\tau_{16} + 2\tau_{13}\tau_{14}\tau_{15})}$$

where $\tau_{11} = \sum_{h=1}^{H} w_h,\ \ \tau_{12} = \sum_{h=1}^{H} w_h\ (logs_{hx}^2)^2, \tau_{13} = \sum_{h=1}^{H} w_h logs_{hx}^2$
$\tau_{14} = \sum_{h=1}^{H} w_h log\bar{x}_h,\ \ \ \tau_{15} = \sum_{h=1}^{H} w_h(log\bar{x}_h)(logs_h^2),\ \tau_{16} = \sum_{h=1}^{H} w_h\ (log\bar{x}_h)^2$
$\mu_{40} = \sum_{h=1}^{H} W_h(log\bar{x}_h)(log\bar{y}_h).\ \mu_{50} = \sum_{h=1}^{H} W_h(logs_h^2)(log\bar{y}_h), \mu_{60} = \sum_{h=1}^{H} W_h log\bar{y}_h$

4.2 Estimation of Variance for Modified Koyuncu and Kadilar (2016) Estimator

The estimator of variance of the modified Koyuncu and Kadilar (2016) Calibration Regression Estimator is derived using large sample approximation (LASAP) method.
Equation (25) can be expressed as:

$$\bar{y}_{New,KK}^{*} = \sum_{h=1}^{H} W_h\ log\bar{y}_h + \hat{\beta}_{h,10}^{*} \sum_{h=1}^{H} W_h \left(log\frac{\bar{X}_h}{\bar{x}_h}\right) + \hat{\beta}_{h,20}^{*} \sum_{h=1}^{H} W_h \left(log\frac{S_{hx}^2}{s_{hx}^2}\right) \tag{26}$$

So that, expressing (26) in the relative error terms gives:

$$\bar{y}_{New,KK}^{*} = \sum_{h=1}^{H} W_h log\left[\bar{Y}_h(1 + e_{hy})\right] + \hat{\beta}_{h,10}^{*} \sum_{h=1}^{H} W_h log(1 + e_{hx})^{-1} + \hat{\beta}_{h,20}^{*} \sum_{h=1}^{H} W_h log(1 + e_{hs})^{-1} \tag{27}$$

Expanding equation (27) gives:

$$\bar{y}_{New,KK}^{*} = \sum_{h=1}^{H} W_h log\bar{Y}_h + \sum_{h=1}^{H} W_h log\left(1 + e_{hy}\right) + \hat{\beta}_{h,10}^{*} \sum_{h=1}^{H} W_h log(1 + e_{hx})^{-1} + \hat{\beta}_{h,20}^{*} \sum_{h=1}^{H} W_h log(1 + e_{hs})^{-1} \tag{28}$$

So that

$$\left(\bar{y}_{New,KK}^{*} - \bar{Y}\right) = \sum_{h=1}^{H} W_h log\left(1 + e_{hy}\right) - \hat{\beta}_{h,10}^{*} \sum_{h=1}^{H} W_h log(1 + e_{hx}) - \hat{\beta}_{h,20}^{*} \sum_{h=1}^{H} W_h log(1 + e_{hs}) \tag{29}$$

where $\bar{Y} = \sum_{h=1}^{H} W_h log\bar{Y}_h$

Now, it is assumed that $|e_{hy}| < 1; |e_{hx}| < 1$ and $|e_{hs}| < 1$ so that expanding $\left(1 + e_{hy}\right)$, $(1 + e_{hx})$ and $(1 + e_{hs})$ as series in power of the $e's$ gives

$$\left(\bar{y}_{New,KK}^{*} - \bar{Y}\right) = \left[\sum_{h=1}^{H} W_h \left(e_{hy} - \frac{e_{hy}^2}{2!} + \frac{e_{hy}^3}{3!} - \cdots\right) - \hat{\beta}_{h,10}^{*} \sum_{h=1}^{H} W_h \left(e_{hx} - \frac{e_{hx}^2}{2!} + \frac{e_{hx}^3}{3!} - \cdots\right)\right.$$

$-\hat{\beta}^{*}_{h,20} \sum_{h=1}^{H} W_h \left(e_{hs} - \frac{e_{hs}^2}{2!} + \frac{e_{hs}^3}{3!} - \cdots \right) \Big]$ (30)

Squaring both sides of (30), multiplying out and retaining terms of the $e's$ to the first degree of approximation gives

$$\left[\bar{y}^{*}_{New,KK} - \bar{Y}\right]^2 = \sum_{h=1}^{H} W_h^2 \left[e_{hy}^2 + \hat{\beta}^{*2}_{h,10} e_{hx}^2 + \hat{\beta}^{*2}_{h,20} e_{hs}^2 - 2\hat{\beta}^{*}_{h,10} e_{hy} e_{hx} - 2\hat{\beta}^{*}_{h,20} e_{hy} e_{hs} + 2\hat{\beta}^{*}_{h,10}\hat{\beta}^{*}_{h,20} e_{hx} e_{hs}\right] \quad (31)$$

Taking expectation of both sides of (31), the variance of the proposed logarithmic calibration regression estimator of population mean in stratified random sampling is obtained as

$$\hat{V}\left[\bar{y}^{*}_{New,KK}\right] = \sum_{h=1}^{H} W_h^2 \gamma_h \left[C_{hy}^2 + \hat{\beta}^{*2}_{h,10} C_{hx}^2 + \hat{\beta}^{*2}_{h,20} C_{hs}^2 - 2\hat{\beta}^{*}_{h,10} \rho_{hyx} C_{hy} C_{hx} - 2\hat{\beta}^{*}_{h,20} \rho_{hys} C_{hy} C_{hs} + 2\hat{\beta}^{*}_{h,10}\hat{\beta}^{*}_{h,20} \rho_{hxs} C_{hx} C_{hs}]\right] \quad (32)$$

4.3 Optimality Conditions for Modified Koyuncu and Kadilar (2016) Estimator

In this section, optimality conditions for optimum performance of the modified Koyuncu and Kadilar (2016) calibration regression estimator is deduced. Thus, the $V\left[\bar{y}^{*}_{New,KK}\right]$ in (32) is minimized by setting $\frac{\partial V[\bar{y}^{*}_{new,KK}]}{\partial \hat{\beta}^{*}_{h,10}} = 0$ and $\frac{\partial V[\bar{y}^{*}_{new,KK}]}{\partial \hat{\beta}^{*}_{h,20}} = 0$ respectively so that:

$$\hat{\beta}^{*}_{h,10} = \frac{\rho_{hyx} C_{hy} - \hat{\beta}^{*}_{h,20} \rho_{hxs} C_{hs}}{C_{hx}} \quad (33)$$

$$\hat{\beta}^{*}_{h,20} = \frac{\rho_{hyx} C_{hy} - \hat{\beta}^{*}_{h,10} \rho_{hxs} C_{hx}}{C_{hs}} \quad (34)$$

Substituting (34) in (33) or (33) in (34), gives the optimum values of $\hat{\beta}^{*}_{h,10,opt}$ and $\hat{\beta}^{*}_{h,20,opt}$ respectively as:

$$\hat{\beta}^{*}_{h,10,opt} = \frac{C_{hy}\left(\rho_{xy} - \rho_{xs}\rho_{sy}\right)}{C_{hx}\left(1 - \rho_{hxs}^2\right)} \quad (35)$$

$$\hat{\beta}^{*}_{h,20,opt} = \frac{C_{hy}\left(\rho_{sy} - \rho_{xs}\rho_{xy}\right)}{C_{hs}\left(1 - \rho_{hxs}^2\right)} \quad (36)$$

Substituting the value of $\hat{\beta}^{*}_{h,10,opt}$ in (35) and $\hat{\beta}^{*}_{h,20,opt}$ in (36) for $\hat{\beta}^{*}_{h,10}$ and $\hat{\beta}^{*}_{h,20}$ in (25), an asymptotically optimum estimator (*AOE*) $\bar{y}^{*}_{New,opt}$ of population mean is obtained for the proposed modified Koyuncu and Kadilar (2016) calibration regression estimator as:

$$\bar{y}^{*}_{New,KK} = \sum_{h=1}^{H} W_h log\bar{y}_h + \frac{C_{hy}\left(\rho_{xy} - \rho_{xs}\rho_{sy}\right)}{C_{hx}\left(1 - \rho_{hxs}^2\right)} \sum_{h=1}^{H} W_h \left(log\bar{X}_h - log\bar{x}_h\right) + \frac{C_{hy}\left(\rho_{sy} - \rho_{xs}\rho_{xy}\right)}{C_{hs}\left(1 - \rho_{hxs}^2\right)} \sum_{h=1}^{H} W_h \left(logS_{hx}^2 - logs_{hx}^2\right) \quad (37)$$

Similarly, substituting the value of $\hat{\beta}^{*}_{h,10,opt}$ in (35) and $\hat{\beta}^{*}_{h,20,opt}$ in (36) for $\hat{\beta}^{*}_{h,10}$ and $\hat{\beta}^{*}_{h,20}$ in (32), variance of the asymptotically optimum estimator (AOE) $\bar{y}^{*}_{New,KK,opt}$ [or minimum variance of $\bar{y}^{*}_{New,KK}$] for the proposed modified Koyuncu and Kadilar (2016) calibration regression estimator is obtained as

$$V_{opt}\left[\bar{y}^{*}_{New,KK}\right] = \sum_{h=1}^{H} W_h^2 \gamma_h C_{hy}^2 \left(1 - \rho_{xs}^2\right)^{-2} \left\{\left(1 - \rho_{xs}^2\right)^2 + \left(\rho_{hxy} - \rho_{hxs}\rho_{hsy}\right)^2 + \left(\rho_{hsy} - \rho_{hxs}\rho_{hxy}\right)^2 - \left(1 - \rho_{xs}^2\right)\left[2\rho_{hxy}\left(\rho_{hxy} - \rho_{hxs}\rho_{hsy}\right) + 2\rho_{hsy}\left(\rho_{hsy} - \rho_{hxs}\rho_{hxy}\right)\right] + 2\rho_{hxs}\left(\rho_{hxy} - \rho_{hxs}\rho_{hsy}\right)\left(\rho_{hsy} - \rho_{hxs}\rho_{hxy}\right)\right\} \quad (38)$$

5. Empirical Study

The data set in Table 1 is used to test the performances of the proposed logarithmic calibration weightings method over existing simple calibration weightings method by Deville and Sarndal (1992). The variance and percent relative efficiency (PRE) are the two measuring criteria used in comparing the performance of each calibration weightings method.

5.1 Cochran (1977) Regression Estimator

The conventional regression estimator in stratified random sampling by Cochran (1977) is given by

$$\bar{y}_{Reg}^{*} = \sum_{h=1}^{H} W_h \bar{y}_h + \beta_h \sum_{h=1}^{H} W_h (\bar{X}_h - \bar{x}_h) \quad (39)$$

with variance estimator given by:

$$V_{opt}\left[\bar{y}_{Reg}^{*}\right] = \sum_{h=1}^{H} W_h^2 \gamma_h \bar{Y}_h^2 C_{hy}^2 \left(1 - \rho_{hxy}^2\right) \quad (40)$$

where $\bar{y}_{st} = \sum_{h=1}^{H} W_h \bar{y}_h$ and $\bar{x}_{st} = \sum_{h=1}^{H} W_h \bar{x}_h$ are the Horvitz-Thompson-type estimators, $\beta_h = \sum_{h=1}^{H} W_h \bar{x}_h \bar{y}_h / \sum_{h=1}^{H} W_h \bar{x}_h^2$ is the regression coefficient and W_h are design weights.

5.2 Percent Relative Efficiency

Let the percent relative efficiency (PRE) of an estimator (say $\bar{y}_i^*$) with respect to the conventional regression estimator in stratified random sampling ($\bar{y}_{Reg}^*$) by Cochran (1977) be defined by

$$PRE = \frac{V(\bar{y}_{Reg}^*)}{V(\bar{y}_i^*)} \times 100 \quad (41)$$

Table 1: Data Statistics

Parameter	Stratum I	Stratum II	Stratum III	Stratum IV
N_h	10	9	26	7
n_h	3	2	5	2
$\bar{X}_h$	11.90	10.38	12.120	11.98
$\bar{Y}_h$	15.72	14.84	13.46	16.32
C_{hy}	1.062	0.986	1.208	1.023
C_{hx}	1.234	1.306	1.032	0.926
ρ_{hxy}	0.940	0.900	0.840	0.890
ρ_{hsy}	0.840	0.880	0.920	0.780
ρ_{hxs}	0.860	0.820	0.760	0.840

Table 2: Variance and PRE of Estimators under the Two Calibration weightings Techniques

S/No.	Estimator	PREs
1.	$\bar{y}_{Reg}^*$	100
2.	$\bar{y}_{KK}^*$	126.0804
3.	$\bar{y}_{New,KK}^*$	191.3110

6. Discussion of results

The percent relative efficiency (PREs) in Table 2 shows that the proposed modified Koyuncu and Kadilar (2016) calibration regression estimator has 91 percent efficiency gains while Koyuncu and Kadilar (2016) calibration regression estimator has 26 percent efficiency gains. This means that the proposed modified Koyuncu and Kadilar (2016) calibration regression estimator is 65 percent more efficient than Koyuncu and Kadilar (2016) calibration regression estimator with respect to Cochran (1977) conventional regression estimator of population mean in stratified random sampling. Therefore, in using the proposed modified Koyuncu and Kadilar (2016) calibration regression estimator one has 65 percent efficiency gains over the Koyuncu and Kadilar (2016) calibration regression estimator and by extension new proposed logarithmic calibration technique. Also, the proposed modified Koyuncu and Kadilar (2016) calibration regression estimator is better than Cochran (1977) conventional regression estimator in terms of efficiency by 91 percent.

7. Conclusion

Koyuncu and Kadilar (2016) used Deville and Sarndal (1992) simple calibration technique to propose a calibration regression estimator of population mean. The present study advocates a modification to Koyuncu and Kadilar (2016) estimator using a new calibration technique (logarithmic calibration weightings). Following the discussion of results, it is concluded that the proposed logarithmic calibration technique gives a better result than the conventional calibration technique by Deville and Sarndal (1992). Therefore, the new calibration technique to statistical estimation theory should be recommended to survey researchers as it gives more precise and consistent estimates of the population parameter(s) of interest.

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The publication does not have any associated conflict of interest.

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